

Main Storyline: How gas gets into galaxies and what it does there.

Nick Gnedin



PART I

Physics of the IGM

1. Starting Point

- Universe expands.

$$\frac{\dot{a}}{a} = \sqrt{\frac{\Omega_R}{a^4} + \frac{\Omega_m}{a^3} + \frac{\Omega_K}{a^2} + \Omega_\Lambda}$$

- Densities fluctuate.

$$\rho(t, \vec{x}) = \bar{\rho}(1 + \delta(t, \vec{x}))$$

- In linear regime they grow in time uniformly on sufficiently large scales.

$$\ddot{\delta}(t, \vec{x}) + 2H\dot{\delta}(t, \vec{x}) = 4\pi G\bar{\rho}_m\delta_m(t, \vec{x}) + O(k)$$

$$\delta_m(t, \vec{x}) = D_+(t)\delta_0(\vec{x})$$

Linear Growing Mode

$$\delta_m(t, \vec{x}) = D_+(t)\delta_0(\vec{x})$$

- $D_+(t)$ is called a *linear growing mode* (notice some ambiguity in normalization).
- In general not an analytic function.
- At early times ($a \ll 1$)

$$\frac{\dot{a}}{a} \approx \sqrt{\frac{\Omega_R}{a^4} + \frac{\Omega_m}{a^3}}$$

- Then

$$D_+(t) \approx a + \frac{2\Omega_R}{3\Omega_m} + \dots$$

- and ... are only important for $z > 1000$.

Linear Power Spectrum

- In the LCDM model the seeds of fluctuations are produced during inflation from quantum noise – hence, they are Gaussian.
- Gaussian random noise is described by one 1D function, the *correlation function* or the *power spectrum*.
- In Fourier space

$$\delta_{\vec{k}}(t) = \int d^3x \delta(t, \vec{x}) e^{i\vec{k}\vec{x}}$$

- Two modes with different wave-vectors are uncorrelated

$$\left\langle \delta_{\vec{k}_1} \delta_{\vec{k}_2}^* \right\rangle = P(k_1) \delta_D^3(\vec{k}_1 - \vec{k}_2)$$

