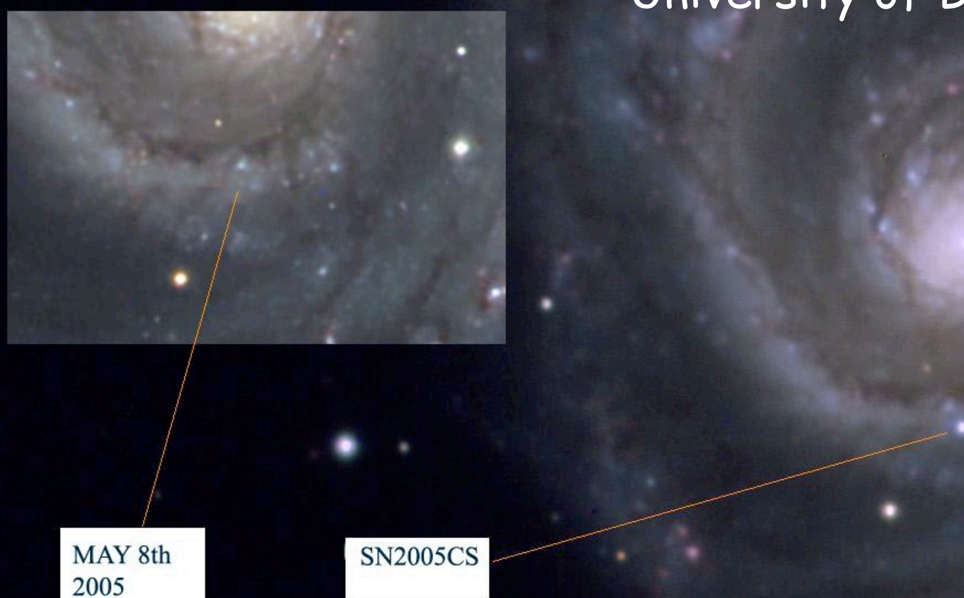
Supernovae Part 4

Numerical Modelling

M. Liebendörfer
Physics Department
University of Basel



- Discretisation of time evolution
- Diffusivity from discrete space
- Solving the Boltzmann equ.
- 3D MHD
- Approximations for ν−transport

SCFA Summerschool 2010, St. Luc

Computer Representation of Equations

UNI BASEL

Equation to be solved:

$$\frac{\partial y}{\partial t} = f\left(y\right)$$

Euler forward differencing:

$$\frac{y\left(t + \Delta t\right) - y\left(t\right)}{\Delta t} = f\left(y\left(t\right)\right)$$

Simple & fast!

Euler backward differencing:

$$\frac{y\left(t + \Delta t\right) - y\left(t\right)}{\Delta t} = f\left(y\left(t + \Delta t\right)\right)$$

Stable & expensive!

(--> implicit demo)

Finite differencing of time evolution

Forward differencing:
evaluate slope with current state vector

Backw. differencing:
evaluate slope with future state

Let’s see...

Explicit finite differencing

Forward differencing: evaluate slope with current state vector

- simple
- accurate for small time steps
- limited by charac-teristic time scale

Go faster...

Explicit finite differencing

Forward differencing: evaluate slope with current state vector

- simple
- inaccurate for large time steps
- even catastrophic!

Think...

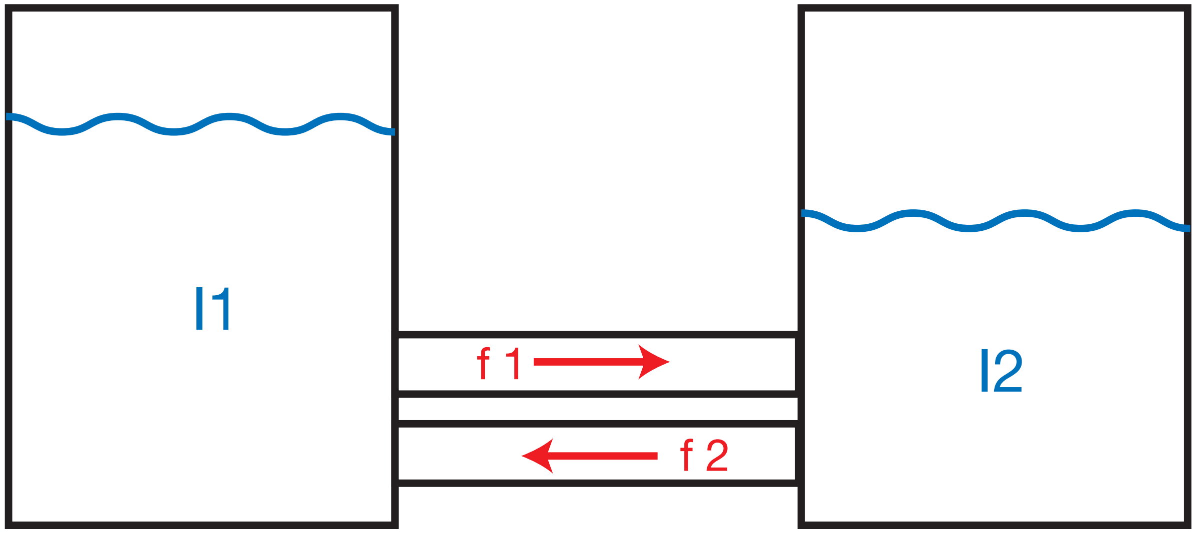
Implicit finite differencing

Backw. differencing: evaluate slope with future state vector

- Long time steps possible
- Follows ‘average’ evolution

- nonlinear system
- computationally expensive!

Finite Differencing in Time



$$\frac{\partial I_1}{\partial t} = -\frac{I_1}{\tau} + \frac{I_2}{\tau} = -\frac{I_1 - I_2}{\tau}$$

$$\frac{\partial I_2}{\partial t} = +\frac{I_1}{\tau} - \frac{I_2}{\tau} = +\frac{I_1 - I_2}{\tau}$$

Explicit finite differencing restricts time step to fastest process, $\Delta t < \tau$

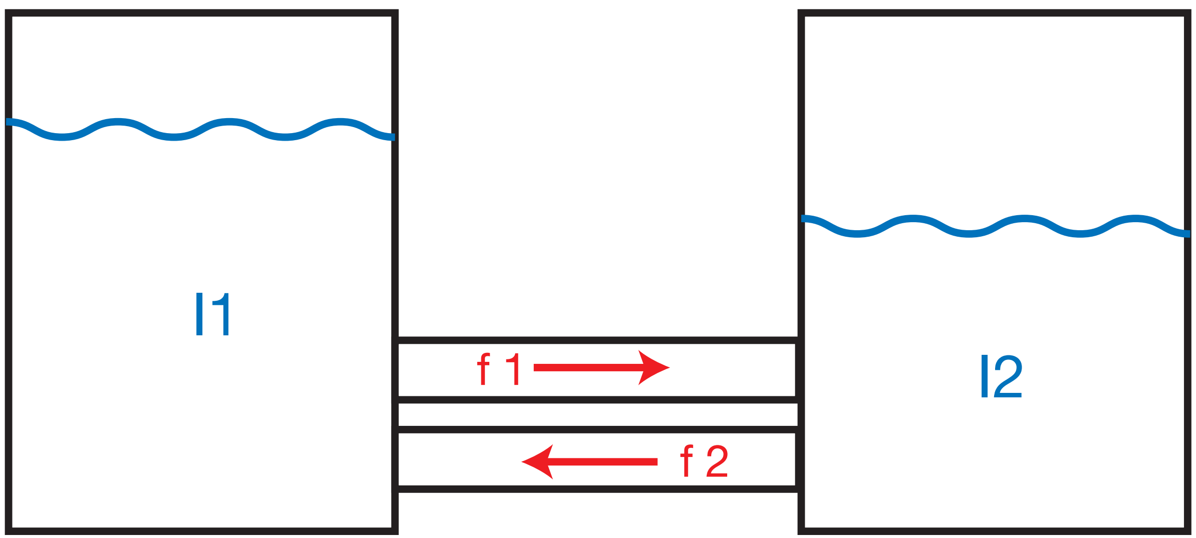
Implicit FD ist unconditionally stable

Analytical solution: x decays exponentially with time

← explicit FD

← implicit FD

Finite Differencing in Time



$$\frac{\partial I_1}{\partial t} = -\frac{I_1}{\tau} + \frac{I_2}{\tau} = -\frac{I_1 - I_2}{\tau}$$

