PRIMA Astrometric Operations: Dispersion Correction

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Abstract

The astrometric mode of PRIMA/VLTI is designed to measure the difference of two interferometric optical path differences (OPD's) —of a primary star (PS) and a secondary star (SS)— and deduce the astrometric angle by trigonometry off a baseline that is to be calibrated. The ground base light ducts lead through "open air." We consider aspects of air refraction on

- determining the differential OPD from knowledge of the auxiliary IR metrology system and temperature/humidity sensors
- second order differential OPD induced by gas density gradients
- the accumulated OPD at arrival at the telescopes

in static (turbulence-free) air density models. Reference: UL-TRE-AOS-15753-0010

1 Refractivity and Differential OPD with the DDL

The differential OPD is the product of the differential OPD by the metrology multiplied by the wavelength dispersion factor $1+\Delta n$. Trivial error analysis: the relative error in the OPD is the absolute error in Δn.

1.1 Air Dispersion

Extrapolation of the PRIMA metrology laser OPD's at 1.3μm to the middle of the K band at 2.2μm has to deal with an error in the "molecular" dispersion of $\Delta n \approx 4.33 \cdot 10^{-7} \pm 8 \cdot 10^{-9}$ caused by the difference between fully theoretical and experimental data. This is already better than the estimated

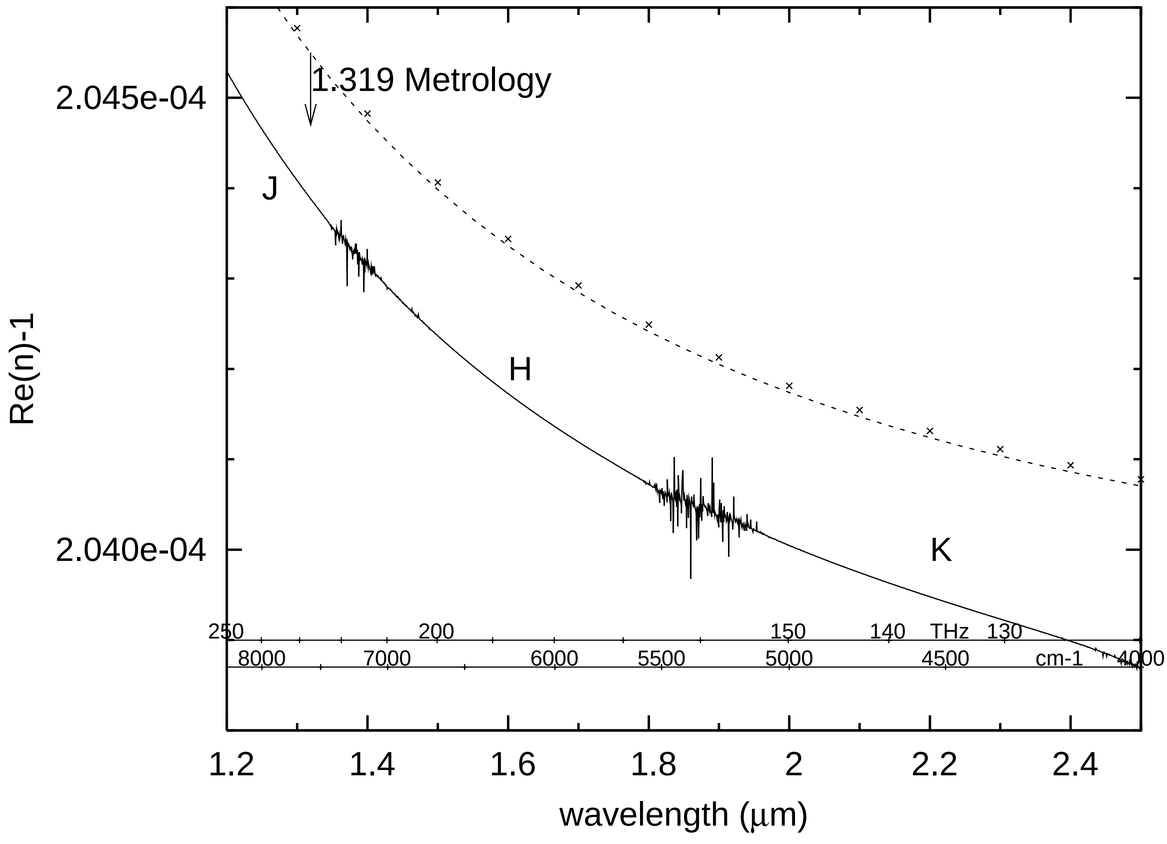

Figure 1: Purely theoretical (solid line) and best fits to experimental humid air refractivities (dashed) differ by about $1.5 \cdot 10^{-7}$ in the H, J, K band.

accuracy of the experimental fit alone ($\approx$ 1–$5 \cdot 10^{-8}$). Accurate long-wavelength effective dispersion needs to take into account (i) molecular cross-polarization (ii) non-ideal gas behavior in the equation of state (iii) temperature effects on the partition sums.