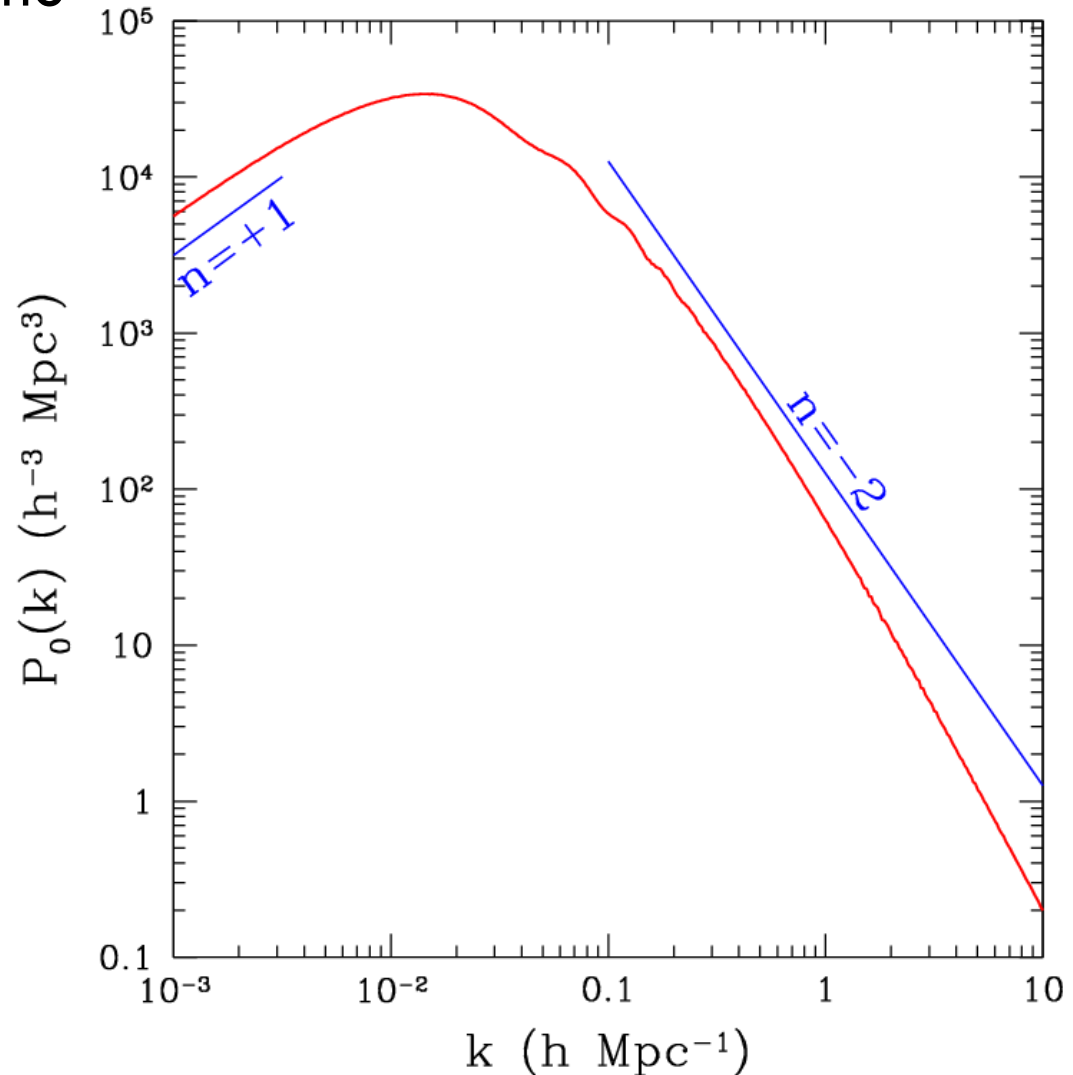
Linear Power Spectrum

- Since in the linear regime

$$\delta_m(t, \vec{x}) = D_+(t)\delta_0(\vec{x})$$

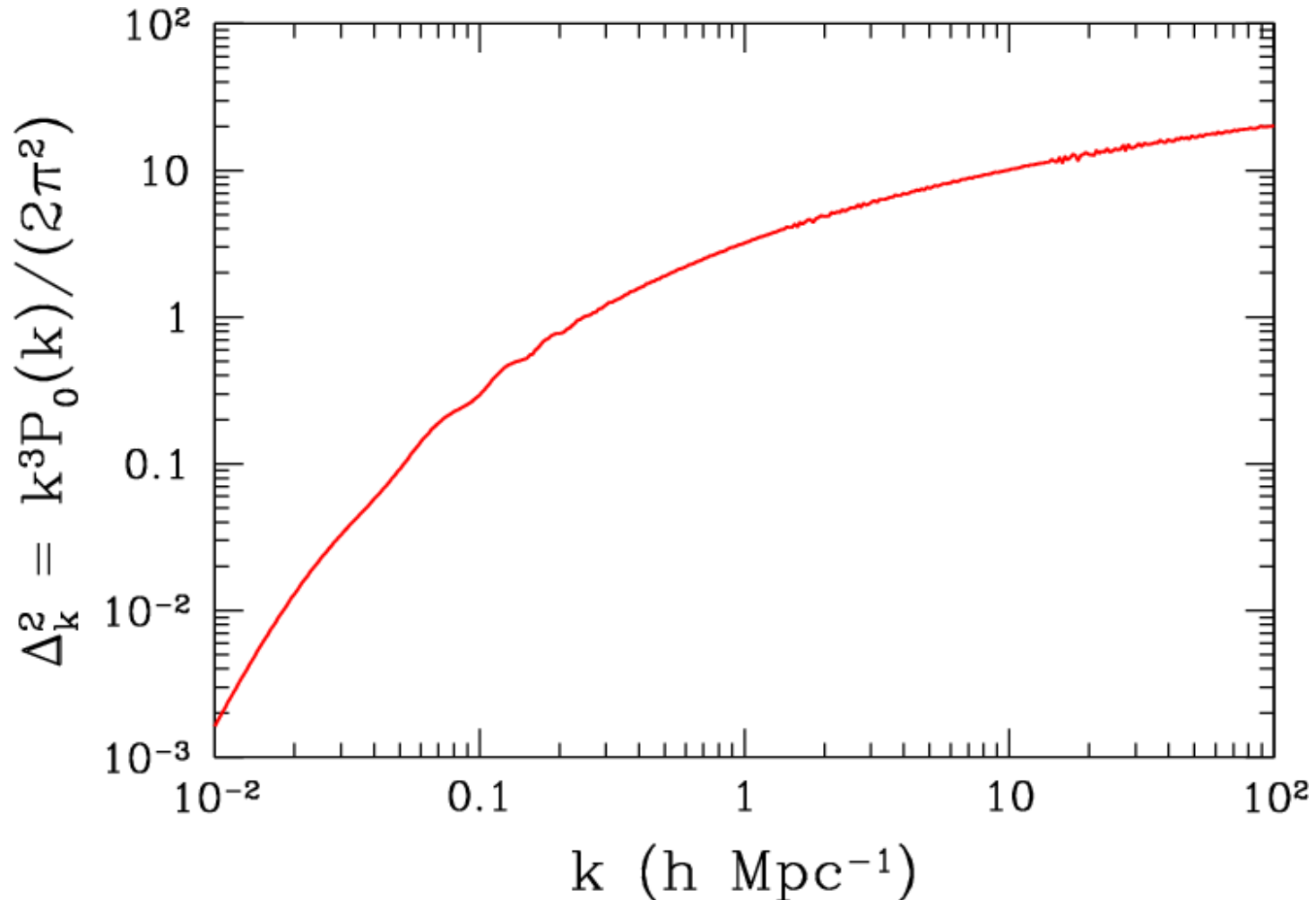
then

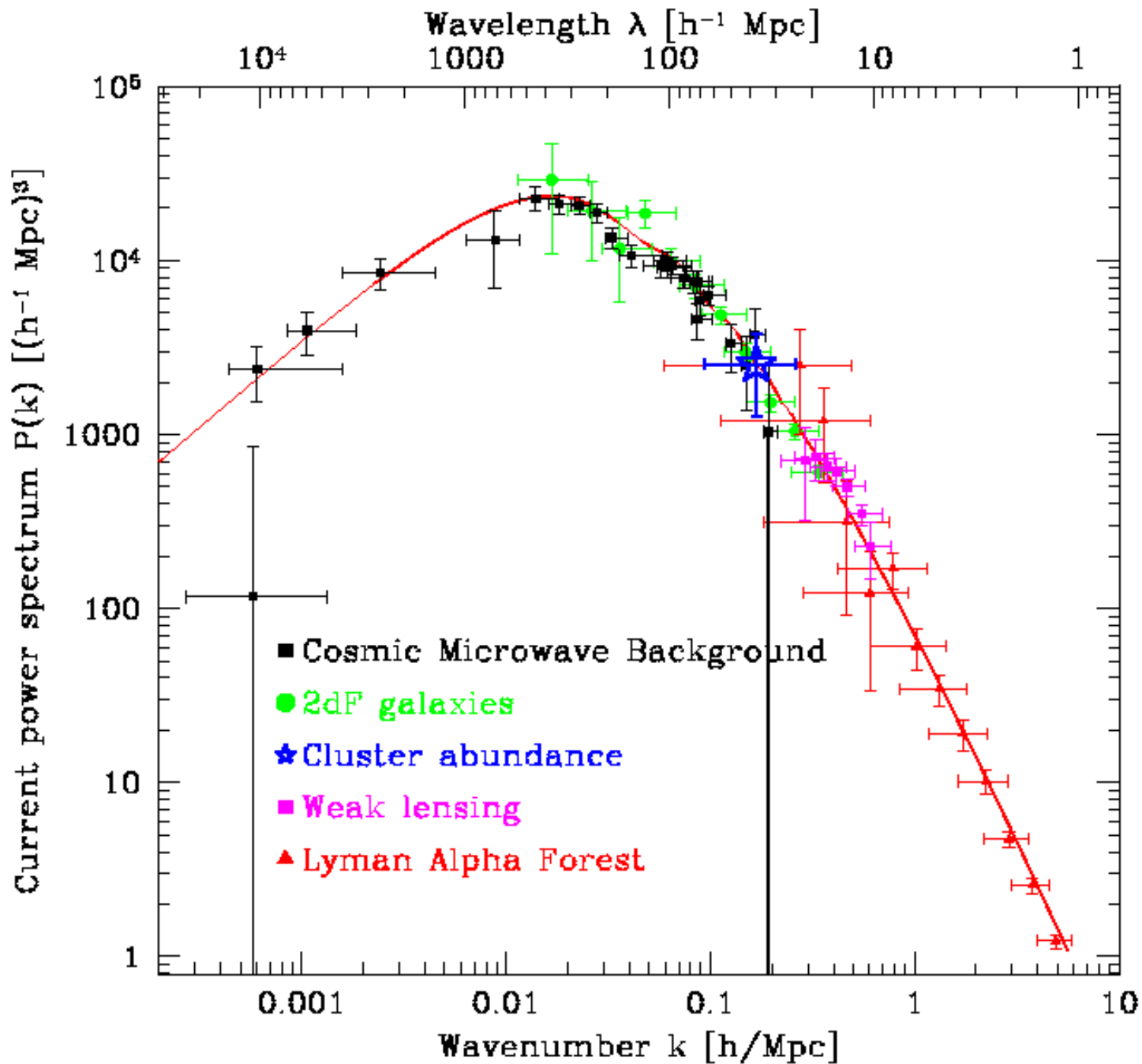
$$P(t, k) = D_+^2(t)P_0(k)$$



Linear Power Spectrum

- Power-per-log-k looks very different!





2. Baryons

- Universe contains baryons.

$$\bar{n}_b = 2.5 \times 10^{-7} a^{-3} \text{cm}^{-3}$$

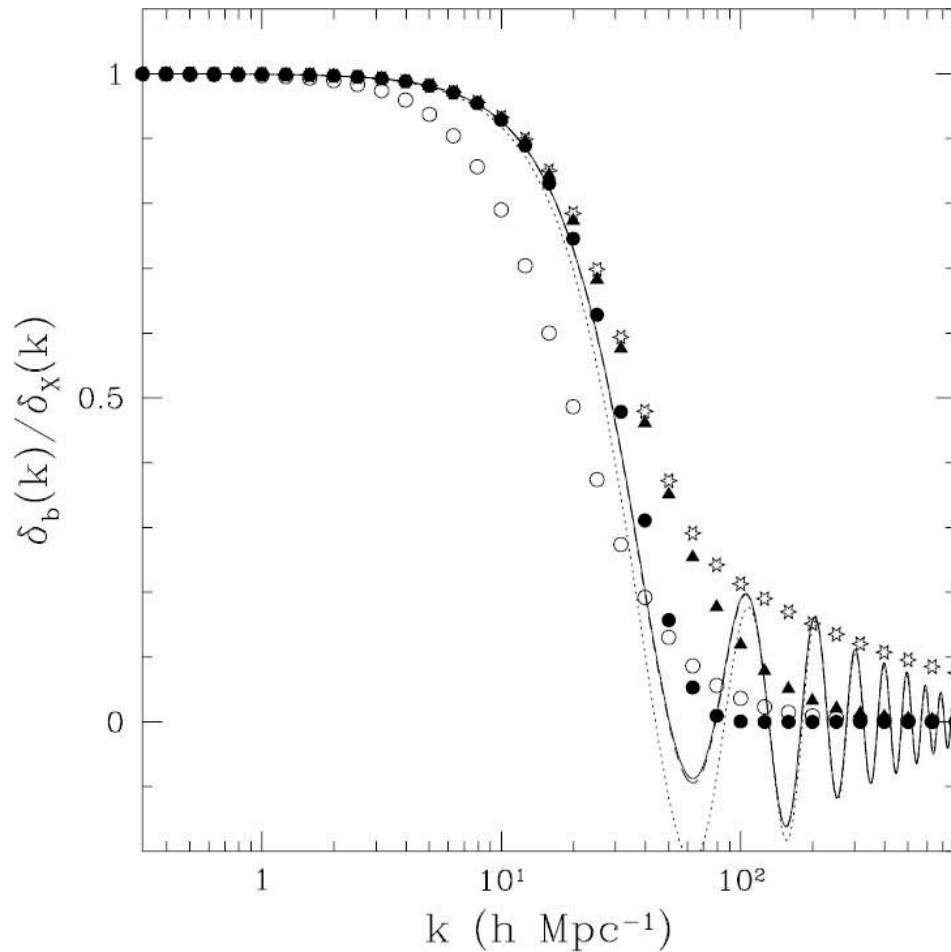
- Primordial abundances are set by BBN, 76% Hydrogen and 24% Helium.
- Gas has pressure.

$$\frac{d^2 \delta_X}{dt^2} + 2H \frac{d\delta_X}{dt} = 4\pi G \bar{\rho} (f_X \delta_X + f_B \delta_B) ,$$

$$\frac{d^2 \delta_B}{dt^2} + 2H \frac{d\delta_B}{dt} = 4\pi G \bar{\rho} (f_X \delta_X + f_B \delta_B) - \frac{c_S^2}{a^2} k^2 \delta_B ,$$

Gas Pressure

- Pressure “suppresses” fluctuations on small scales.



Filtering Scale

- At late times ($z < 5$) baryons trace dark matter on large scales. On small scales baryons are suppressed relative to the dark matter, hence to the first order in k^2

$$\frac{\delta_b(t, k)}{\delta_X(t, k)} = 1 - \frac{k^2}{k_F^2}$$

- We call k_F the *filtering scale* because it is not generally equal to the Jeans scale.

$$k_J = \frac{a}{c_S} \sqrt{4\pi G \bar{\rho}}$$

Filtering Scale

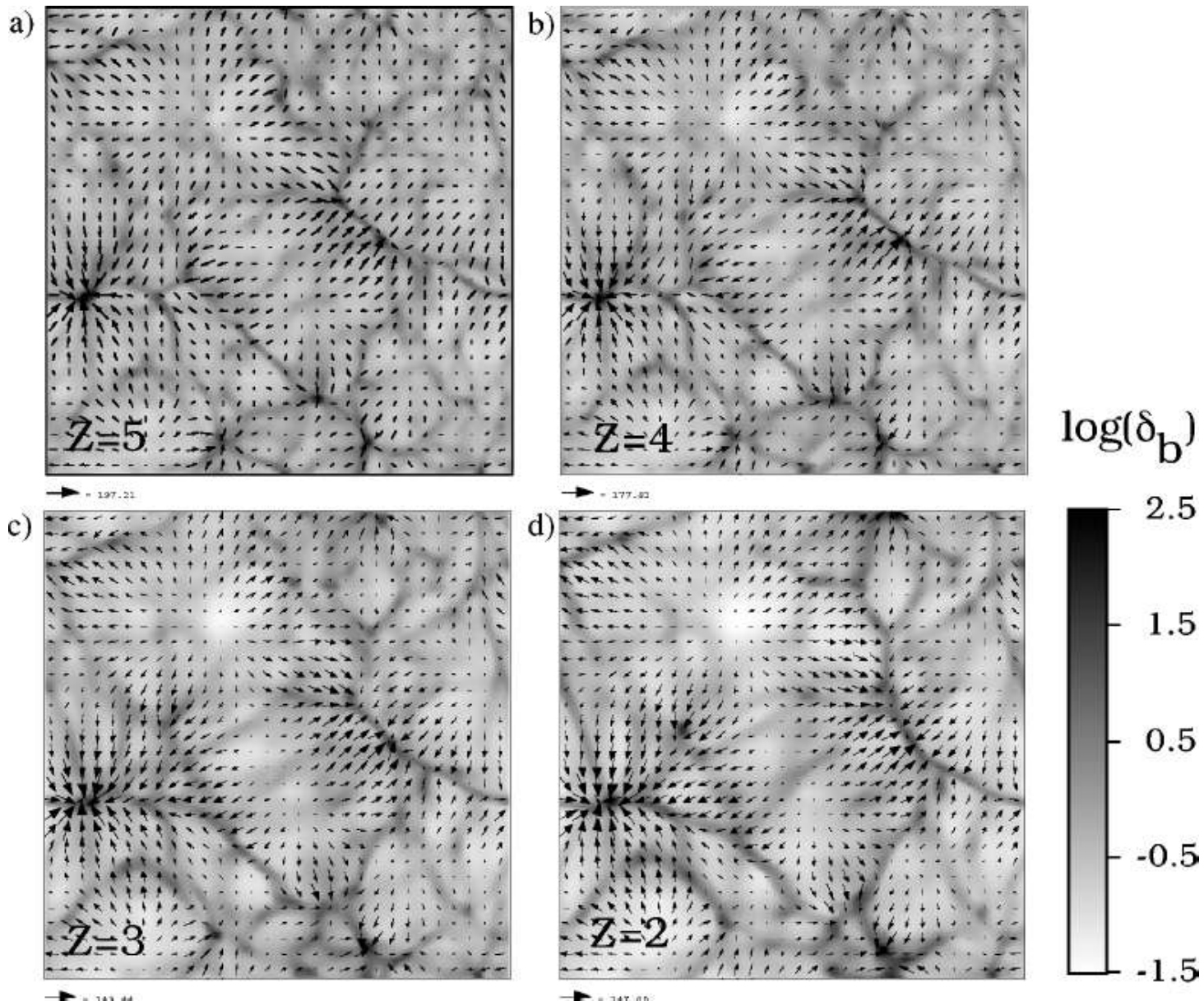
- One can solve for the *filtering scale* analytically in the limit $f_b \ll 1$:

$$\frac{1}{k_F^2(t)} = \frac{1}{D_+(t)} \int_0^t dt' a^2(t') \frac{\ddot{D}_+(t') + 2H(t')\dot{D}_+(t')}{k_J^2(t')} \int_{t'}^t \frac{dt''}{a^2(t'')},$$

- At high redshifts and smaller scales expressions for k_F become progressively more and more scarier.
- A “rule of thumb” filtering scale is $1/2$ of the Jeans scale:

$$k_F \approx 2 \times k_J$$

Filtering Scale In Action



Filtering Scale In Action

Quiz: When pressure dominates in hydrodynamics, all fluctuations separate into a bunch of sound waves. In ideal gas sound waves do not dissipate.

? *Why are fluctuations in baryons suppressed?*

- A. They are not, baryons always trace the dark matter.
- B. They are suppressed by the force of gravity.
- C. Angular momentum is conserved.
- D. Fluctuations in dark matter grow with time.
- E. Fluctuations in baryons decay with time.

3. Ionization

- “Then God said, “Let there be light,” and there was light.”
(And the light was of all frequencies!).
- Consider a region of the IGM with some density fluctuations
(not necessarily linear)

$$n_H(\vec{x}) = \bar{n}_H(1 + \delta(\vec{x}))$$

- **Empirical fact:** IGM at lower redshifts ($z < 6$) is highly ionized (we’ll come back to how we know that).
- Space is filled with ionizing radiation called *Cosmic Ionizing Background*.