$$\frac{\partial I_2}{\partial t} = +\frac{I_1}{\tau} - \frac{I_2}{\tau} = +\frac{I_1 - I_2}{\tau}$$

$$\frac{\partial}{\partial t}(I_1 + I_2) = 0$$

$$\frac{\partial}{\partial t}(I_1 - I_2) = -\frac{2}{\tau}(I_1 - I_2)$$

Explicit finite differencing restricts time step to fastest process, $\Delta t < \tau$

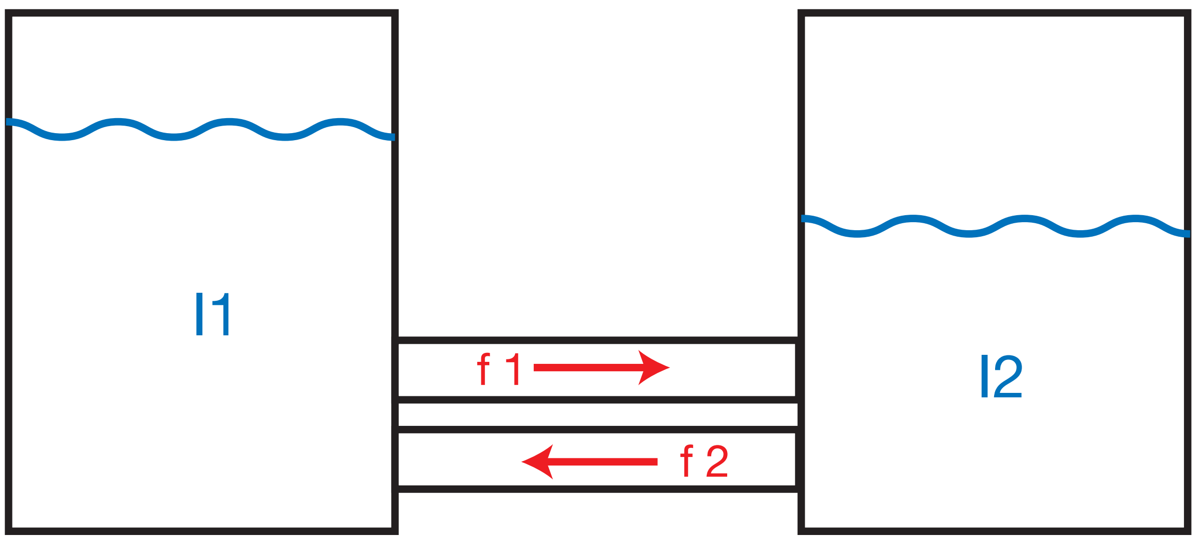
Implicit FD ist unconditionally stable

Analytical solution: x decays exponentially with time

← explicit FD

← implicit FD

Finite Differencing in Time



$$\frac{\partial I_1}{\partial t} = -\frac{I_1}{\tau} + \frac{I_2}{\tau} = -\frac{I_1 - I_2}{\tau}$$

$$\frac{\partial I_2}{\partial t} = +\frac{I_1}{\tau} - \frac{I_2}{\tau} = +\frac{I_1 - I_2}{\tau}$$

$$\frac{\partial}{\partial t}(I_1 + I_2) = 0$$

$$\frac{\partial}{\partial t}(I_1 - I_2) = -\frac{2}{\tau}(I_1 - I_2)$$

$$x := I_1 - I_2 \Rightarrow \frac{x^{n+1} - x^n}{\Delta t} = -\frac{2}{\tau} x^n \Rightarrow \left|\frac{x^{n+1}}{x^n}\right| > 1 \text{ if } \Delta t > \tau$$

$$\frac{x^{n+1} - x^n}{\Delta t} = -\frac{2}{\tau} x^{n+1} \Rightarrow \left|\frac{x^{n+1}}{x^n}\right| < 1 \checkmark$$

Explicit finite differencing restricts time step to fastest process, $\Delta t < \tau$

Implicit FD ist unconditionally stable

Analytical solution: x decays exponentially with time

← explicit FD

← implicit FD

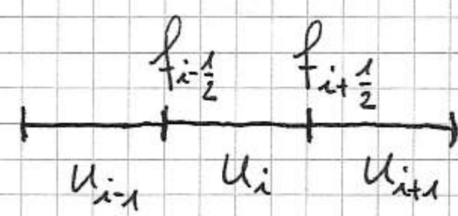
Finite Differencing in Space

$u(t,x)$ ... function of time and space

$a = \text{const.}$ ... advection speed

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(a u) = 0$$

Discretisation



$$(*) \qquad \frac{u_i^{n+1} - u_i^n}{\Delta t} + \frac{f_{i+\frac{1}{2}} - f_{i-\frac{1}{2}}}{\Delta x} = 0$$

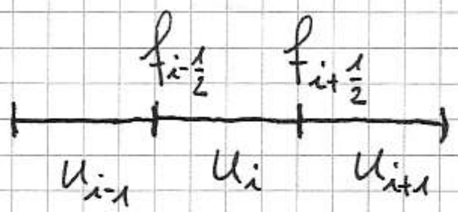
Finite Differencing in Space

$u(t,x)$ ... function of time and space

$a = \text{const.}$ ... advection speed

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(a u) = 0$$

Discretisation



$$(*) \qquad \frac{u_i^{n+1} - u_i^n}{\Delta t} + \frac{f_{i+\frac{1}{2}} - f_{i-\frac{1}{2}}}{\Delta x} = 0$$

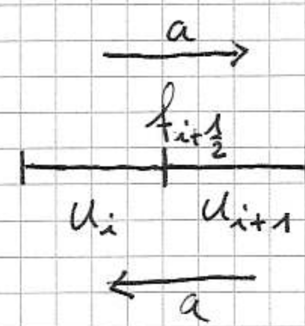
Naive choice of fluxe on cell-interfaces

$$f_{i+\frac{1}{2}} = a \, \frac{1}{2}(u_{i+1} + u_i)$$

accurate, but
not stable!