1.2 Sensor Readings

The refractivity Δn is internally build from the product of “molecular” polarizabilities and the gas densities (measured as pressures and temperatures), so the relative error becomes the sum of these. Typically 90% of the dispersion from the metrology wavelength to the science wavelength are caused by the N_2, O_2, CO_2 “dry” components, the rest caused by H_2O. (Note the difference to the mid-IR!) Example: At a differential OPD of 7 cm a change in humidity from 0 to 50% would change the doubly differential OPD (from 1.3 to 2.2 μ) by no more than 3 nm.

Summary: presumed the differential OPD in the metrology light is “exact,” extrapolation into the K band can reach nm accuracy without further ado. In contrary, determination of the 200+ m baseline to nm accuracy with such a wavelength extrapolation would need gas molecule density inputs to $2 \cdot 10^{-7}$ accuracy and can *not* be done.

1.3 Density Gradients and Beam Curvature

“Parabolic” beam propagation is caused by air density gradients perpendicular to the beam direction. Benchmark calculations with temperature gradients of 2°C/100 m yield transverse beam displacements of up some ten μm, in accordance with UT light duct experiments.

Lateral beam displacement will be dominated by the vertical temperature gradient in the VLTI DL tunnel. It is quasi-static and could be caught by alignment (IRIS).