Ionization Balance

$$\dot{n}_{HI} - 3Hn_{HI} = -n_{HI}\Gamma + R(T)n_en_{HII}$$

- It is easier to use the neutral fraction x instead

$$\dot{x} = -x\Gamma + R(T)n_e(1 - x)$$

- Ionization equilibrium

$$x = \frac{R(T)}{\Gamma}n_e(1 - x)$$

- If the universe is highly ionized, $x \ll 1$

$$x = \frac{R}{\Gamma}(n_H + 2n_{He}) = \frac{R}{\Gamma}(\bar{n}_H + 2\bar{n}_{He})(1 + \delta)$$

(denser gas is more neutral)

4. Ly- α Forest

- Consider a light bulb somewhere in the universe (a quasar, a galaxy, a search light of an alien spaceship).
- We see it at a cosmic redshift z_e .
- As a photon of wavelength λ_e propagates towards us, it gets redshifted. At redshift $z_a < z_e$ (from our point of view) it has a wavelength of

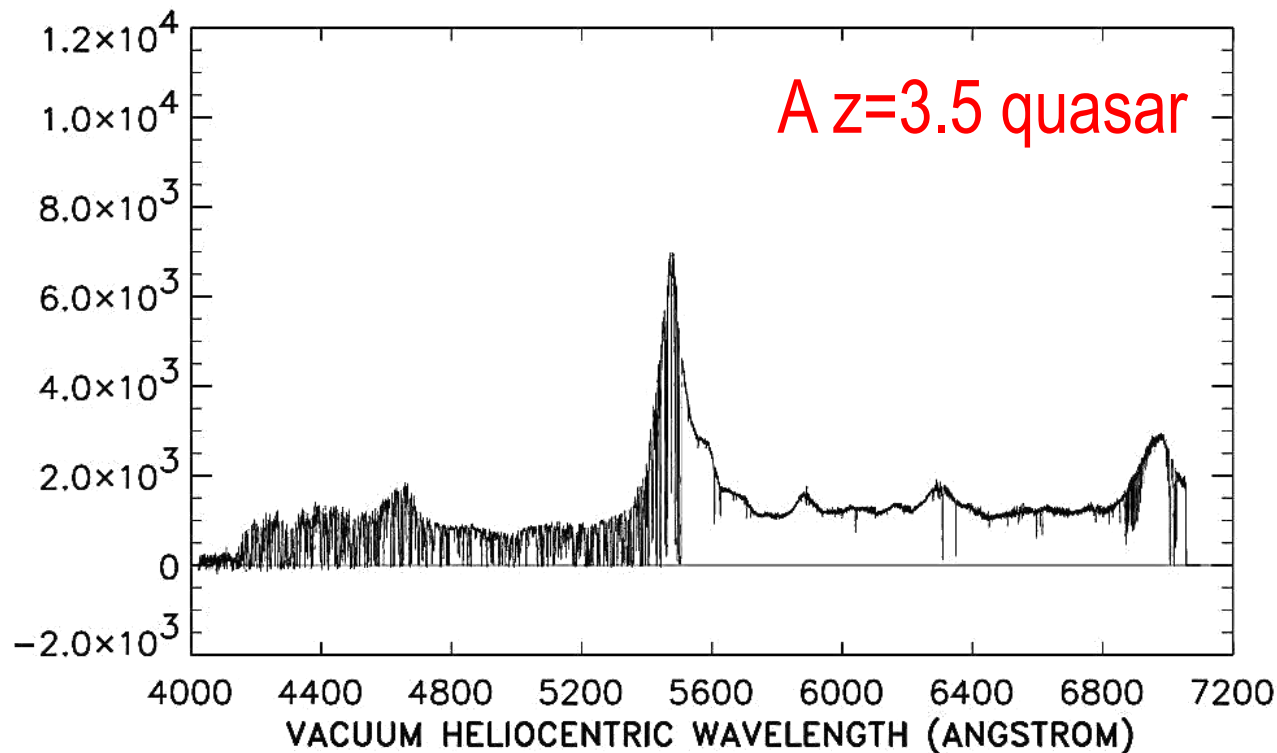
$$\lambda_e \frac{1 + z_e}{1 + z_a}$$

- For any $1216A/(1 + z_e) < \lambda_e < 1216A$ there is such z_a that

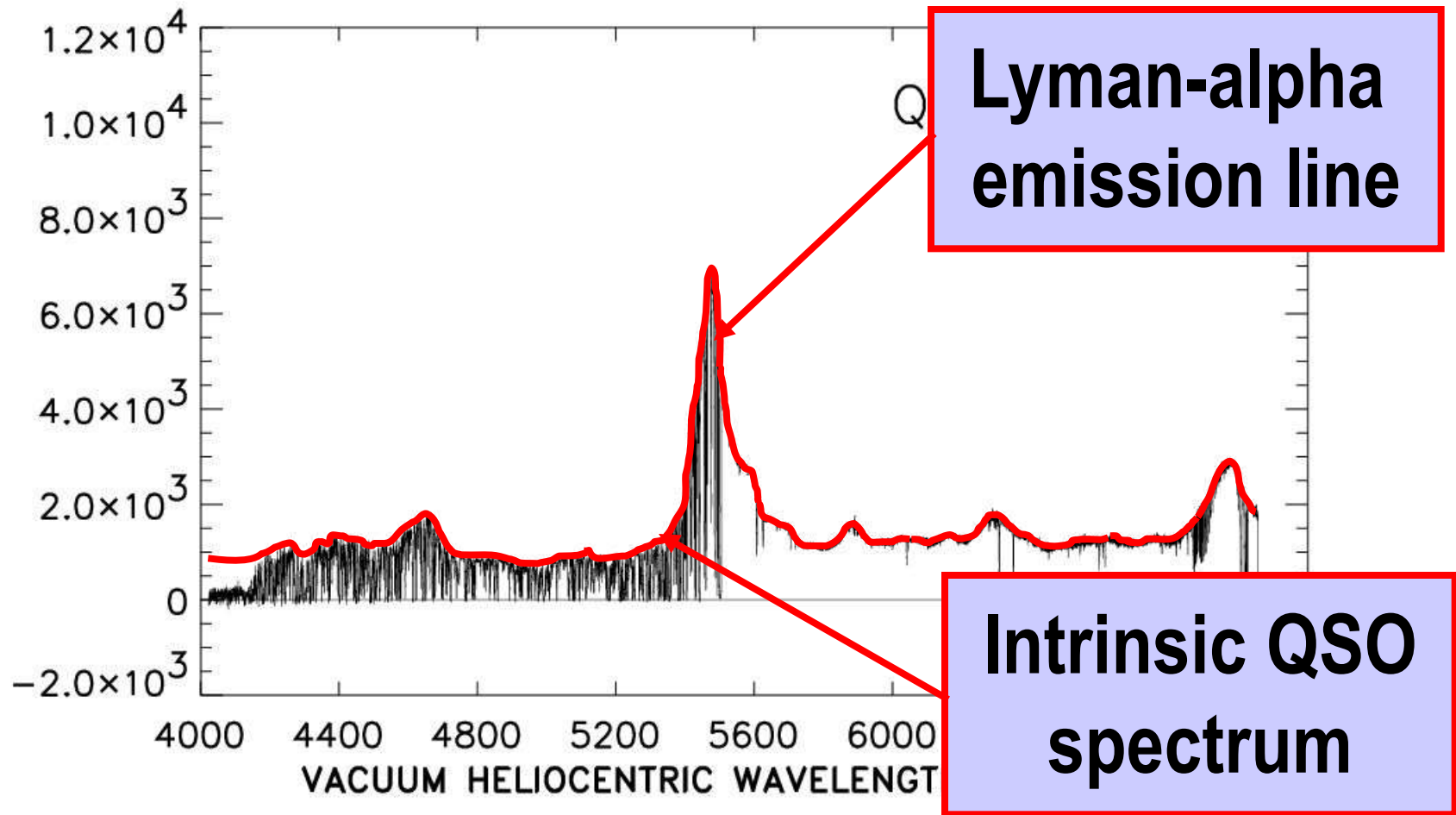
$$\lambda_e \frac{1 + z_e}{1 + z_a} = 1216A$$

Ly- α “Absorption”

- When a 1216Å (=Ly- α) photon hits a neutral hydrogen atom, it can get absorbed and excite the atom to $n=2$ level.
- Does this ever happen?



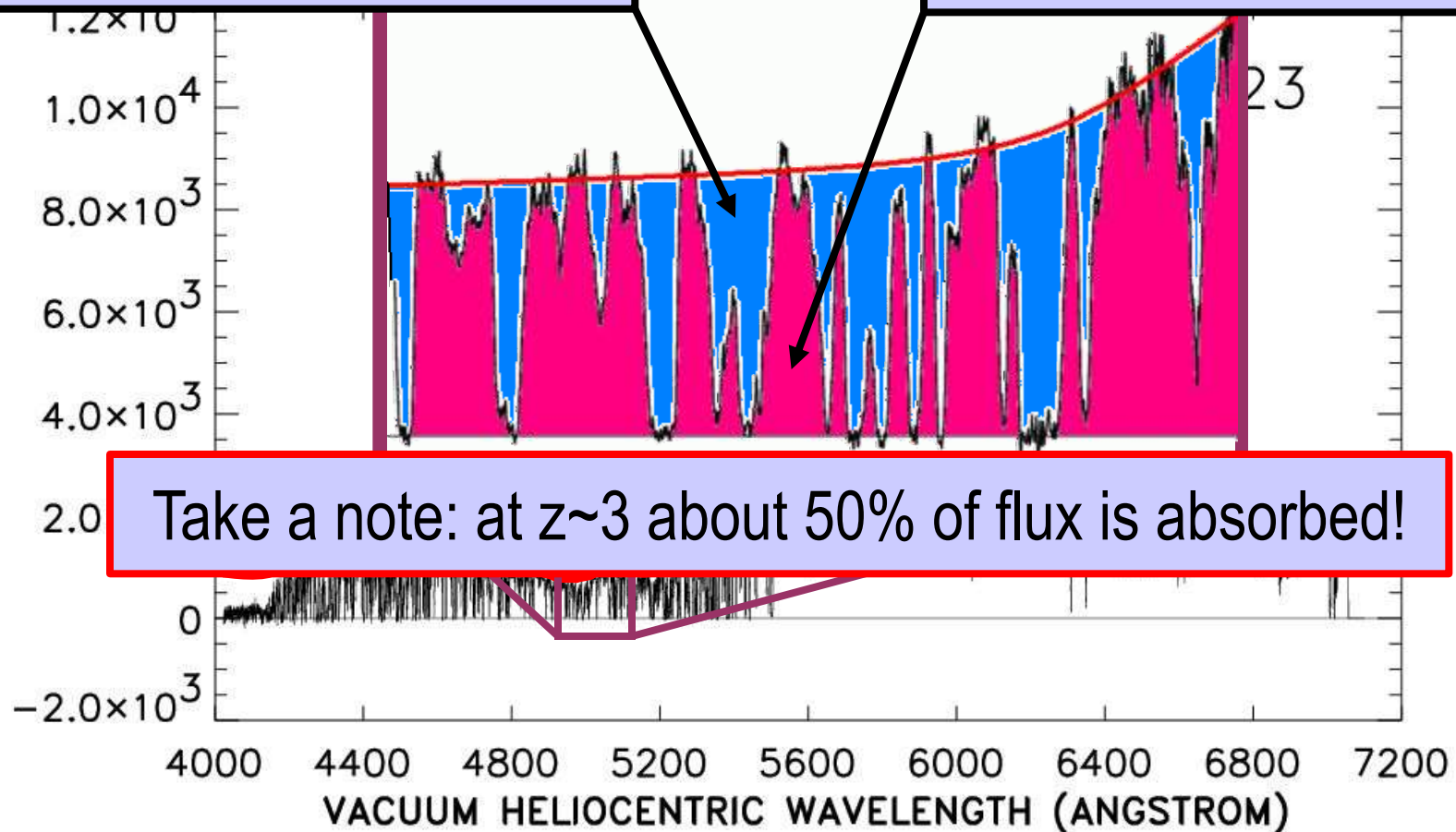
Spectral Anatomy 101



Spectral Anatomy 101

Absorbed flux

Transmitted flux



Ly- α Forest

- Numerous Ly- α absorption lines in the spectra of distant quasars are called “Ly- α Forest”.
- ? ***Wait! Hydrogen atoms don't sit forever in $n=2$ state, they decay back to $n=1$ and a Ly- α photon is created back. There can not be any forest! Can it?***

- A. Ly- α forest is indeed an optical illusion.
- B. It is cold in space, atoms freeze in $n=2$ state.
- C. Emission is isotropic.
- D. Angular momentum is conserved.
- E. $n=2$ state also decays in a two-photon process, truly destroying a Ly- α photon.

