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PRIMA Astrometric Observations

Polarization effects

Technical Report

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1 Introduction

The PRIMA, VLT interferometer optics employs various reflection optics. They produce different phase and amplitude of light from the astronomical object between two polarization components (ie. S- and P-polarizations). The instrumental polarization may reduce the visivility and the accuracy of astrometry and, therefore, has to be considered. In this analysis, properties of the instrumental polarization of PRIMA optics including VLT telescope optics, and effect of polarization in measurement of the phase delay and in astrometry will be studied.

We first have to establish a method of analysis. The operation principle of the fringe sensor unit (FSU) will be explained in the chapter 2. Then we will make a model of PRIMA, VLTI optics, which will be described in the chapter 3.1. The details of polarization properties of all optics will be studied. Since VLT is an Az-El telescope, the time evolution of polarization effect may be important. In addition, the difference of polarization properties between the reference star and the object will be analyzed.

2 Polarization effects on fringe detection

The FSU is placed after the differencial delay line (DDL). A schematic of the optics is shown in Fig.??. It is based on Alenia's concept (see VLT-TRE-ALS-15740-0004). The beam 2 is incident to an achromatic $\frac{\lambda}{4}$ wavelength retarder (K prism) and the P-polarization will have a phase delay of $\frac{\pi}{2}$ to the S-polarization. The beam 1 is incident to a compensator to keep the same phase delay as the beam 2. The two beams are combined by a beam combiner (BC) and the combined beams will be separated into S- and P-polarizations by the polarizing beam splitter (PBS). The 4 signals identified as A, B, C and D are incident to a dispersor. The dispersor divides into 3 sub-bands of K1, K2 and K3 with different wavelength band width. The K1 and K3 of longer and shorter sub-bands with a narrow band width, are used for slow fringe scan ($f < 100$ Hz) for accurate fringe detection and the K2 is used for fast fringe scan ($f \sim 8$ kHz). The color properties will be described in the further version. Here we conside only one wavelength.

2.1 The original ABCD algorithm

Let the S- and P-polarizations of beam 1 and 2 be S_i and P_i ($i = 1, 2$), respectively. The complex amplitude of the A, B, C, and D beams are

$$\begin{aligned} E_A &= -\beta(P_1 - P_2), && (1)\\ E_B &= \beta(S_1 + S_2), && (2)\\ E_C &= \beta(P_1 + P_2), && (3)\\ E_D &= -\beta(S_1 - S_2) && (4)\\ & && (5)\end{aligned}$$

Where β is the reflection (or transmission) coefficient of the BC. To simplify, the reflection coefficient is assumed to be the same as the transmission one (ie. $-\tau = \rho = \beta$, see also Fig.??). Let the optical path length of the beam 1 and 2 be $L_{opd,i}$ ($i = 1, 2$). Here, assume the identical polarization properties between the beam 1 and 2. The polarization statuses are

$$S_1 = expi\left(kL_{opl,1}\right), \quad (6)$$

$$
\begin{aligned}
S_2 &= expi\left(kL_{opl,2}\right), &(7)\\
P_1 &= expi\left(kL_{opl,1}\right), &(8)\\
P_2 &= expi\left(kL_{opl,2}+\frac{\pi}{2}\right). &(9)\\
& &(10)
\end{aligned}
$$

Where L_{opd} is now the optical path difference between the beam 1 and 2 and $L_{opd} = L_{opl,1} - L_{opl,2}$. The A, B, C and D signals are expressed as

$$
\begin{aligned}
I_A &= 2|\beta|^2\{1+\sin\left(kL_{opd}\right)\}, &(11)\\
I_B &= 2|\beta|^2\{1+\cos\left(kL_{opd}\right)\}, &(12)\\
I_C &= 2|\beta|^2\{1-\sin\left(kL_{opd}\right)\}, &(13)\\
I_D &= 2|\beta|^2\{1-\cos\left(kL_{opd}\right)\}. &(14)\\
& &(15)
\end{aligned}
$$

The phase delay ϕ and the visibility V are derived as below:

$$
\begin{aligned}
\phi &= kL_{opd} = \arctan\left(\frac{I_A - I_C}{I_B - I_D}\right), &(16)\\
V &= \frac{1}{2}\left(I_A + I_B + I_C + I_D\right). &(17)
\end{aligned}
$$

The advantage of this method is that the phase delay ϕ and the visibility V can be obtained using simple equations.