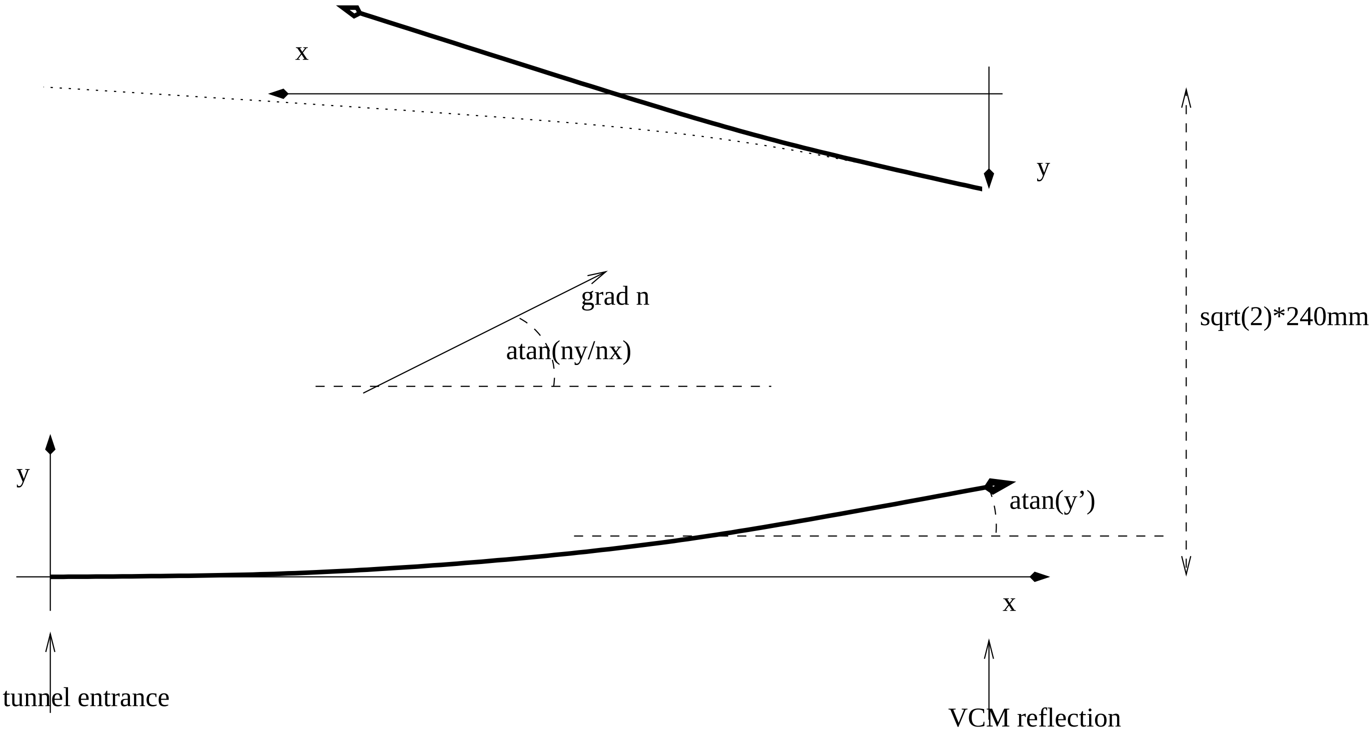

Figure 2: Sketch of a "free flight" leg of a beam that enters at the lower left, bends towards a gradient in n, and is symmetrically reflected at the VCM at the right.

Prediction of this displacement effect will be only marginally possible, as the sensor placements seem to be fully dedicated to temperature/humidity measurement *along* the beam axis, leaving no data on the perpendicular component of the gradients.

The dual-beam arrangement in a *horizontal* pair means that differential effects are only introduced by the *horizontal* component of the gradient, assumed to be smaller. Fortunately, the additional OPD introduced by the prolonged beam path equivalent to the curvature remains typically in the sub-nm region (Fermat principle), and is automatically incorporated by the metrology "calibration" which—by design—takes almost exactly the same detour.

Summary: beam curvature is in essence an alignment problem and it needs to be seen how it scales to detector pixels (which might divide the intensity differently). The additional OPD accumulated along the curved geometry is not important.

2 Spherical Model of the Earth Atmosphere

Refractivity calculations with Snell's law are "well known" from 1-dish, 1-target configurations, neither special to ground-based interferometry nor to astrometry. The challenge is posed by the length scale of the earth radius over the PRIMA OPD accuracy, 10 nm $\div 6340$ km $\approx 2 \cdot 10^{-15}$ which cannot be done in naïve "double precision" arithmetics.

The geometrical effects originate from the different four hit angles of the rays at the high atmopshere.