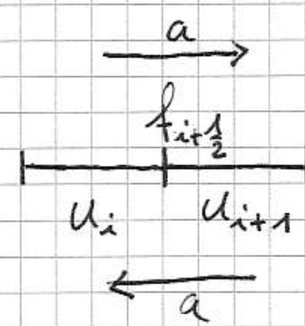
Upwind Differencing

$$f_{i+\frac{1}{2}} = \begin{cases} a u_i & \text{if } a > 0 \\ a u_{i+1} & \text{if } a \leq 0 \end{cases}$$



Upwind Differencing

$$f_{i+\frac{1}{2}} = \begin{cases} a u_i & \text{if } a > 0 \\ a u_{i+1} & \text{if } a \leq 0 \end{cases}$$



Same f written differently:

$$f_{i+\frac{1}{2}} = a \frac{1}{2}(u_{i+1} + u_i) - |a| \frac{1}{2}(u_{i+1} - u_i)$$

accurate choice

stabilising correction

Artificial Diffusivity

Insert into equation (*)

$$\frac{u_i^{n+1}-u_i^n}{\Delta t} + a\,\frac{\frac{1}{2}(u_{i+1}+u_i)-\frac{1}{2}(u_i+u_{i-1})}{\Delta x} - |a|\,\frac{\frac{1}{2}(u_{i+1}-u_i)-\frac{1}{2}(u_i-u_{i-1})}{\Delta x} = 0$$

$$\underbrace{\frac{u_i^{n+1}-u_i^n}{\Delta t} + a\,\frac{u_{i+1}-u_{i-1}}{2\Delta x}}_{\text{2nd order accurate}} - \underbrace{\frac{|a|\Delta x}{2}}_{\text{diffusivity depending on advection speed and resolution}} \cdot \frac{u_{i+1}-2u_i+u_{i-1}}{\Delta x^2} = 0$$

Important to compare numerical diffusivity with physical diffusivity!

Implement Conservation Laws!

If the resolution/ diffusivity cannot meet the physical requirements?

-> Conservation laws make errors local and set bounds to their size.

-> Always guarantee that finite differenced equations guarantee conservation laws!

Implement Conservation Laws!

Conservation laws:

- Baryon number
- Lepton number
- Energy
- Momentum
- Magnetic flux

Conditions:

- Nuclear statistical equilibrium (NSE)
- Charge neutrality
- Detailed balance
- div(B) = 0

Conservation laws are for computational physicists what ropes are for the rock climber: First you think you can survive by just being careful,...

If the resolution/ diffusivity cannot meet the physical requirements?

-> Conservation laws make errors local and set bounds to their size.

-> Always guarantee that finite differenced equations guarantee conservation laws!

Implement Conservation Laws!

… but in astrophysics you always meet the situation where they are indispensable!

If the resolution/ diffusivity cannot meet the physical requirements?

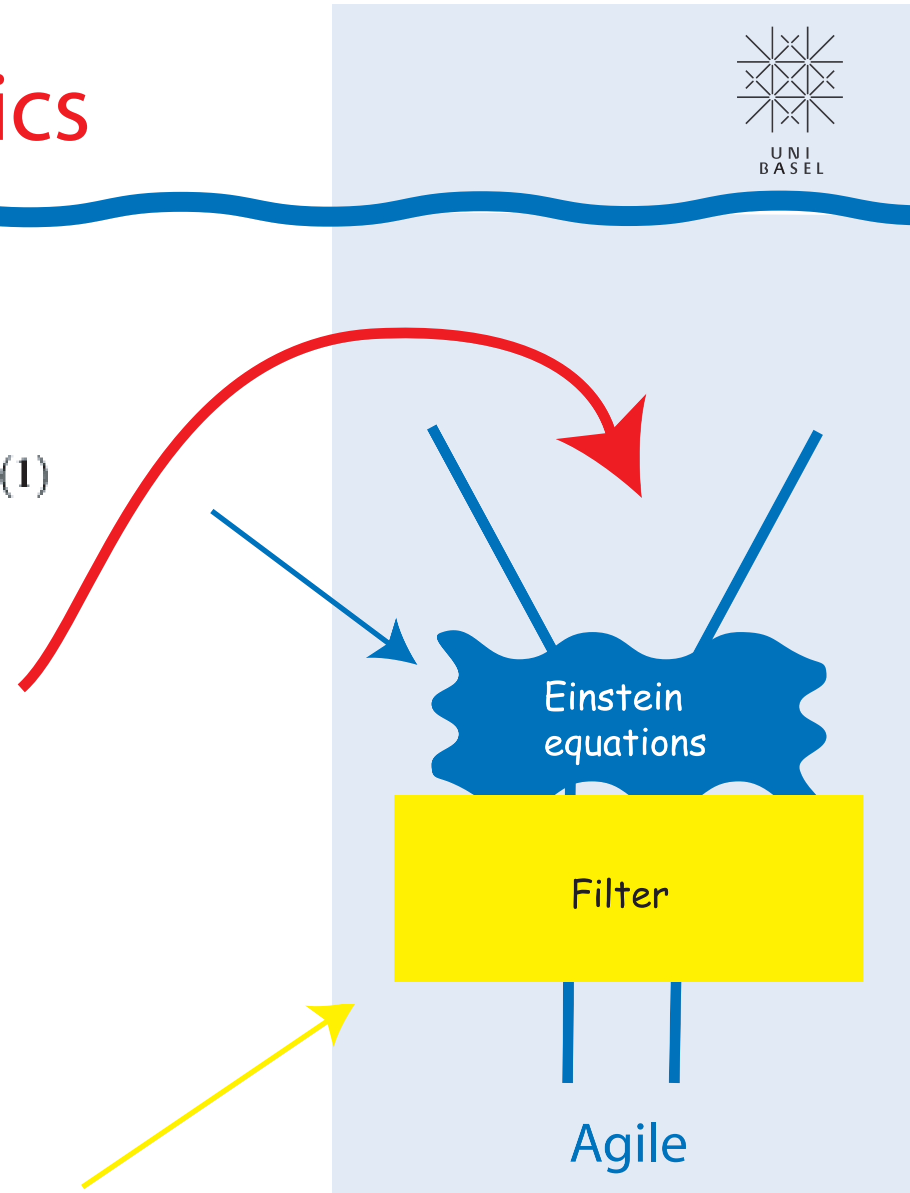
-> Conservation laws make errors local and set bounds to their size.

-> Always guarantee that finite differenced equations guarantee conservation laws!

Physics <--> Model <--> Observation

Physics	Challenges	Astrophysical modelling:
Reaction network	• uncertainties • stiff partial diff'eqs.	local
Magneto-hydrodynamics	• resolution • time scales	semi-local
Gravity	• NR: elliptic equations • GR: metric/horizons	non-local
Radiative transfer	• dimensionality • non-locality	

Modeling: hydrodynamics



Metric in spherical symmetry:

$$ds^2 = -\alpha^2 dt^2 + \left(\frac{r'}{\Gamma}\right)^2 da^2 + r^2\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right), \quad (1)$$

Stress-energy tensor:

$$T^{tt} = \rho(1 + e + J),$$

$$T^{ta} = T^{at} = \rho H,$$

$$T^{aa} = p + \rho K,$$

$$T^{\vartheta\vartheta} = T^{\varphi\varphi} = p + \frac{1}{2}\rho(J - K). \quad (2)$$

Conservation quantities:

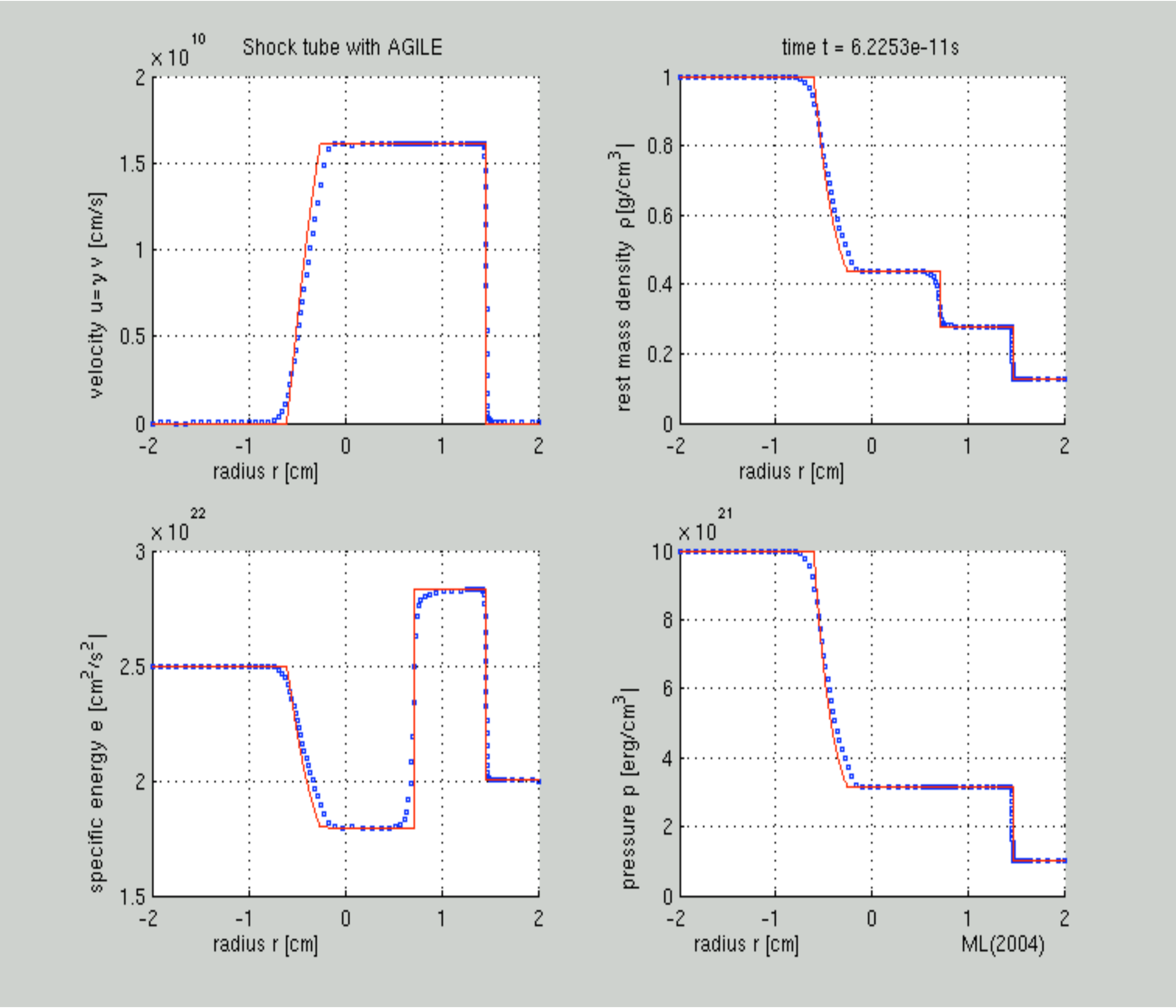
$$\frac{1}{D} = \frac{\Gamma}{\rho}, \quad (3)$$

$$\tau = \Gamma(e + J) + \frac{2}{\Gamma + 1}\left(\frac{1}{2}u^2 - \frac{m}{r}\right) + uH, \quad (4)$$

$$\frac{\partial}{\partial t}\left(\frac{1}{D}\right) = \frac{\partial}{\partial a}\left(4\pi r^2 \alpha u\right), \quad (6)$$

$$\frac{\partial \tau}{\partial t} = -\frac{\partial}{\partial a}\left[4\pi r^2 \alpha\left(up + u\rho K + \Gamma\rho H\right)\right], \quad (7)$$

Hydrodynamics: Adaptive Mesh Refinement



AGILE:

- general relativistic hydrodynamics
- concentric shells
- adaptive mesh
- 2nd order TVD
- fully implicit

(Liebendörfer et al. 2002)

Methods of neutrino transport

Multi-Group Flux-Limited Diffusion (MGFLD)

- solution of one equation for f(t,r,E)
- flux limiter required
- flux factor by geometric estimate
- no characteristics

(Arnett 1966, Myra & Bludman 1989, Bruenn 1985/2001)

+ efficient
+ accurate @ opaque
- ad hoc @ transparent
- multi-D extension?

Methods of neutrino transport

Multi-Group Flux-Limited Diffusion (MGFLD)
- solution of one equation for f(t,r,E)
- flux limiter required
- flux factor by geometric estimate
- no characteristics

(Arnett 1966, Myra & Bludman 1989, Bruenn 1985/2001)

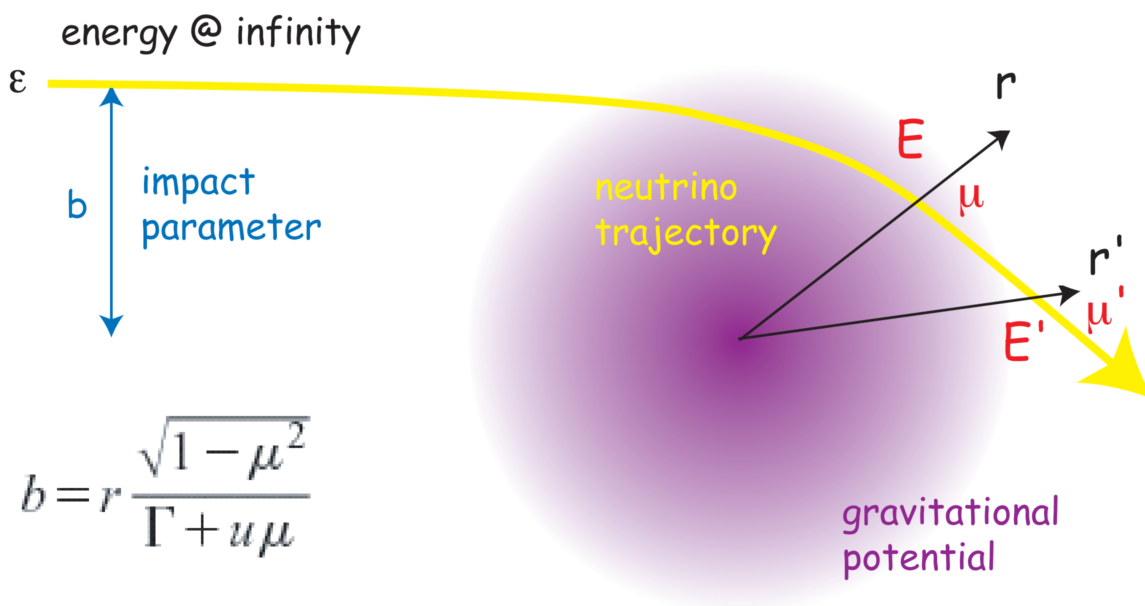
+ efficient
+ accurate @ opaque
- ad hoc @ transparent
- multi-D extension?

