# Saas Fee Winterschool 2013

## Lectures on high-performance computing and numerical modeling

# Hands-On Set 2

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## 2 Simulating a fluid instability and a blast wave

In this exercise, we use the Eulerian mesh-code ATHENA by Jim Stone (Princeton) and collaborators to carry out a few hydrodynamical test calculations. This is instructive for getting a better intuitive understanding of typical numerical results for basic flow phenomena, as well as for learning the first steps in using a modern code like ATHENA in practice.

### 2.1 Kelvin-Helmholtz instability

For definiteness, we consider a 2D domain of extension $[0, L] \times [0, L]$ with periodic boundaries on the left and right sides, and reflecting boundaries on the top and bottom. Let the upper half of the box be filled with gas ($\gamma = 5/3$) at density $\rho_1 = 1.0$, pressure $P_1 = 1.0$, and velocity $u_1 = 0.3$ in the $x$-direction (i.e. to the right). The lower half has density $\rho_2 = 2.0$, the same pressure $P_2 = P_1$, and moves with velocity $u_2 = -0.3$ to the left. In order to avoid a perfectly sharp boundary in the initial conditions between these two phases (which is prone to triggering secondary instabilities at grid corners) we introduce a small transition region that smoothly connects them:

$$\rho(x, y) = \rho_1 + \frac{\rho_2 - \rho_1}{1 + \exp\left[(y - 0.5)/\sigma\right]}, \tag{1}$$

and similarly

$$u(x, y) = u_1 + \frac{u_2 - u_1}{1 + \exp\left[(y - 0.5)/\sigma\right]}, \tag{2}$$

with $\sigma = 0.01$. In these unperturbed initial conditions, we now impose a seed perturbation in the velocity in the $y$-direction of the form

$$v(x, y) = A\,\cos(kx)\,\exp(-k|y - 0.5|), \tag{3}$$

with wavenumber $k = 2 \times (2\pi/L)$ and perturbation amplitude $A = 0.05$. For simplicity, we refrain from imparting a perturbation in $\rho$ and $u$ as well that would be consistent with the velocity perturbation in the $y$-direction at the linear theory level, kind of hoping that we get away with this on the grounds that the perturbation should anyway grow (which is of course expected if the shear flow is indeed unstable against arbitrarily small transverse perturbations).

(a) We want to simulate this problem with the ATHENA mesh code developed by the group of Jim Stone (Princeton University). You can download the current public version 4.1 of this code from
`http://www.astro.princeton.edu/~jstone/downloads/athena/athena4.1.tar.gz`
Then, unpack the code with the command: `tar -zxvf athena4.1.tar.gz`

We want to run the problem with ATHENA until time $t = 3.0$ and create an image of the resulting density field at the end. To this end, you need to implement appropriate initial conditions in a problem generator, and then compile the code appropriately. For the problem generator, you can download the file
`http://www.mpa-garching.mpg.de/~volker/kelvin.c` and place it into the subdirectory `src/prob` of ATHENA. Edit the file to finish off the implementation of the initial conditions (there are primarily three lines to fill out – see the comments in the file). Then configure the code as

```
./configure --with-problem=kelvin --with-gas=hydro --with-eos=adiabatic
--with-flux=roe
```

followed by the compilation step with `make all`.

Next, you also need to setup a parameterfile that is passed to ATHENA at run time. This sets things such as the resolution you want to use, the number and times of outputs you want to have, the desired simulation time span, etc. You can try the parameterfile supplied at
`http://www.mpa-garching.mpg.de/~volker/kelvin.param`
to get started, which you may modify as you see fit (for example to change the resolution or the parameters of the initial conditions generator). Then run the code with
`./bin/Athena -i <parameterfile>`,
where you replace the name of the parameterfile with your file `kelvin.param`.

At the final time, you should get a ".ppm" image file displaying a slice of the density field, e.g. `kh.0060.d.ppm`. Load this into an image view program of your choice. Carry out a series of simulations with different resolutions, equal to 64 × 64, 128 × 128 and 256 × 256 mesh cells, and produce images for them at the same nominal pixel resolution, for example 512 × 512 pixels, by enlarging the images accordingly. Compare them visually and discuss.

You could also make a little movie of the evolving KH instability. The simplest would be to use the `ffmpeg` program, if available. A suitable command for this could be, e.g.: `ffmpeg -qscale 1 -i kh.%04d.d.ppm movie.mp4`

(b) We now want to check whether we can verify the linear growth rate of the perturbation. The growth rate of the amplitude of the $y$-displacement of a single mode $k$ is given by $\propto \exp(\omega t)$, with

$$\omega = k|u_1 - u_2|\sqrt{\rho_1 \rho_2}/(\rho_1 + \rho_2) \tag{4}$$

Make a plot of the log of the mean kinetic energy in the $y$-direction as a function of time (you can get this quantity from the history output in `kh.hst`, column 1 has the time, column 9 the kinetic energy in the $y$-direction), and overplot a growth line reflecting the above timescale. Why is the growth initially slower than expected based on equation (4)? What could be the reason that there is a large slow-down at late times?

(c) Now repeat the Kelvin-Helmholtz simulation of (a) but add a constant velocity of $\Delta u = 5.0$ everywhere to the initial conditions. Compare the results you obtain at $t = 3$ with the ones you got in (a) and discuss the origin of the differences, if any.

## 2.2 Sedov-Taylor blast wave

We now consider a two-dimensional strong explosion, again simulated with ATHENA in 2D. We consider a periodic domain of extension $[L, L]$, subdivided into $N \times N$ cells. The background gas is initially at rest, has homogeneous density $\rho = 1.0$, and a low pressure of $P = 0.00001$. We inject a large amount of energy $E = 1$ into a single cell at the center of the domain, and evolve the explosion until time $t = 0.1$.

(a) Set-up a problem generator for this test, which you can most easily do by copying your file `src/prob/kelvin.c` to `src/prob/sedov.c`, and then by making some small changes in `sedov.c`. Be careful about the distinction between converved and primitive variables (e.g. between energy density of a cell and *energy of a cell*), and the notation the ATHENA code uses for these quantitites. You then need to configure ATHENA again, by doing something like

```
./configure --with-problem=sedov --with-gas=hydro --with-eos=adiabatic
--with-flux=roe
```

followed by the compilation step with `make clean; make all`. You will also need a parameterfile. Here again, it will be simplest if you copy `kelvin.param` to `sedov.param` and make some appropriate changes there. In particular you might want to adjust the final time, and the frequency with which log-file output is created.

(b) Produce a plot in which you show the simulated density profile at the final time as a function of distance from the explosion site. Each cell's density value should be shown as a single point or symbol. In order to do this, you need to access the "sedov.XXXX.tab" file at the final time, which contains a table of all cell coordinates and their corresponding fluid quantities. Produce two such plots, one for a resolution of $64^2$ cells, and one for $256^2$ cells.

(c) Make a rough estimate where you expect the shock at this time based on a dimensional analysis (note that the problem is treated in 2D here), and mark the scale in the figures.