2.2 A modified ABCD algorithm

However, the above original algorithm is assumed to have the identical polarization status between the two beams. Here assume the P-polarization has a different property from the S-polarization but those of the beam 1 are still identical to those of the beam 2. The equations (6)–(9) are now

$$
\begin{aligned}
S_1 &= S_1 expi\left(kL_{opl,1}\right), &(18)\\
S_2 &= S_1 expi\left(kL_{opl,2}\right), &(19)\\
P_1 &= P_1 expi\left(kL_{opl,1}\right), &(20)\\
P_2 &= P_1 expi\left(kL_{opl,2}+\frac{\pi}{2}\right). &(21)
\end{aligned}
$$

The equations (11)–(14) become

$$
\begin{aligned}
I_A &= 2|\beta P_1|^2\{1+\sin\left(kL_{opd}\right)\}, &(22)\\
I_B &= 2|\beta S_1|^2\{1+\cos\left(kL_{opd}\right)\}, &(23)\\
I_C &= 2|\beta P_1|^2\{1-\sin\left(kL_{opd}\right)\}, &(24)\\
I_D &= 2|\beta S_1|^2\{1-\cos\left(kL_{opd}\right)\}. &(25)
\end{aligned}
$$

The visibility can be expressed the same equation as (17. But the phase dealy ϕ should be modified as below

$$
\phi = kL_{opd} = \arctan\left(\frac{I_A - I_C}{I_A + I_C}\frac{I_B + I_D}{I_B - I_D}\right). \qquad (26)
$$

The visibility V is the same as eq. (17).

The advantage of the ABCD algorithm is that the phase delay measurement is not affected by the polarization status of the reference beam.

Now we consider a more general case, that the beam 2 has a different complex amplitude and a different phase in both S- and P-polarizations from those of the beam 1. The complex amplitude of beam 1 and 2 are now

$$\begin{aligned} S_1 &= S_1 expi\left(kL_{opl,1}\right), & (27)\\ P_1 &= P_1 expi\left(kL_{opl,1}\right). & (28)\\ S_2 &= S_0 S_1 expi\left(kL_{opl,2} + \phi_S\right), & (29)\\ P_2 &= P_0 P_1 expi\left(kL_{opl,2} + \phi_P + \frac{\pi}{2}\right). & (30) \end{aligned}$$

The A, B, C and D signals become

$$\begin{aligned} I_A &= |\beta P_1|^2\{1 + P_0^2 + 2P_0 \sin\left(kL_{opd} + \phi_P\right)\}, & (31)\\ I_B &= |\beta S_1|^2\{1 + S_0^2 + 2S_0 \cos\left(kL_{opd} + \phi_S\right)\}, & (32)\\ I_C &= |\beta P_1|^2\{1 + P_0^2 - 2P_0 \sin\left(kL_{opd} + \phi_P\right)\}, & (33)\\ I_D &= |\beta S_1|^2\{1 + S_0^2 - 2S_0 \cos\left(kL_{opd} + \phi_S\right)\}. & (34) \end{aligned}$$

Therefore the original ABCD algorithm described in the previous chapter does not provide a correct phase delay. However, there is still a method to measure the phase delay. This method requires another sampling with a phase step of $\frac{\pi}{2}$ in each beam to compensate the polarization intensity. In the case of P-polarization, we will get

$$\begin{aligned} I_{A0} &= |\beta P_1|^2\{1 + P_0^2 + 2P_0 \sin\left(kL_{opd} + \phi_P\right)\}, & (35)\\ I_{A1} &= |\beta P_1|^2\{1 + P_0^2 + 2P_0 \cos\left(kL_{opd} + \phi_P\right)\}, & (36)\\ I_{C0} &= |\beta P_1|^2\{1 + P_0^2 - 2P_0 \sin\left(kL_{opd} + \phi_P\right)\}, & (37)\\ I_{C1} &= |\beta P_1|^2\{1 + P_0^2 - 2P_0 \cos\left(kL_{opd} + \phi_P\right)\}. & (38) \end{aligned}$$

The phase delay Φ_P, therefore, can be obtained as

$$\Phi_P = kL_{opd} + \phi_P = \arctan\left(\frac{I_{A0} - I_{C0}}{I_{A1} + I_{C1}}\right). \tag{39}$$

As the same mannter, we can get the value for S-polarization

$$\Phi_S = kL_{opd} + \phi_S = \arctan\left(\frac{I_{B0} - I_{D0}}{I_{B1} + I_{D1}}\right). \tag{40}$$

The visibility V is still same as the eq. (17).