Atomic Absorption 101

- The cross-section for an atom at rest to absorb a photon in the frequency range from ν to $\nu + \Delta\nu$:

$$\sigma(\nu) = \frac{\pi e^2}{m_e c \nu_0} f \phi(\nu) = \sigma_0 \phi(\nu)$$

- Line profile

$$\phi(\nu) = \frac{1}{\pi} \frac{w \nu_0}{(\nu - \nu_0)^2 + w^2} \approx \nu_0 \delta_D(\nu - \nu_0)$$

- (w is a (small) *natural line width*.)
- $\sigma_0 = 4.5 \times 10^{-18} \text{cm}^2$ – “center-of-the-line” cross-section.

Atomic Absorption 101

- Atoms are social creatures, they tend to live together. If we have a cloud of gas with density n and size L , we have to integrate over all atoms:

$$\tau(\nu) = nL \int \sigma_0 \phi(\nu') \frac{1}{\sqrt{\pi}b} e^{-\frac{(\nu - \nu')^2}{b^2}} d\nu'$$

- with

$$\nu' = \nu_0 \left(1 + \frac{v'}{c}\right)$$

$$\nu = \nu_0 \left(1 + \frac{v_\nu}{c}\right)$$

Atomic Absorption 101

- If natural line width w is neglected,

$$\tau(\nu) = \sigma_0 n L \frac{c}{\sqrt{\pi} b} e^{-\frac{v_\nu^2}{b^2}}$$

- Some definitions:

$$b = \left(2 \frac{k_B T}{m_H} \right)^{1/2} \quad \text{is the Doppler parameter.}$$

$$N = nL \quad \text{is the column density.}$$

Atomic Absorption 101

- In an expanding universe with the continuous density distribution we integrate along the line-of-sight (LOS):

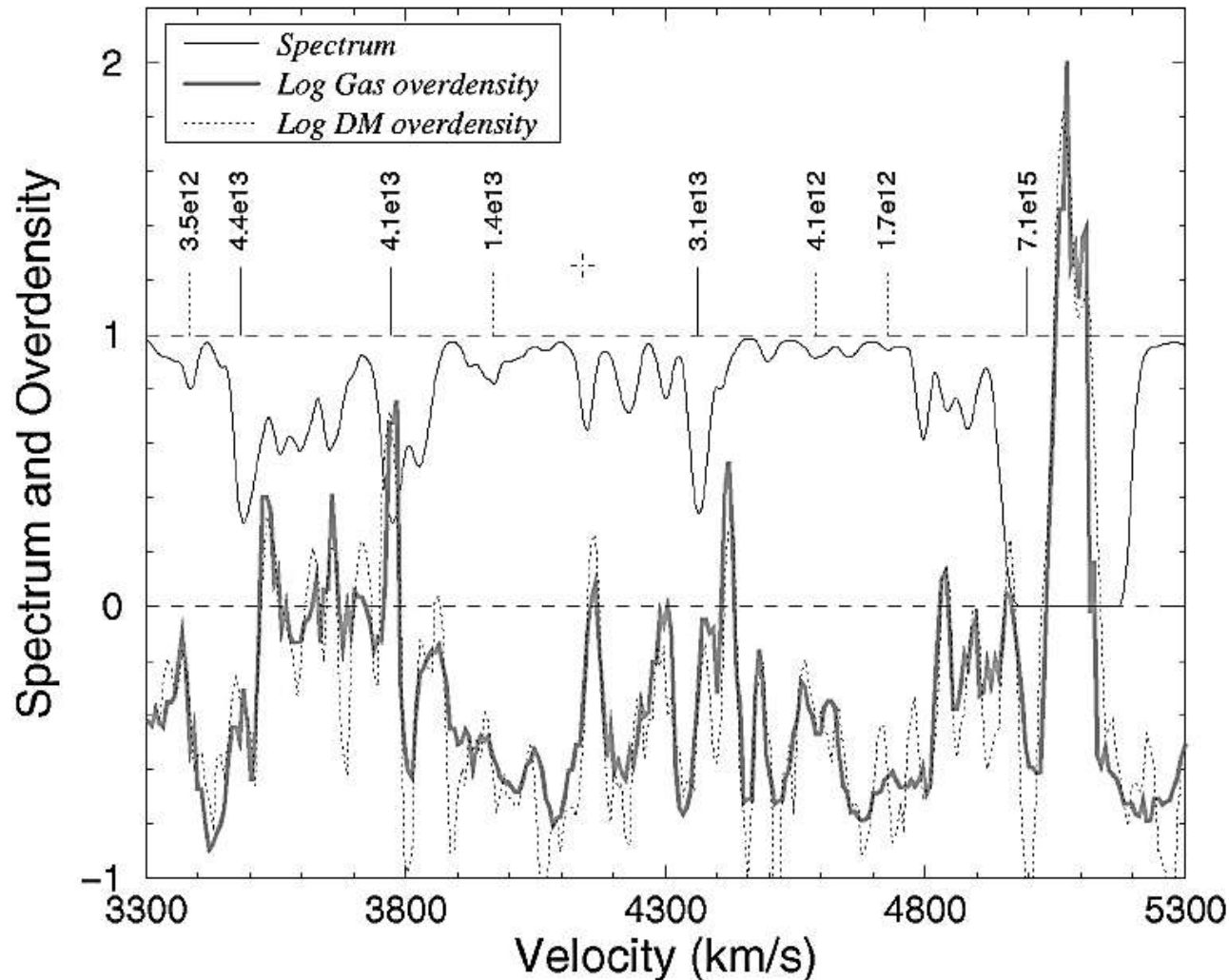
$$\tau(\lambda) = \sigma_0 \int n(x) \frac{c}{\sqrt{\pi} b_x} e^{-\frac{(v_x - v_\lambda)^2}{b_x^2}} \frac{dx}{1 + z_x}$$

- with

$$\lambda = \lambda_0 \left(1 + z_x + \frac{v_\lambda}{c} \right)$$

Ly- α Forest: Example

- Optical depth of Ly- α absorption line (Doppler profile):



Why IGM Is Ionized

- **Empirical fact:** IGM at lower redshifts ($z < 6$) is highly ionized
– time to figure out why.

- Average level of absorption in the forest at $z=2-4$ is

$$\langle \tau \rangle = 0.5 - 1.$$

$$\tau = 4.5_{-18} \text{cm}^2 \frac{c}{\sqrt{\pi b}} [x 2_{-7} 4^3 (4a)^{-3} \text{cm}^{-3}] 3_{23} 0.25 (4a) \text{cm}$$

$$= 4.3 \frac{3_5 \text{km/s}}{\sqrt{\pi} 10 \text{km/s}} \frac{x}{(4a)^2}$$

$$= 7 \times 10^4 \frac{x}{(4a)^2}$$

- Hence $x \sim 10^{-5}$.

5. Temperature

- The final component in modeling IGM is to know what the temperature of the gas is.
- Two facts:
 - Universe is highly ionized at $z < 6$.
 - Universe was neutral until $z \sim 30$ (ask CMB folks why).
- Corollary: ***universe was re-ionized between $z=30$ and $z=6$.***

Photo-heating

- When a high energy photon hits an atom, 13.6eV goes on ionizing it, the rest goes into the energy of the ejected electron, which eventually thermalizes and adds this energy to the thermal energy of the gas (at least for $E_\gamma < 50\text{eV}$)

$$\frac{3}{2} \frac{d}{dt}(nk_B T) = \dots + cn_{HI} \int_{E_0}^{\infty} (E - E_0) \sigma_{HI}(E) n_E dE$$

- Recall ionization balance: $\dot{x} = -x\Gamma + R(T)n_e(1 - x)$

- Photoionization rate $\Gamma = c \int_{E_0}^{\infty} \sigma_{HI}(E) n_E dE$



Photo-heating

- Forget about Helium for a minute:

$$n_e = (1 - x)n_H$$

$$n = n_H + n_e = (2 - x)n_H$$

- Then

$$\frac{3}{2} \frac{d}{dt} ((2 - x)k_B T) = x\Gamma \langle \Delta E \rangle$$

$$\frac{d}{dt} x = -x\Gamma + Rn_H(1 - x)^2$$

$$\langle \Delta E \rangle = \frac{\int (E - E_0) \sigma_{\text{HI}}(E) n_E dE}{\int \sigma_{\text{HI}}(E) n_E dE}$$

Photo-heating

- In the ionization equilibrium:

$$x_{eq} = \frac{Rn_H}{\Gamma}(1 - x_{eq})^2$$

- Hence, if $x \gg x_{eq}$ then

$$x\Gamma \gg Rn_H(1 - x)^2$$

- Before reionization we start with the neutral cold IGM ($x = 1, T = 0$). Let's assume that reionization is instantaneous

$$\Gamma \propto \theta(t - t_R)$$

Photo-heating

- Solution:

$$x(t) = e^{-\Gamma(t-t_R)}$$

$$(2 - x)k_B T = \frac{2}{3} \langle \Delta E \rangle [1 - e^{-\Gamma(t-t_R)}]$$

- Hence when x gets small,

$$T = \frac{\langle \Delta E \rangle}{3k_B}$$

- **Lesson #1:** After instantaneous ionization the gas becomes isothermal, and the value of the temperature is independent of the strength of ionizing radiation, but depends *only* on its spectrum.



Photo-heating

? ***We know reionization was not instantaneous – does this lesson ever apply in real life?***

- A. Yes
- B. No

- **Lesson #2:** Ionization fronts in the IGM are rather sharp – hence, while the whole universe is reionized gradually, each parcel of gas is ionized instantaneously, just different parcels are ionized at different times.
- **Lesson #3:** You cannot heat up the IGM by cranking up the ionization source, only by making it *harder!*

How Hot Is It?

$$T = \frac{\langle \Delta E \rangle}{3k_B}$$

- Take a power-law spectrum just beyond the Lyman edge

$$n_E \propto E^{-\alpha} \quad \sigma_E \propto E^{-3}$$

- Hence

$$\langle \Delta E \rangle = \frac{\int (E - E_0) \sigma_{\text{HI}}(E) n_E dE}{\int \sigma_{\text{HI}}(E) n_E dE} = \frac{E_0}{1 + \alpha}$$

- And (basically, 10,000K give-or-take a factor of 2)

$$T = \frac{52,000\text{K}}{1 + \alpha} = \begin{cases} 26,000\text{K} (\alpha = 1) \\ 5,000\text{K} (\alpha = 9) \end{cases}$$

What's Next?