Discrete ordinate (from neutron transport)
- solution of one equation for f(t,r,m,E)
- tuned for important expectation values
- short characteristics

(Wilson 1971, Mezzacappa & Bruenn 1993, Liebendörfer et al. 2001/2004, Sumiyoshi et al. 2005)

+ comprehensive
+ local (adaptive)
- expectation values
- angular resolution

Spherical Boltzmann transport



$$b = r\frac{\sqrt{1-\mu^2}}{\Gamma + u\mu}$$

$$\varepsilon = (\Gamma + u\mu)E.$$

distribution function $f(t, r, b, \varepsilon)$

$$\frac{\partial f}{\alpha \partial t} + \mu \Gamma \frac{\partial f}{\partial r} \approx \Omega(f)$$

partial derivatives at constant b, ε

(comoving frame --> Lindquist, Ann. Phys. 1966)

Comoving metric:

$$ds^2 = -\alpha^2 dt^2 + \left(\frac{1}{\Gamma}\frac{\partial r}{\partial a}\right)^2 + r^2\left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right)$$

Stress-energy tensor:

$$T^{tt} = \rho\left(1 + e + J\right)$$
$$T^{ta} = T^{at} = \rho H$$
$$T^{aa} = p + \rho K$$
$$T^{\vartheta\vartheta} = T^{\varphi\varphi} = p + \frac{1}{2}\rho\left(J - K\right)$$

Radiation moments:

$$J = \frac{4\pi}{(hc)^3}\int F d\mu E^2 dE$$
$$H = \frac{4\pi}{(hc)^3}\int F \mu d\mu E^2 dE$$
$$K = \frac{4\pi}{(hc)^3}\int F \mu^2 d\mu E^2 dE$$

Solving the Boltzmann equation

$$\frac{\partial F}{\alpha c \partial t} + \frac{\partial \left(4\pi r^2 \alpha \rho \mu F\right)}{\alpha \partial m}$$

$$= \frac{j}{\rho} - \tilde{\chi} F$$

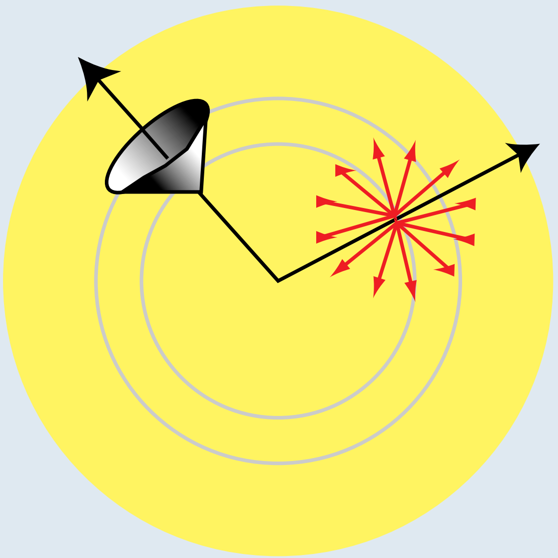
$$\frac{\partial Y_e}{\partial t} = -\frac{2\pi m_B}{h^3 c^2} \int E^2 dE d\mu \left(\frac{j}{\rho} - \tilde{\chi} F\right) \quad \frac{\partial e}{\partial t} = \dots \quad \frac{\partial u}{\partial t} = \dots$$

(Mezzacappa & Bruenn 1993, Liebendörfer 2000, Liebendörfer et al. 2004)

Evolution of specific neutrino distr. function:

F(t,m,μ,E) = f(t,r,μ,E)/ρ

=> 3D implicit problem



Solving the Boltzmann equation

$$\frac{\partial F}{\alpha c \partial t} + \frac{\partial \left(4\pi r^2 \alpha \rho \mu F\right)}{\alpha \partial m}$$

$$\begin{aligned}
= \ & \frac{j}{\rho} - \tilde{\chi} F + \frac{1}{h^3 c^4} E^2 \int d\mu' R_{is}\left(\mu, \mu', E\right) F\left(\mu', E\right) \\
- \ & \frac{1}{h^3 c^4} E^2 F \int d\mu' R_{is}\left(\mu, \mu', E\right) \\
+ \ & \frac{1}{h^3 c^4} \left[\frac{1}{\rho} - F\left(\mu, E\right)\right] \int E'^2 dE' d\mu' \tilde{R}^{in}_{nes}\left(\mu, \mu', E, E'\right) F\left(\mu', E\right) \\
- \ & \frac{1}{h^3 c^4} F\left(\mu, E\right) \int E'^2 dE' d\mu' \tilde{R}^{out}_{nes}\left(\mu, \mu', E, E'\right) \left[\frac{1}{\rho} - F\left(\mu', E'\right)\right]
\end{aligned}$$

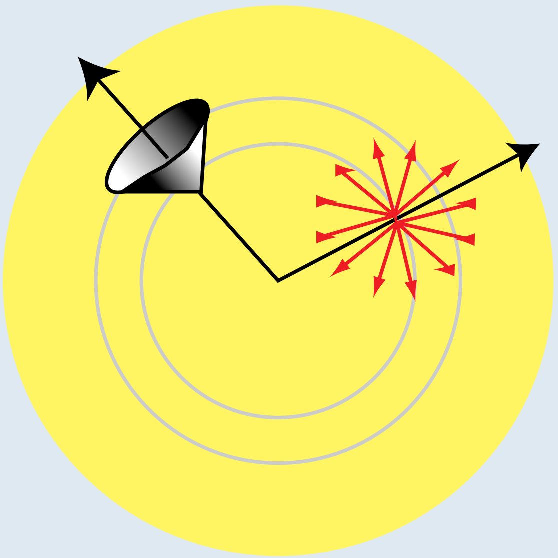
$$\frac{\partial Y_e}{\partial t} = -\frac{2\pi m_B}{h^3 c^2} \int E^2 dE d\mu \left(\frac{j}{\rho} - \tilde{\chi} F\right) \quad \frac{\partial e}{\partial t} = \ldots \quad \frac{\partial u}{\partial t} = \ldots$$

(Mezzacappa & Bruenn 1993, Liebendörfer 2000, Liebendörfer et al. 2004)

Evolution of specific neutrino distr. function:

F(t,m,μ,E) = f(t,r,μ,E)/ρ

=> 3D implicit problem



Solving the Boltzmann equation

$$\frac{\partial F}{\alpha c \partial t} + \frac{\partial\left(4\pi r^2 \alpha\rho\mu F\right)}{\alpha \partial m} + \Gamma\left(\frac{1}{r} - \frac{\partial\alpha}{\alpha\partial r}\right)\frac{\partial\left[\left(1-\mu^2\right)F\right]}{\partial\mu}$$

$$+ \left[\qquad - \mu\Gamma\frac{\partial\alpha}{\alpha\partial r}\right]\frac{1}{E^2}\frac{\partial\left(E^3 F\right)}{\partial E}$$

$$= \frac{j}{\rho} - \tilde{\chi}F + \frac{1}{h^3c^4}E^2\int d\mu' R_{is}\left(\mu,\mu',E\right)F\left(\mu',E\right)$$

$$- \frac{1}{h^3c^4}E^2 F\int d\mu' R_{is}\left(\mu,\mu',E\right)$$

$$+ \frac{1}{h^3c^4}\left[\frac{1}{\rho} - F\left(\mu,E\right)\right]\int E'^2 dE' d\mu' \tilde{R}^{in}_{nes}\left(\mu,\mu',E,E'\right)F\left(\mu',E\right)$$

$$- \frac{1}{h^3c^4}F\left(\mu,E\right)\int E'^2 dE' d\mu' \tilde{R}^{out}_{nes}\left(\mu,\mu',E,E'\right)\left[\frac{1}{\rho} - F\left(\mu',E'\right)\right]$$

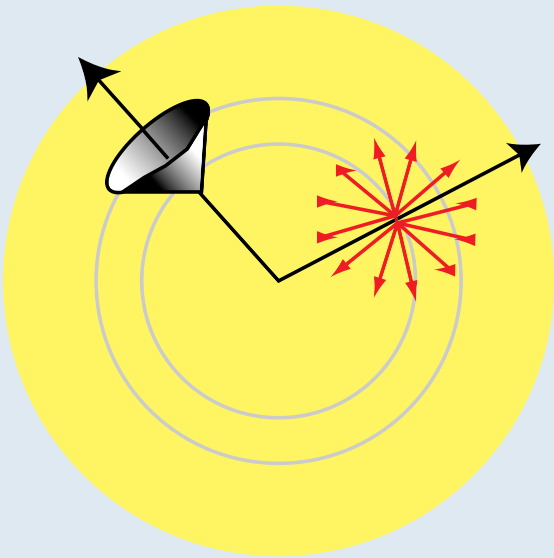
$$\frac{\partial Y_e}{\partial t} = -\frac{2\pi m_B}{h^3c^2}\int E^2 dE d\mu\left(\frac{j}{\rho} - \tilde{\chi}F\right) \quad \frac{\partial e}{\partial t} = \ldots \quad \frac{\partial u}{\partial t} = \ldots$$