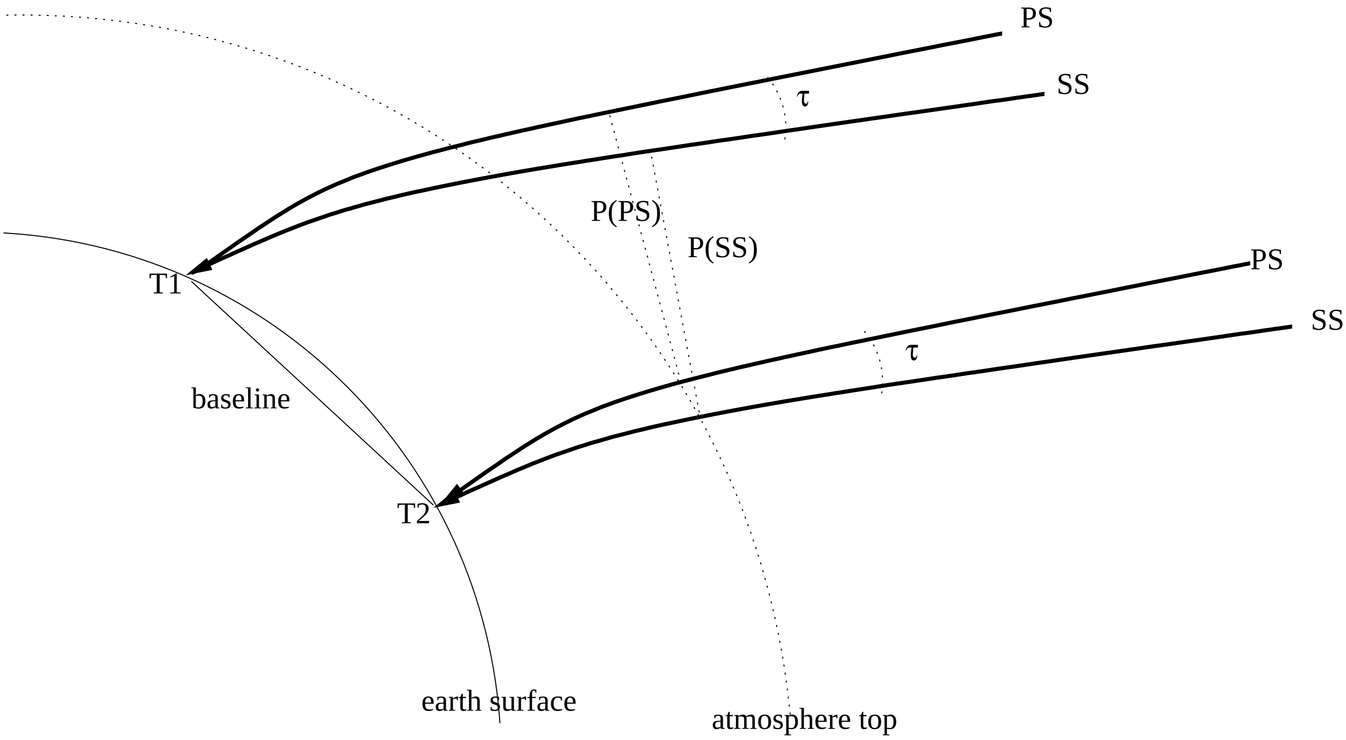

Figure 3: Ground based astrometry deals with one geometric baseline vector b, four beams, one astrometric angle τ, two projected baseline vectors P, two OPD's D, and two effective baselines $b^{*2} = P^2 + D^2$.

The atmosphere acts like a lens which enlarges the baseline––different from models of the earth as a planar disk.

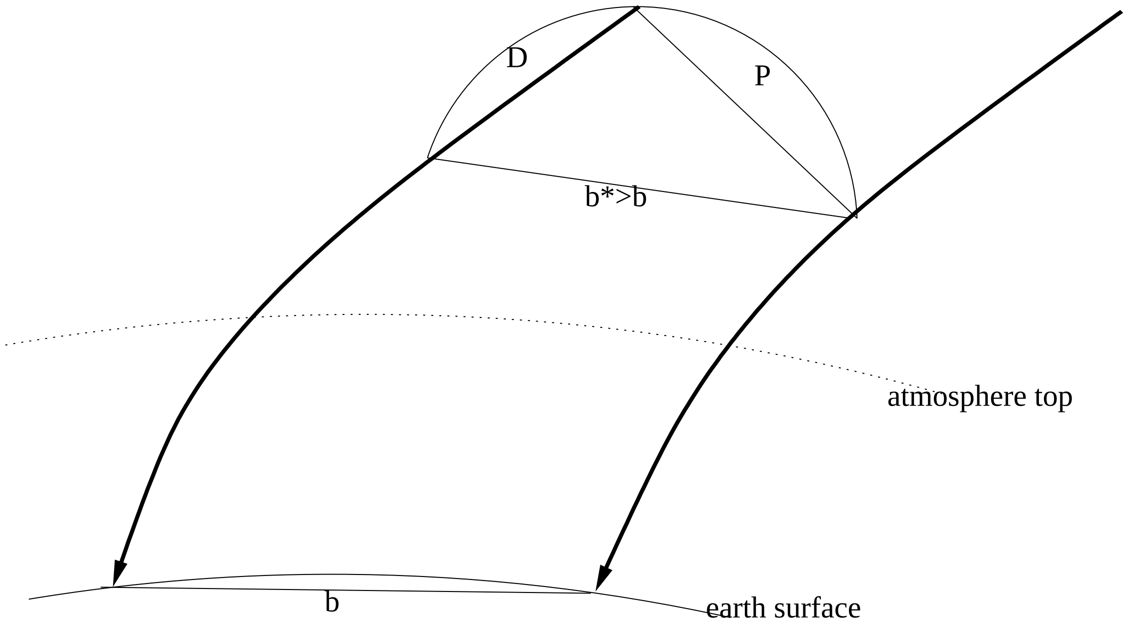

Figure 4: Above the atmosphere, the effective baseline b^* is longer than b and tilted.

One aspect of the art of Baseline Calibration is to de-correlate the measured OPD D and variable atmospheric parameters (static pressure, humidity) to consistent "rock-solid" geometric baseline vectors b on the ground: This is *not* PRIMA related but generic interferometry.

It remains to be seen how much of the incomplete knowledge of the atmospheric layer density parameters makes it into the astrometric (4-beam differential) error budget.

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