- After ionization equilibrium is established, another important effect is the plain adiabatic expansion:

$$\frac{dT}{dt} = T \frac{2\dot{n}_H}{3n_H} + \frac{\langle E \rangle}{3k_B} x_{eq} \Gamma = T \frac{2\dot{n}_H}{3n_H} + \frac{\langle E \rangle}{3k_B} R n_H$$

- Finally,

$$R(T) \approx 4.3 \times 10^{-13} T_4^{-0.7} \text{cm}^3/\text{s}$$

$$n_H = \bar{n}_H (1 + \delta) = \frac{\bar{n}_{H,0}}{a^3} (1 + \delta)$$

- In a matter-dominated regime this equation can (almost) be solved analytically.

Temperature-Density Relation

- At late times:

$$T_4 \approx \text{const} \left(\frac{a_R}{a} \right)^{1.5/1.7} (1 + \delta)^{1/1.7}$$

- It turns out, a power-law approximation is good at all times,

$$T(\rho) \approx T_0(1 + \delta)^{\gamma-1}$$

- Both T_0 and γ are functions of t (or a):
 - $\gamma = 1$ right after the instantaneous reionization, and
 - $\gamma = 1.62$ at late times.
- (notice, it is 1.62, not $2/3$!!!)

Temperature-Density Relation

- What about radiative cooling?
- In a fully ionized gas the dominant radiative cooling mechanism is recombination cooling:

$$\frac{dU}{dt} = -\Lambda_{RC}(T)n_en_{HII}$$

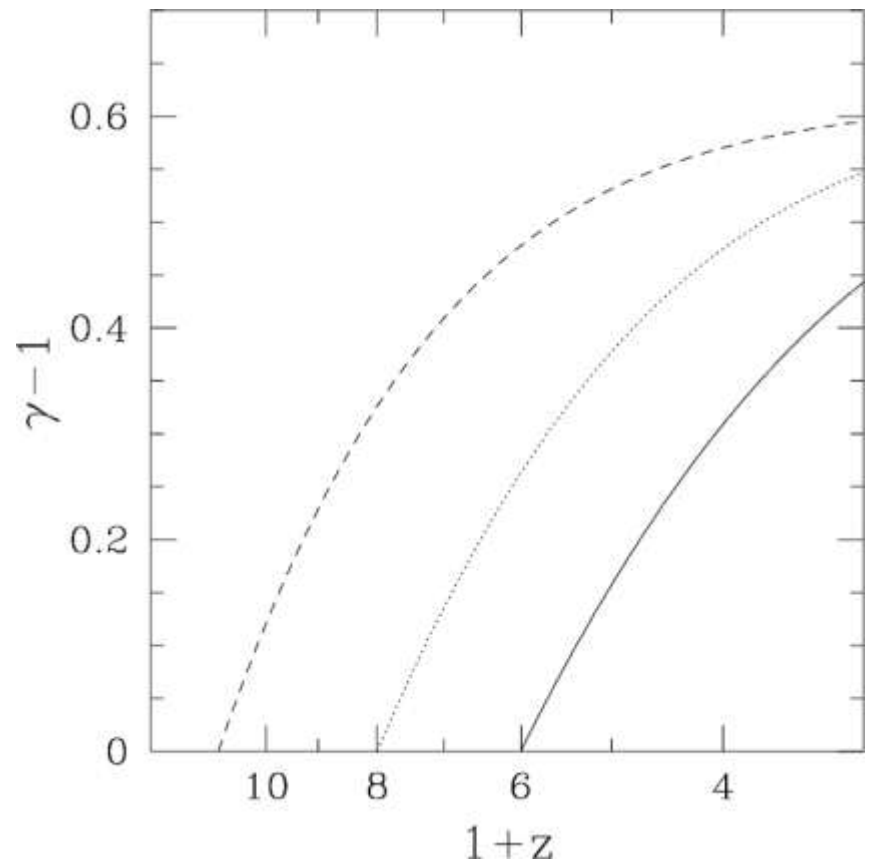
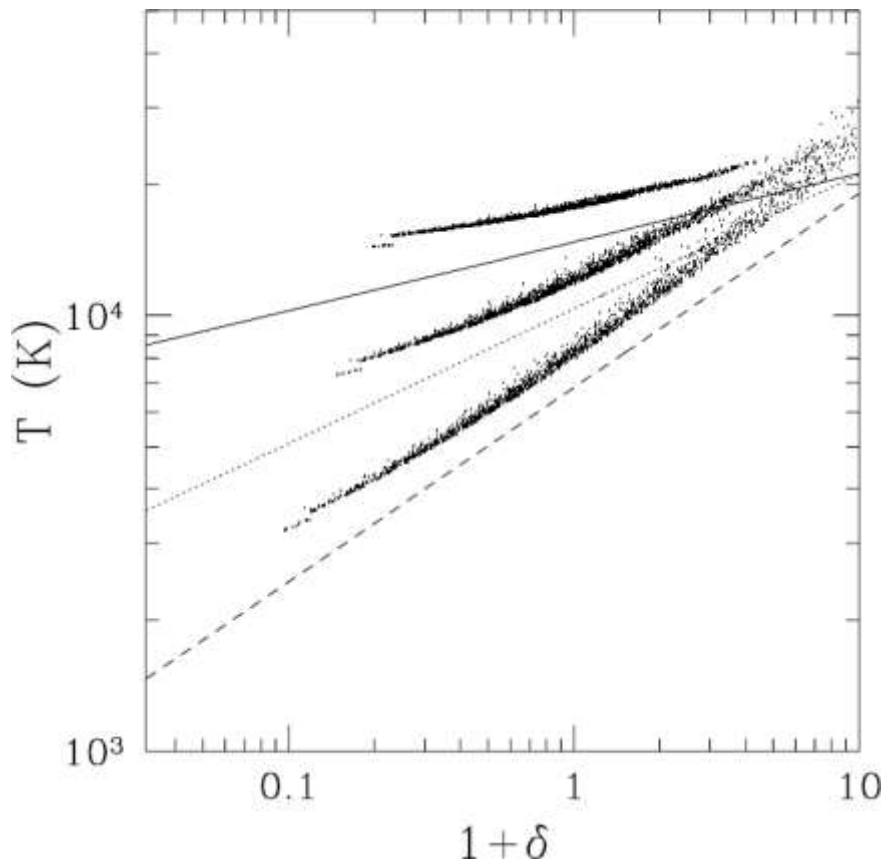
- and

$$\Lambda_{RC}(T) \approx \frac{3}{4}k_B T R(T)$$

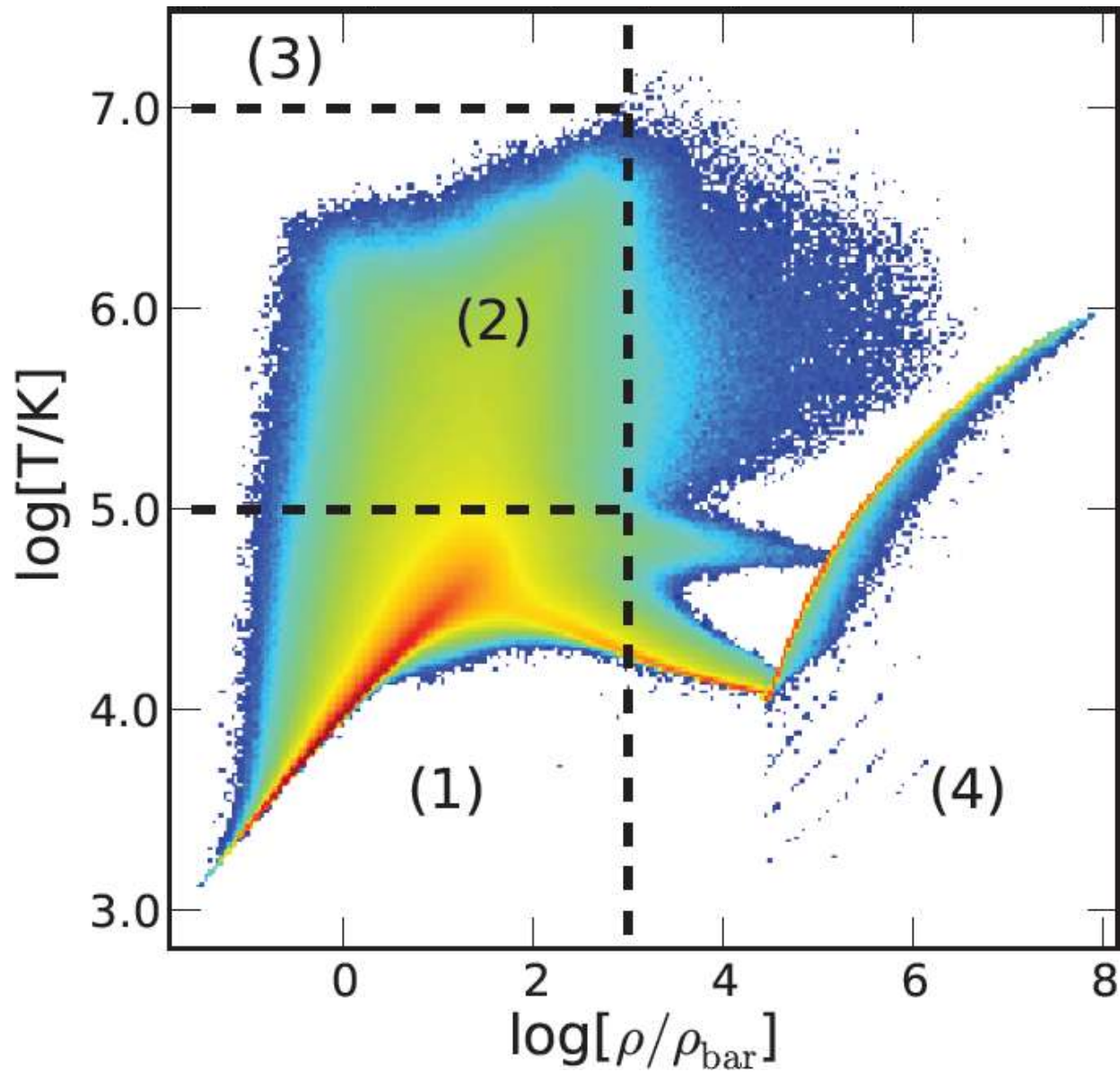
- **Exercise #1**: estimate how important this is.