(Mezzacappa & Bruenn 1993, Liebendörfer 2000, Liebendörfer et al. 2004)

Evolution of specific neutrino distr. function:

F(t,m,μ,E) = f(t,r,μ,E)/ρ

=> 3D implicit problem



Solving the Boltzmann equation

$$\frac{\partial F}{\alpha c \partial t} + \frac{\partial\left(4\pi r^2 \alpha\rho\mu F\right)}{\alpha \partial m} + \Gamma\left(\frac{1}{r} - \frac{\partial \alpha}{\alpha \partial r}\right)\frac{\partial\left[\left(1-\mu^2\right)F\right]}{\partial \mu}$$

$$+ \left(\frac{\partial \ln \rho}{\alpha c \partial t} + \frac{3}{r}\frac{u}{c}\right)\frac{\partial\left[\mu\left(1-\mu^2\right)F\right]}{\partial \mu}$$

$$+ \left[\mu^2\left(\frac{\partial \ln \rho}{\alpha c \partial t} + \frac{3}{r}\frac{u}{c}\right) - \frac{1}{r}\frac{u}{c} - \mu\Gamma\frac{\partial \alpha}{\alpha \partial r}\right]\frac{1}{E^2}\frac{\partial\left(E^3 F\right)}{\partial E}$$

$$= \frac{j}{\rho} - \tilde{\chi}F + \frac{1}{h^3c^4}E^2\int d\mu' R_{is}\left(\mu,\mu',E\right)F\left(\mu',E\right)$$

$$- \frac{1}{h^3c^4}E^2 F\int d\mu' R_{is}\left(\mu,\mu',E\right)$$

$$+ \frac{1}{h^3c^4}\left[\frac{1}{\rho} - F\left(\mu,E\right)\right]\int E'^2 dE' d\mu' \tilde{R}^{in}_{nes}\left(\mu,\mu',E,E'\right)F\left(\mu',E\right)$$

$$- \frac{1}{h^3c^4}F\left(\mu,E\right)\int E'^2 dE' d\mu' \tilde{R}^{out}_{nes}\left(\mu,\mu',E,E'\right)\left[\frac{1}{\rho} - F\left(\mu',E'\right)\right]$$

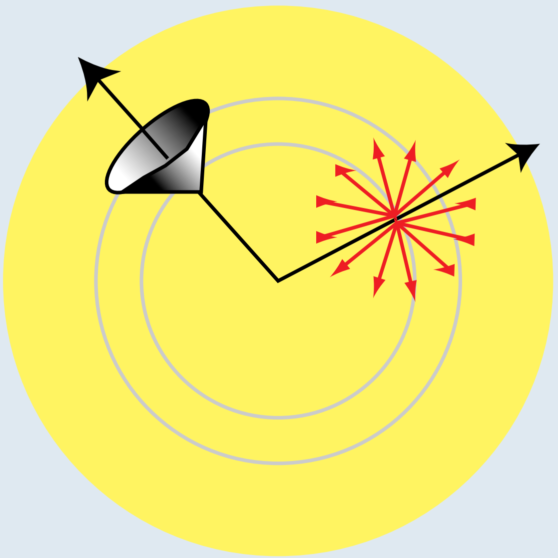
$$\frac{\partial Y_e}{\partial t} = -\frac{2\pi m_B}{h^3c^2}\int E^2 dE d\mu\left(\frac{j}{\rho} - \tilde{\chi}F\right) \qquad \frac{\partial e}{\partial t} = \ldots \qquad \frac{\partial u}{\partial t} = \ldots$$

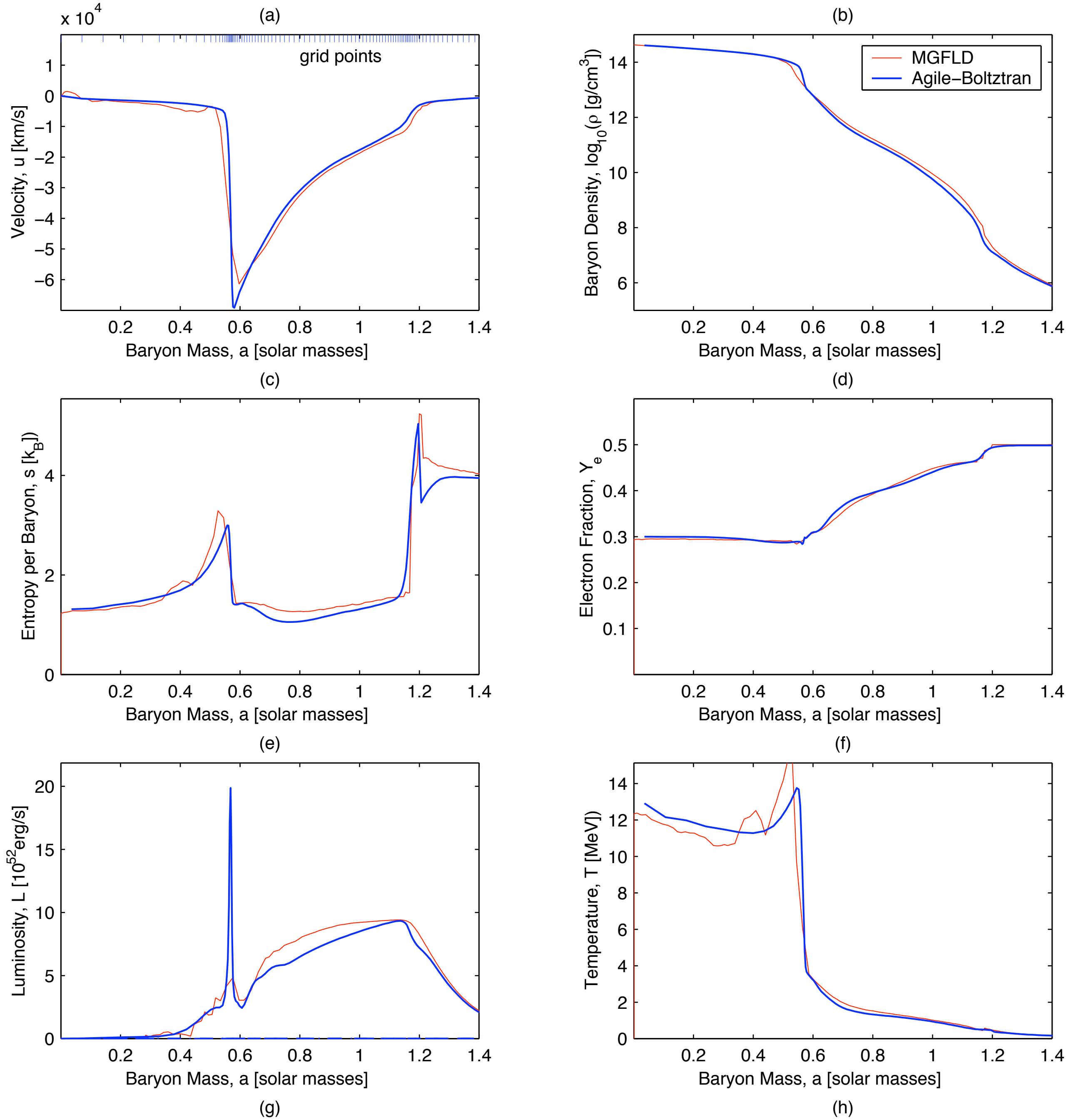
(Mezzacappa & Bruenn 1993, Liebendörfer 2000, Liebendörfer et al. 2004)