As shown in the eq.(39) and (40), we can get the phase delay. But we can not derive the phase delay by the instrumental polarization or we can not principally correct the phase delay by the instrumental polarization. We may take the phase delay of $kL_{opd} + \frac{1}{2}(\phi_S + \phi_P)$ and this will give the error between the S- and P-polarizations of $|\phi_S - \phi_P|$.

2.3 Impact on astrometry

Above discussions are for only the reference beam. Here, we consider effects of instrumental polarization on the visibility. For a convenience, we use electric field vector instead of the Stokes parameters to analyze the visibility properties. Science instrument may get the visibility like below

$$V =< |E_{S,1} + E_{S,2} + E_{P,1} + E_{P,2}|^2 > \tag{41}$$

Where $< * >$ is the ensemble average. Expand this formula, we get

$$\begin{aligned} V \;=\; & < |E_{S,1}|^2 > + < |E_{S,2}|^2 > + < |E_{P,1}|^2 > + < |E_{P,2}|^2 > \\ & + < E_{S,1}E^*_{S,2} > + < E^*_{S,1}E_{S,2} > \\ & + < E_{S,1}E^*_{P,1} > + < E^*_{S,1}E_{P,1} > \\ & + < E_{S,1}E^*_{P,2} > + < E^*_{S,1}E_{P,2} > \\ & + < E_{S,2}E^*_{P,1} > + < E^*_{S,2}E_{P,1} > \\ & + < E_{S,2}E^*_{P,2} > + < E^*_{S,2}E_{P,2} > \\ & + < E_{P,1}E^*_{P,2} > + < E^*_{P,1}E_{P,2} > \end{aligned} \tag{42}$$

The first four terms are the intensity of each beam of each polarization and remains are cross correlations. Now we itend to assume that the object beam has an elliptic polarization. The S-polarization of the beam 1 is assumed to be the reference light. The phase difference of the P-polarization of the beam 1, that of S- and P-polarizations of the beam 2 are ϕ'_P, ϕ'_S and ϕ'_{SP}, respectively. The amplitude of the beam 1 and 2 are, S_1, P_1, S_2 and P_2, respectively. The optical path lengths of each beam are $L'_{opl,1}$ and $L'_{opl,2}$.

$$E_{S,1} = S_1 expi(kL'_{opl,1}), \tag{43}$$
$$E_{S,2} = S_2 expi(kL'_{opl,2} + \phi'_S), \tag{44}$$
$$E_{P,1} = P_1 expi(kL'_{opl,1} + \phi'_{SP}), \tag{45}$$
$$E_{P,2} = P_2 expi(kL'_{opl,2} + \phi'_{SP} + \phi'_P). \tag{46}$$

The cross correlations are

$$< E_{S,1}E^*_{S,2} > + < E^*_{S,1}E_{S,2} > = 2 < S_1S_2 \cos(kL'_{opd} - \phi'_S) >, \tag{47}$$
$$< E_{S,1}E^*_{P,1} > + < E^*_{S,1}E_{P,1} > = 2 < S_1P_1 \cos(\phi'_{SP}) >, \tag{48}$$
$$< E_{S,1}E^*_{P,2} > + < E^*_{S,1}E_{P,2} > = 2 < S_1P_2 \cos(kL'_{opd} - \phi'_{SP} - \phi'_P) >, \tag{49}$$
$$< E_{S,2}E^*_{P,1} > + < E^*_{S,2}E_{P,1} > = 2 < S_2P_1 \cos(kL'_{opd} + \phi'_{SP} - \phi'_S) >, \tag{50}$$
$$< E_{S,2}E^*_{P,2} > + < E^*_{S,2}E_{P,2} > = 2 < S_2P_2 \cos(\phi'_{SP} + \phi'_P - \phi'_S) >, \tag{51}$$
$$< E_{P,1}E^*_{P,2} > + < E^*_{P,1}E_{P,2} > = 2 < P_1P_2 \cos(kL'_{opd} - \phi'_P) > . \tag{52}$$