Temperature-Density Relation

- Example: $Z_R = 6, z = 4, 3, 2.$



Real (Virtual) Life

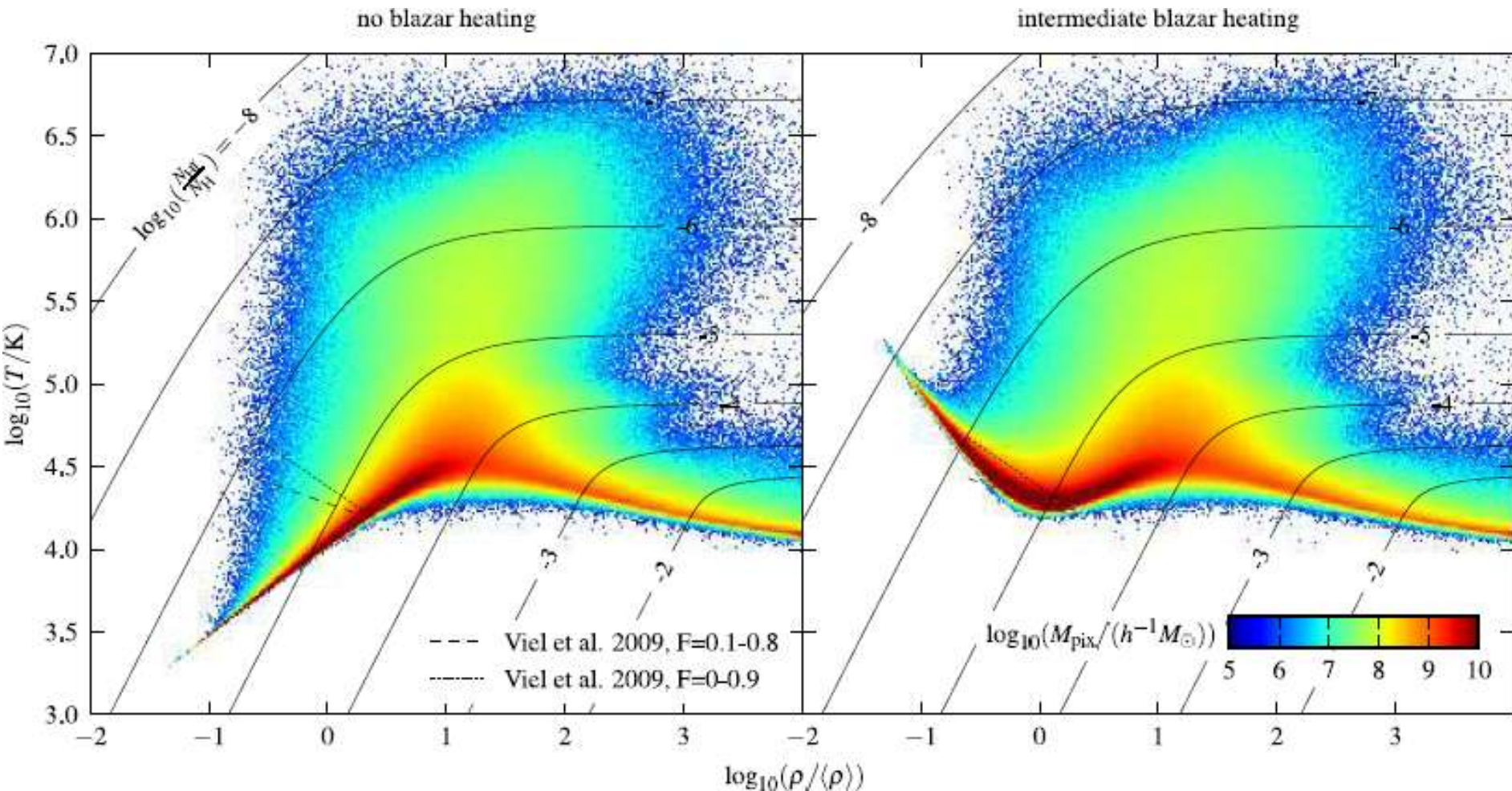


Caveats:

- The $T - \rho$ relation is an *approximation*, with ~5-10% scatter at low densities and larger scatter at higher densities, because it misses a major hydro effect – shocks!
- The only heating mechanism included is photo-heating, and the only cooling mechanism is adiabatic cooling. Other cooling mechanism may exist.
- It assumes that gas is optically thin to heating radiation (may not be such a good assumption at $z \sim 3$, but that is a different story).
- The $T - \rho$ relation is *not* an equation of state as it is sometimes incorrectly called.

Caveats In Action

- Blazar heating controversy: there may be other heating mechanism indeed...



6. Modeling

- The most straightforward model of Ly- α forest is a hydro simulation with ionization balance.
- In the 1990-ties several approximate methods have been developed
 - Log-normal approximation,
 - Zel'dovich approximation,
 - pure N-body simulation,
 - Hydro-Particle-Mesh (HPM) simulation.
- They are not competitive any more.

Simulating IGM

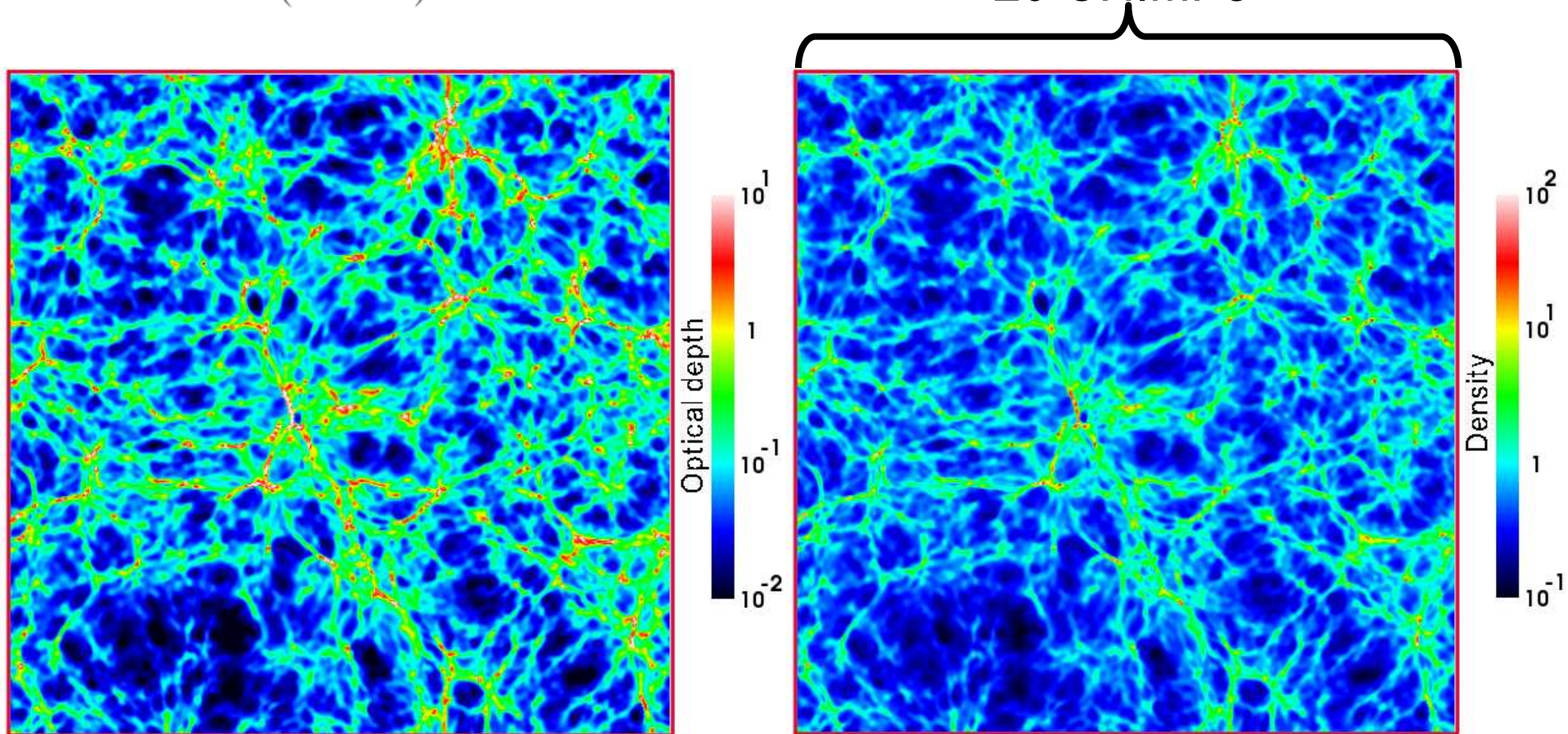
- The assumption of the ionization equilibrium is very good in the IGM, but it breaks down in a few cases (QSO proximity zones, He reionization, ...).
- The most accurate simulation: includes (a model of) Cosmic Ionizing Background, RT, non-equilibrium ionization, separate fields for species (usually overkill).
- Standard approach: includes CIB and ionization equilibrium, radiative heating and cooling are computed “on-the-fly”.
- For simulations of forest alone, the temperature-density relation may be assumed (usually not worth it).

Finding Your Way In The Forest

- Optical depth is closely correlated with the gas density

$$\tau \propto (1 + \delta)^{\sim 1.5}$$

20 CHIMPs



FGPA

- The correlation between the optical depth and density is highly approximate and is a function of scale – the larger the scale, the better it is.
- On small scales one has to account for the fact that the spectrum is in the velocity space, which is different from the physical space.
- An ansatz $\tau(\rho)$ is called *Fluctuating Gunn-Peterson (optical depth) Approximation*, or FGPA.
- It is useful for modeling large-scale clustering in the Ly- α forest.

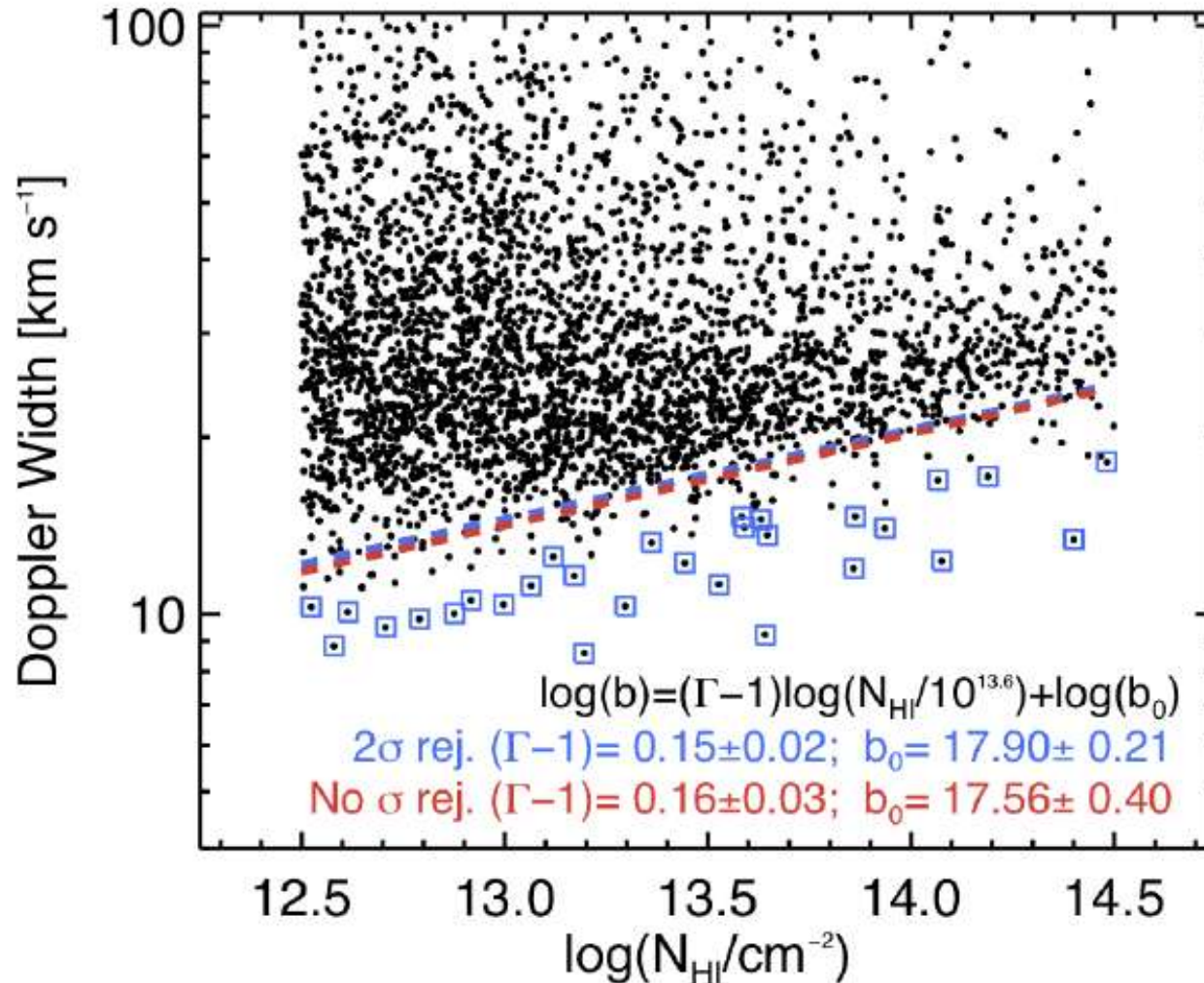
7. Observations

(or, rather examples of)

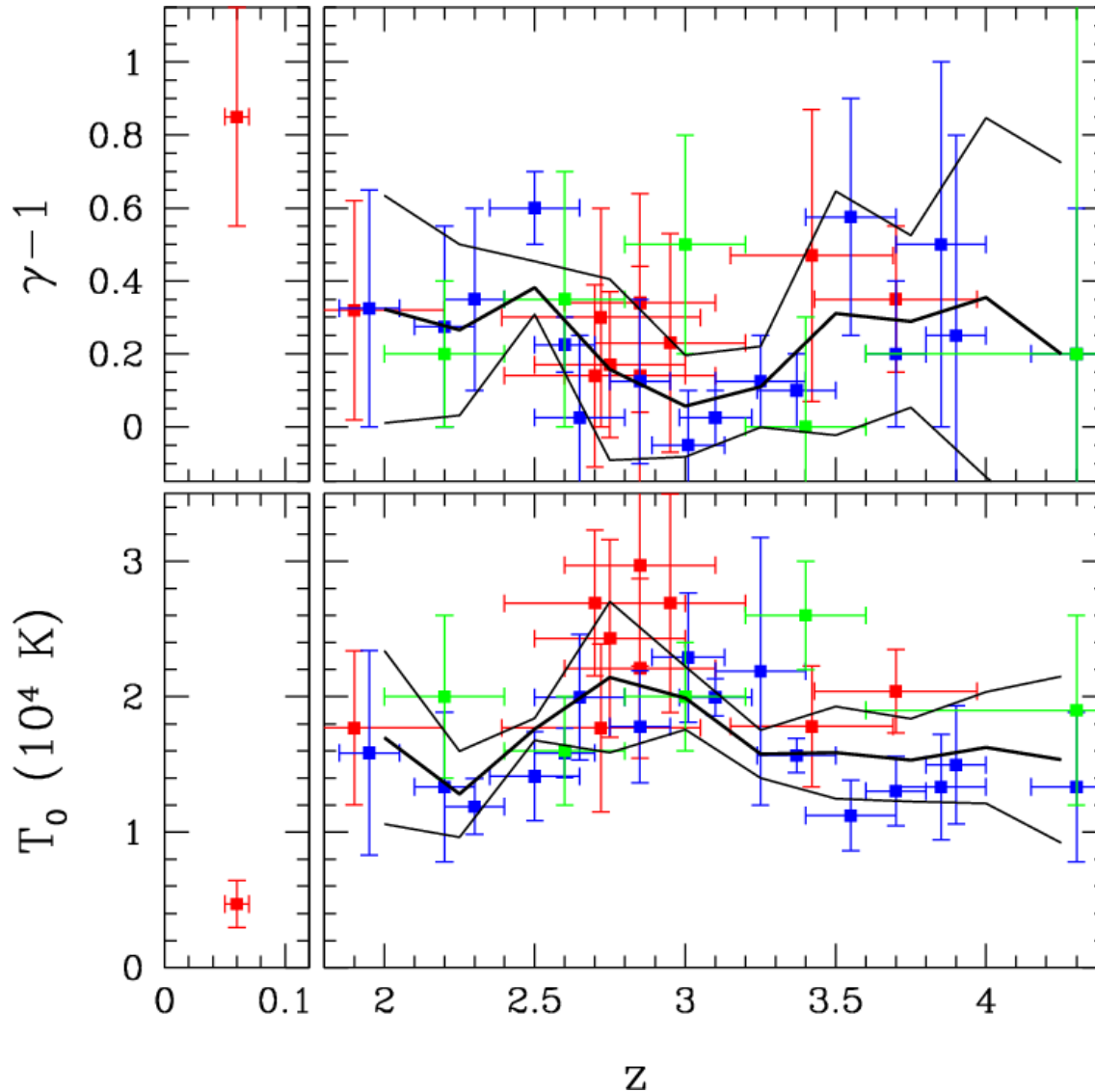
- Historically, Ly- α forest was considered to be a set of individual lines from discrete clouds. Such a spectrum can be uniquely represented by a set of pairs (N, b) for each line.
- In reality, the density field is continuous, and the (N, b) decomposition becomes non-unique.
- The modern view is to consider the $\tau(\lambda)$ as a continuous field and operate on that field.

(N,b) Decomposition

- Is still useful for measuring the $T - \rho$ relation.



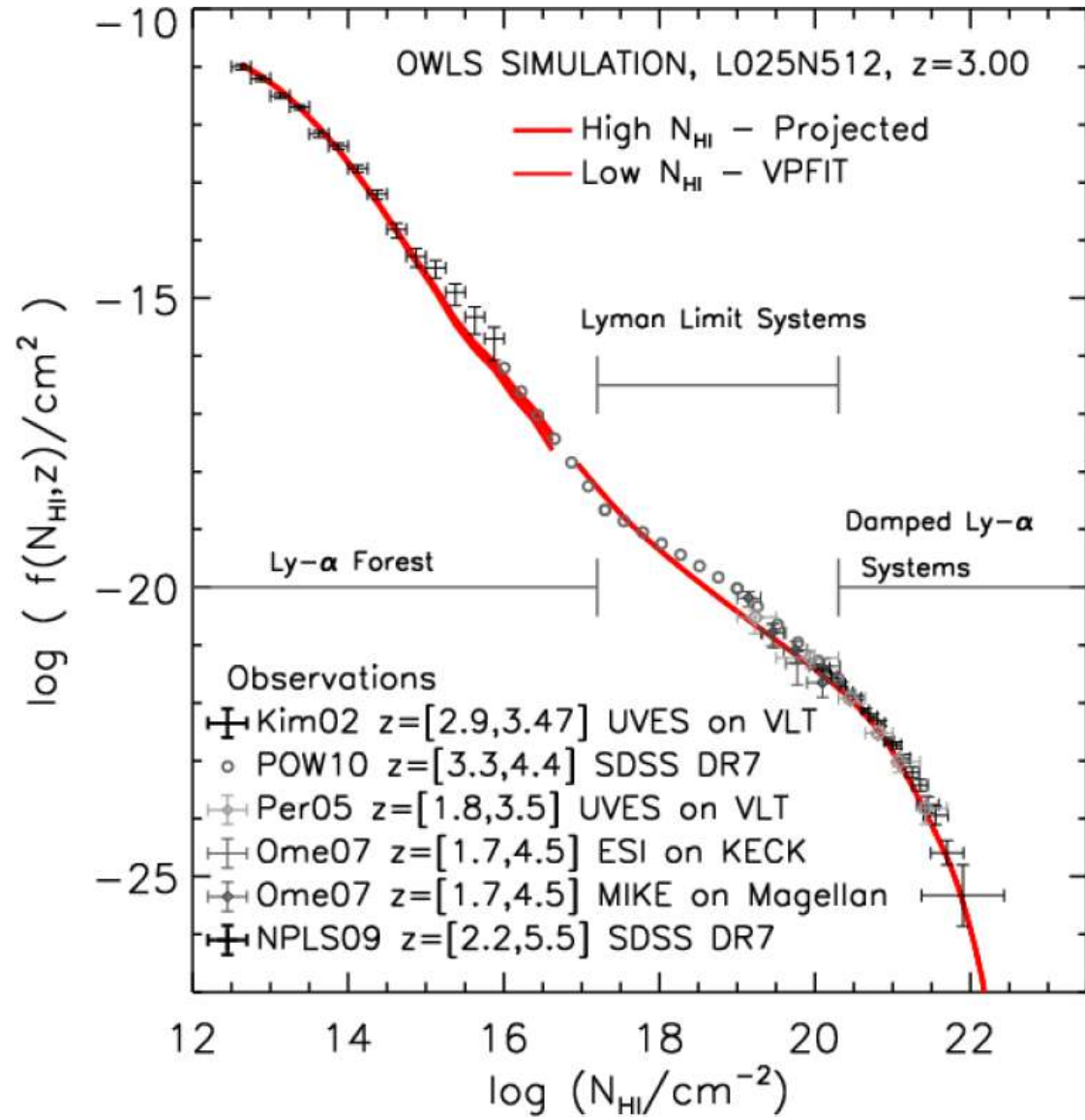
Temperature-Density Relation



- Three sets of observational measurements: one point – one QSO
- Line is the mean over 5 sets of measurements
- Can you see a peak in T_0 and a dip in γ at $z \sim 3$?

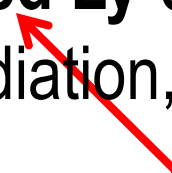
Column Density Distribution

- Another commonly used quantity is the *column density distribution* – a histogram of all N_{HI} (in weird units due to historical legacy).



Classification of Ly- α Absorbing Systems

- $N_{\text{HI}} < 10^{17} \text{ cm}^{-2}$: **Ly- α forest** (optically thin to ionizing radiation, live in the IGM)
- $10^{17} \text{ cm}^{-2} < N_{\text{HI}} < 10^{20} \text{ cm}^{-2}$: **Ly-limit systems** (mostly ionized, but have non-negligible opacity to ionizing radiation, live in the CGM?)
- $10^{20} \text{ cm}^{-2} < N_{\text{HI}}$: **Damped Ly- α systems** (neutral, totally opaque to ionizing radiation, live in the ISM and are the Warm Neutral ISM)



Because the natural line width in the line profile becomes important.

Quiz

- The photo-ionization cross-section for neutral hydrogen at the ionization edge (13.6eV) is $\sigma_{\text{ion}} = 6.3 \times 10^{-18} \text{cm}^2$. Hence, a column density of $N_{\text{HI}} = 1.7 \times 10^{17} \text{cm}^{-2}$ has an optical depth of

$$\tau_{\text{ion}} = \sigma_{\text{ion}} N_{\text{HI}} = 1$$

Quiz: Why do Ly-a absorbers remain ionized all the way to $N_{\text{HI}} \sim 10^{20} \text{cm}^{-2}$?