Where L'_{opd} is the optical path difference of the object between the beam 1 and 2, and is expressed $L'_{opd} = L'_{opl,1} - L'_{opl,2}$. If the object has unpolarized light, the term with ϕ'_{SP} will be 0. Therefore, only eq.(47) and (52) will remain. Since the FSU corrects the phase delay of $kL_{opd} + \frac{1}{2}(\phi_S + \phi_P)$, the residual phase delay becomes $k(L_{opd} - L'_{opd}) + \{(\phi_S - \phi_P) - (\phi'_S - \phi'_P)\}$. The first term is the thing which gives the astrometry and the second term is the error by instrumental polarization of PRIMA optics. Therefore, the difference polarization between the the reference and the object and between the beam 1 and 2 produce the error in astrometry. The correlation between S- and P-polarizations may change the visibility if elliptic polarization component exists and that affect the astrometry. To avoid this, science instrument should take correlations with the same polarization components (eg. S-polarization between beam 1 and 2) as the FSU does.

3 Polarization properties of PRIMA, VLTI optics

3.1 The basic concept of the polarization model analysis

Any optical componet can change the polarization status of incident light and can be regarded as a "polarization optics" like a wave retarder or a polarizer. Let the polarization status of the incident light $\mathbf{S}_o$=(I,Q,U,V), and that of each optical element with a Mueller matrix $\mathbf{M}$. The Stokes parameters of the output light is

$$\mathbf{S}_o = \mathbf{M}\ \mathbf{S}_i. \tag{53}$$

The final polarization status $\mathbf{S}_f$ can be expressed using a product of the Mueller matrices of all optics $\mathbf{M}_i$ (i=1,2,...,N),

$$\mathbf{S}_f = \mathbf{M}_N\ \mathbf{M}_{N-1}\textbf{.....}\mathbf{M}_1\ \mathbf{S}_*. \tag{54}$$

To analyze the instrumental polarization of PRIMA, all of the optical components shall be considered and the Mueller matrices of all optics shall be derived. It is worth that optics will be devided into some groups. For examples, only telescope optics with M1 to M9 depends on the azimuth and the elevation of the telescope pointing and these optics will be expressed with a Mueller matrix like

$$\mathbf{M}_{tel}(Az(h), El(h), r, \theta, \lambda, St). \tag{55}$$

Where (r, θ), λ and St are the offset of star from the telescope pointing, the wavelength and identifer of telescope station, respectively. In case of the object, the separation may be (0, 0). The star separator (StS) optics and the base line optics may be

$$\mathbf{M}_{StS}(r, \theta, \lambda),$$
$$\mathbf{M}_{BL}(\lambda, St),$$

respectively. Since the main delay line and the differential delay line, the reference beam takes almost identical optical path to the object does, their optics can be ignored at the first instance. Therefore, the Stokes parameters of the reference beam and the object beam are

$$\mathbf{S}_{f,ref} = \mathbf{M}_{BL}(\lambda, St)\mathbf{M}_{StS}(r, \theta, \lambda)\mathbf{M}_{tel}(Az(h), El(h), r, \theta, \lambda, St)\mathbf{S}_{*,ref}, \tag{56}$$
$$\mathbf{S}_{f,obj} = \mathbf{M}_{BL}(\lambda, St)\mathbf{M}_{StS}(0, 0, \lambda)\mathbf{M}_{tel}(Az(h), El(h), 0, 0, \lambda, St)\mathbf{S}_{*,obj}. \tag{57}$$

From the results of $\mathbf{S}_{f,ref}$ and $\mathbf{S}_{f,obj}$, the phase differences between the reference and the object and those between different telescope stations. In the future version, we will present the results of properties of each Mueller matrices.

3.2 VLT telescope optics

3.3 StS optics

3.4 Base line optics

3.5 Beam combiner

4 Conclusion

5 Acknowledgement

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