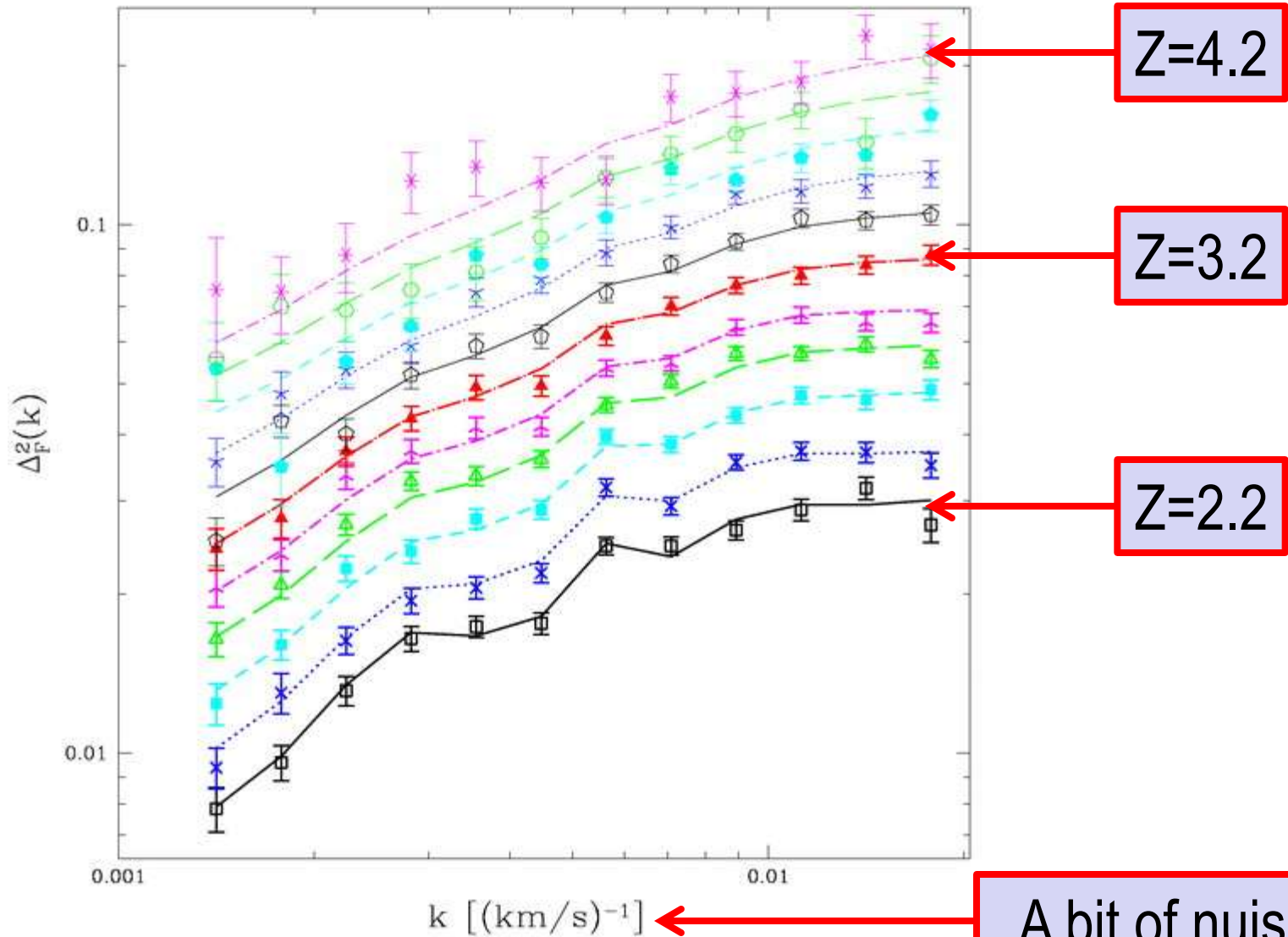
Ly- α Power Spectrum

- In 1998 Rupert Croft and David Weinberg (with Neal Katz and Lars Hernquist) realized that Ly- α forest is a perfect example of a *locally-nonlinear field*.
- **Theorem:** for any locally non-linear field $f(\rho)$, on large scales field f is linearly biased wrt the density field,

$$P_f(k) = b_f^2 P(k)$$

- Hence, to measure the matter (3D) PS one has to
 - A. measure the (1D) PS of Ly- α flux on large scales,
 - B. convert from a 1D PS to a 3D PS,
 - C. determine the bias (say, from simulations), and
 - D. declare ***A BIG VICTORY!!!!***

Ly- α Power Spectrum From SDSS



$Z=4.2$

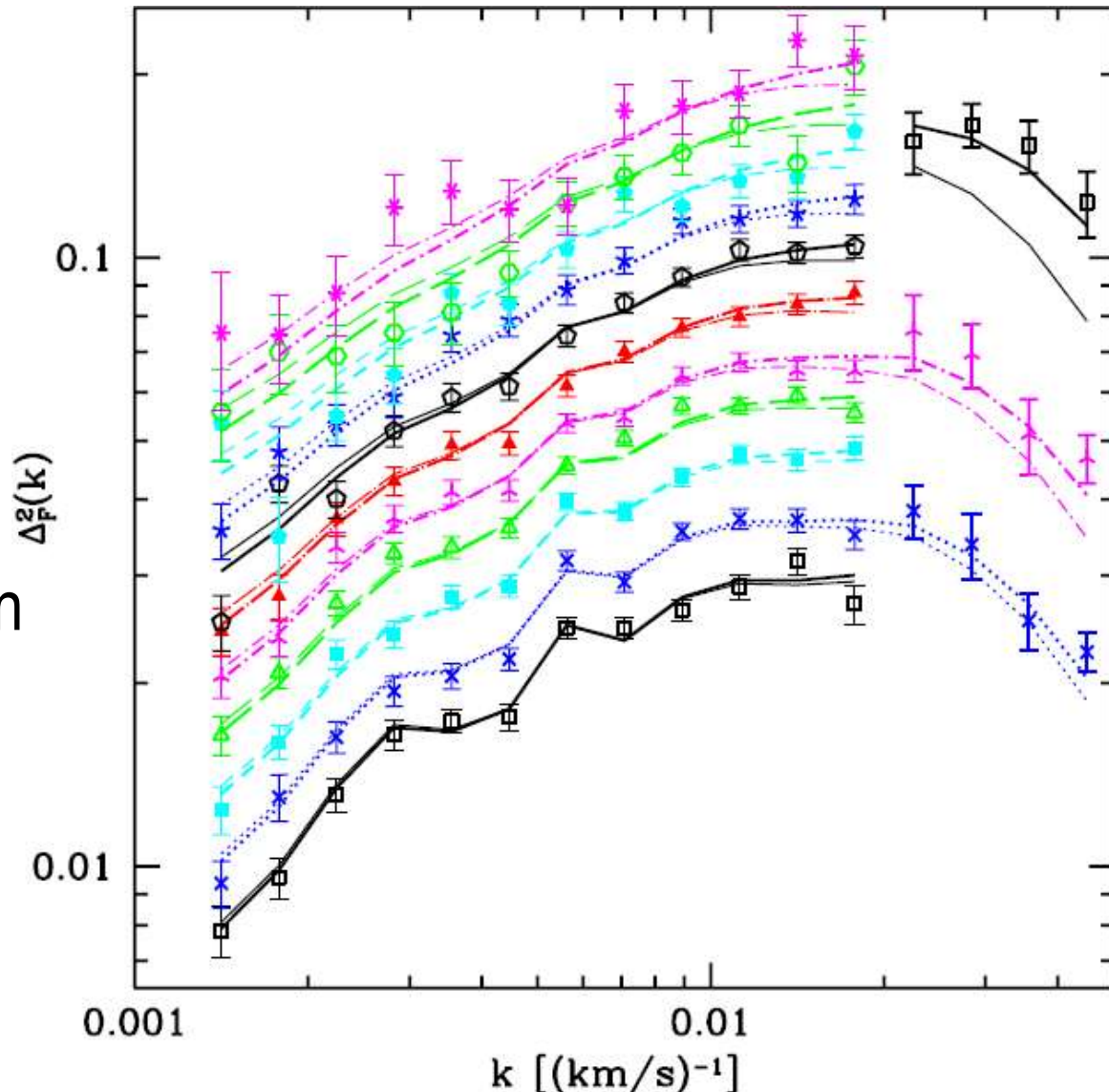
$Z=3.2$

$Z=2.2$

A bit of nuisance

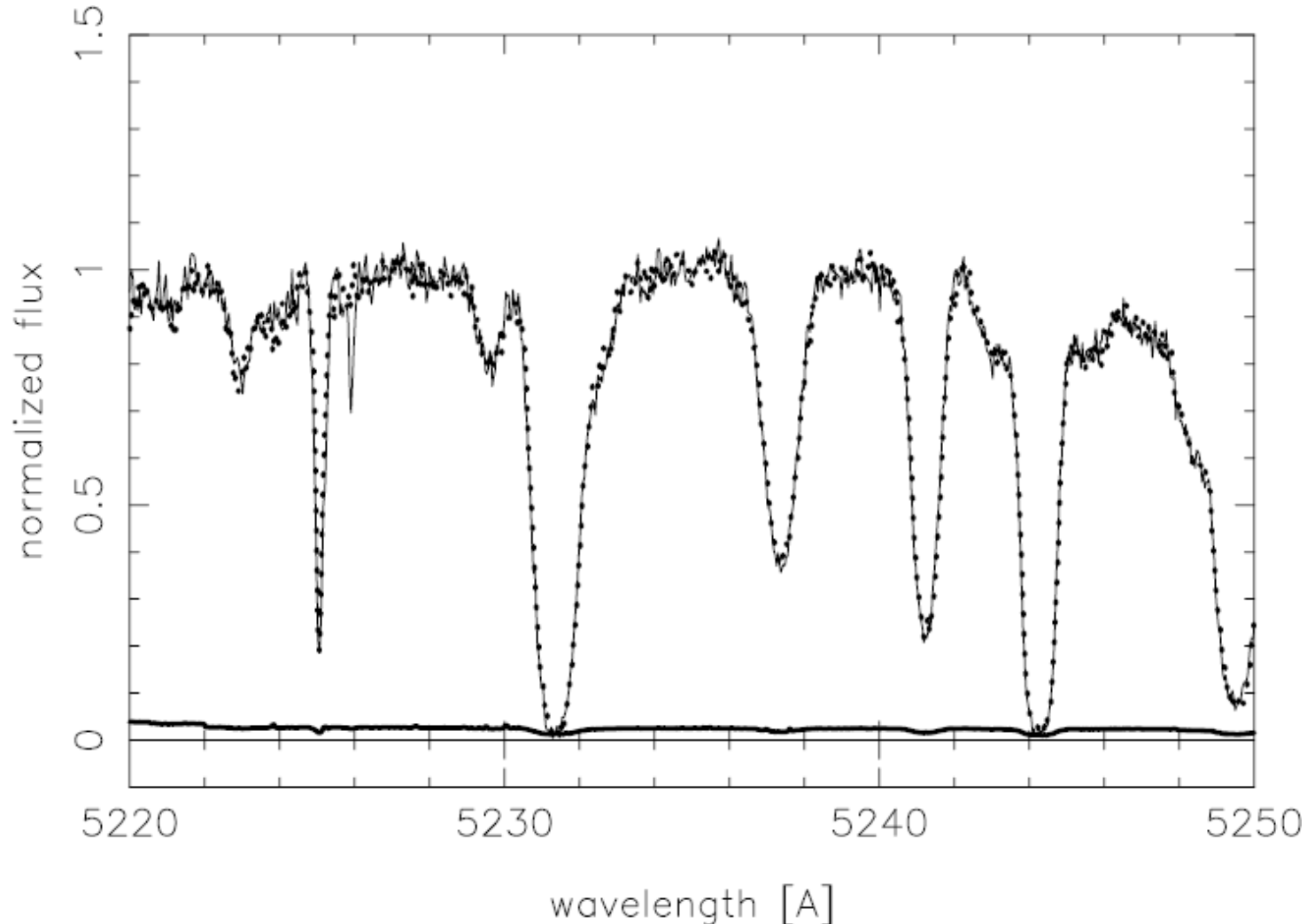
8. Where Forest Ends

- On scales below the filtering scale the fluctuations are suppressed.
- Is this a true end of the forest?
- (Reynold's number in the forest is $> 10^6$ where is the turbulence?)



The Small-Scale Structure In The Forest

- Find 10 differences:



The Small-Scale Structure In The Forest

- Double quasar Q1422+231.
- From cross-correlating two lines of sight:
 $\sqrt{\langle (\Delta \ln \rho)^2 \rangle} < 3 \times 10^{-2}$ for $\langle \Delta x \rangle = 0.6$ kpc, or

$$\sqrt{\left\langle \left(\frac{\Delta \ln \rho}{\Delta x} \right)^2 \right\rangle} < 0.05 \text{ kpc}^{-1}$$

- **Quiz:** Reynolds number in the forest is $> 10^6$ - where is the turbulence?

The End