Evolution of specific neutrino distr. function:

F(t,m,μ,E) = f(t,r,μ,E)/ρ

=> 3D implicit problem





(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
x 10^4
grid points
Velocity, u [km/s]
Baryon Density, log10(ρ [g/cm^3])
MGFLD
Agile–Boltztran
Entropy per Baryon, s [kB]
Electron Fraction, Ye
Luminosity, L [10^52 erg/s]
Temperature, T [MeV]
Baryon Mass, a [solar masses]

Methods of neutrino transport

UNI BASEL

Multi-Group Flux-Limited Diffusion (MGFLD)
- solution of one equation for f(t,r,E)
- flux limiter required
- flux factor by geometric estimate
- no characteristics

(Arnett 1966, Myra & Bludman 1989, Bruenn 1985/2001)

+ efficient
+ accurate @ opaque
- ad hoc @ transparent
- multi-D extension?

Discrete ordinate (from neutron transport)
- solution of one equation for f(t,r,m,E)
- tuned for important expectation values
- short characteristics

(Wilson 1971, Mezzacappa & Bruenn 1993, Liebendörfer et al. 2001/2004, Sumiyoshi et al. 2005)

+ comprehensive
+ local (adaptive)
- expectation values
- angular resolution

Variable Eddington Factor Method
- solution of moments equation
- separate calculation of Eddington factor
- long characteristics

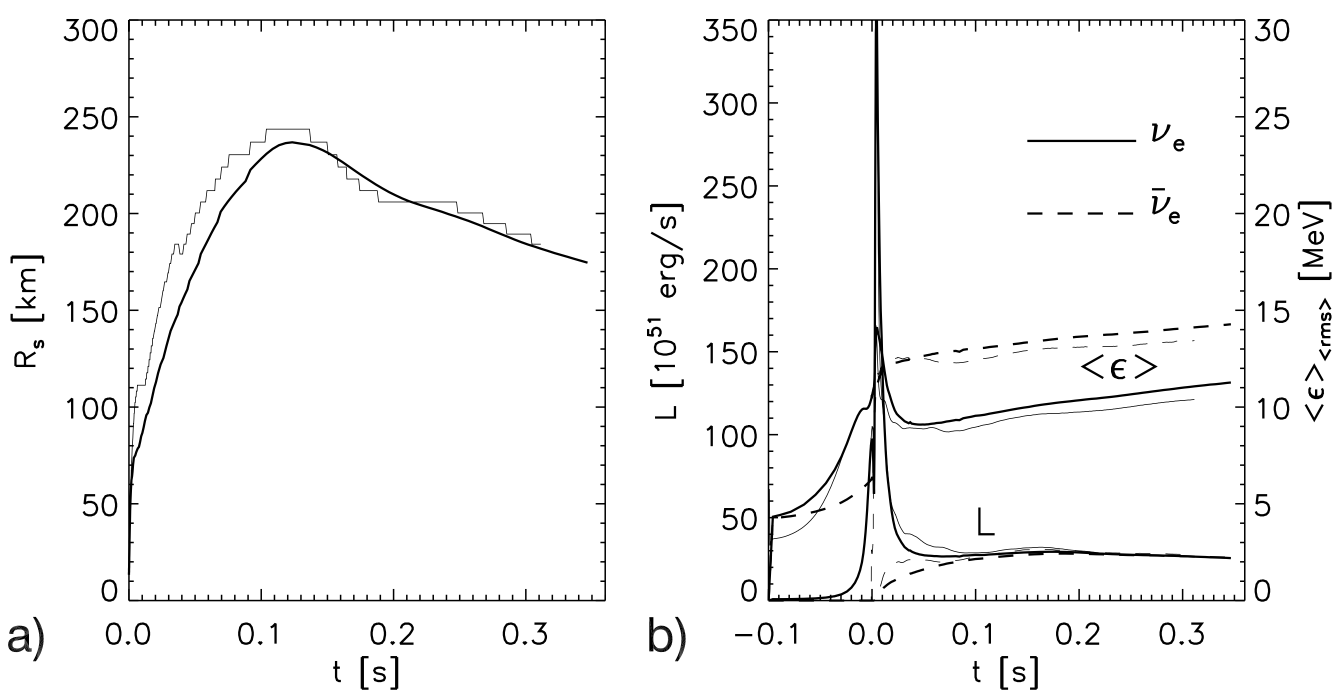
(Burrows et al. 2000, Rampp & Janka 2000/2002, Thompson et al. 2003)

+ angular resolution
+ expectation values
- many equations
- global connection

Boltzmann transport comparison

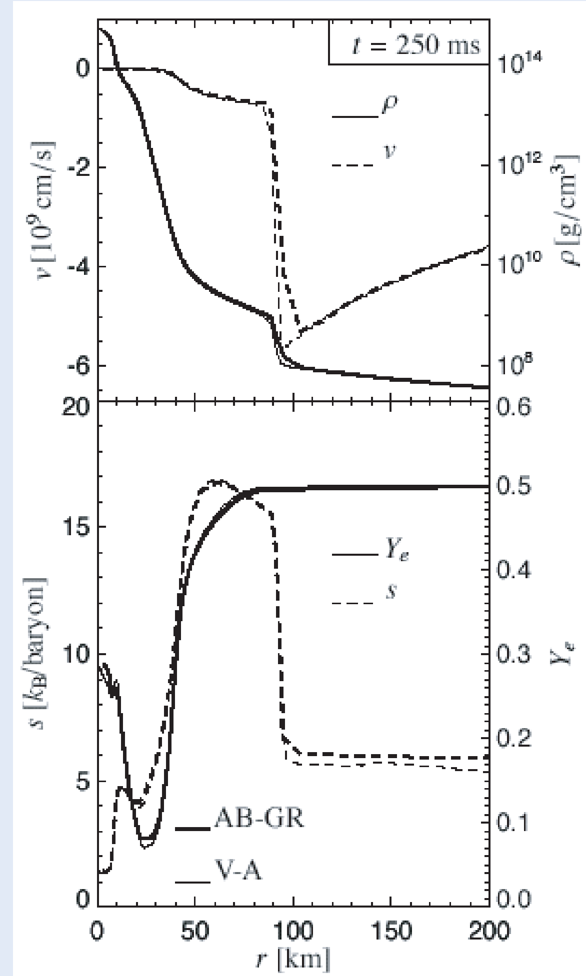
Comparison of spherically symmetric simulations: Oak Ridge/Basel group and Garching group

Liebendörfer, Rampp, Janka, Mezzacappa, ApJ 620 (2005)



datafiles.tar.gz of simulation in ApJ electronic edition

excellent agreement:



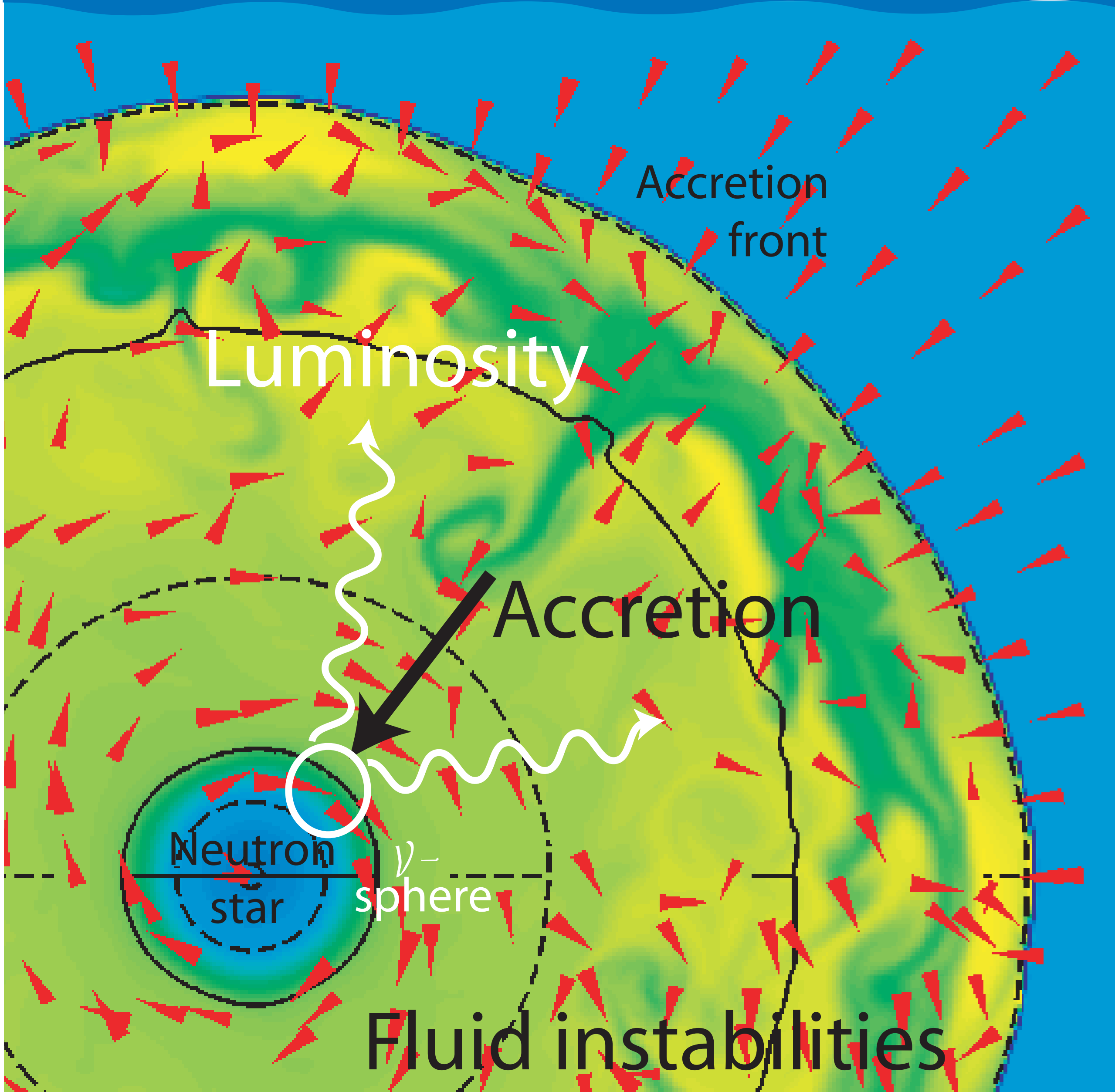
(Marek et al., A&A 2006)

SN model minimum requirements

- shock-proof hydrodynamics
  - energy conservation for dissipation on small scales
- radial GR effects (as in TOV equation)
- transport of electron neutrino and antineutrino
  - hydrodynamic limit of neutrino gas (pdV term!)
  - comoving-frame diffusion limit
  - spectral decoupling in semi-transparent regime
  - non-local determination of flux factor
- emission of μ/τ neutrinos and antineutrinos
- equation of state
  - advection of composition at low density
  - nuclear statistical equilibrium at high density
- dominant weak interactions in each phase
  - accurate detailed balance
  - implicit finite differencing to obtain equilibria

- Good approximations can be more accurate if the full problem is computationally very challenging
- But, it is difficult to quantify their accuracy without a solution of the full problem

Multi-dimensional radiation-hydrodyn.



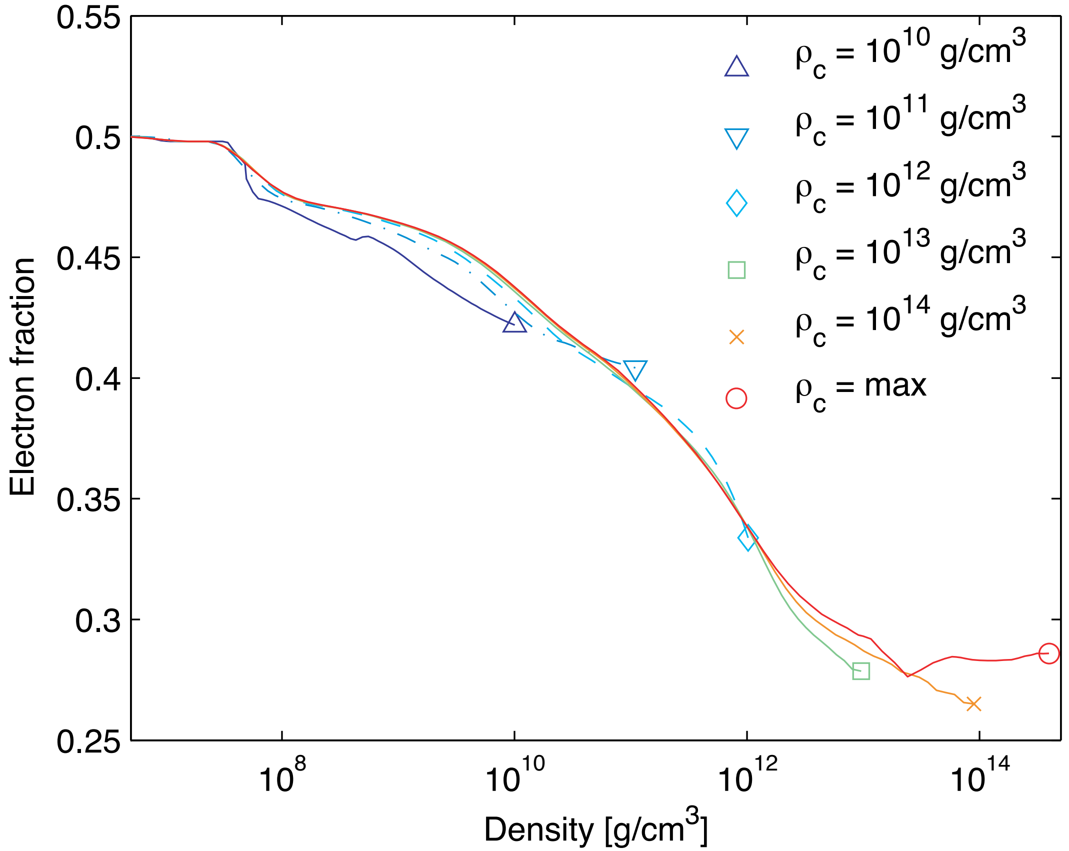
- accretion controls >50% of luminosity
- up- or downflow absorb and emit neutrinos differently
- luminosity controls the accretion rate

=> 3D nonlocal transport problem: full solution not feasable!

Buras et al. (2003-2005)
Livne et al. (2004)
Walder et al. (2004)
Cardall & Mezzacappa (2005)
Fryer & Warren (2004)
Myra & Swesty (2005)

Parameterised ν-physics before bounce

Electron fraction in spherical runs can be parameterised



Entropy changes and neutrino stress can be derived:

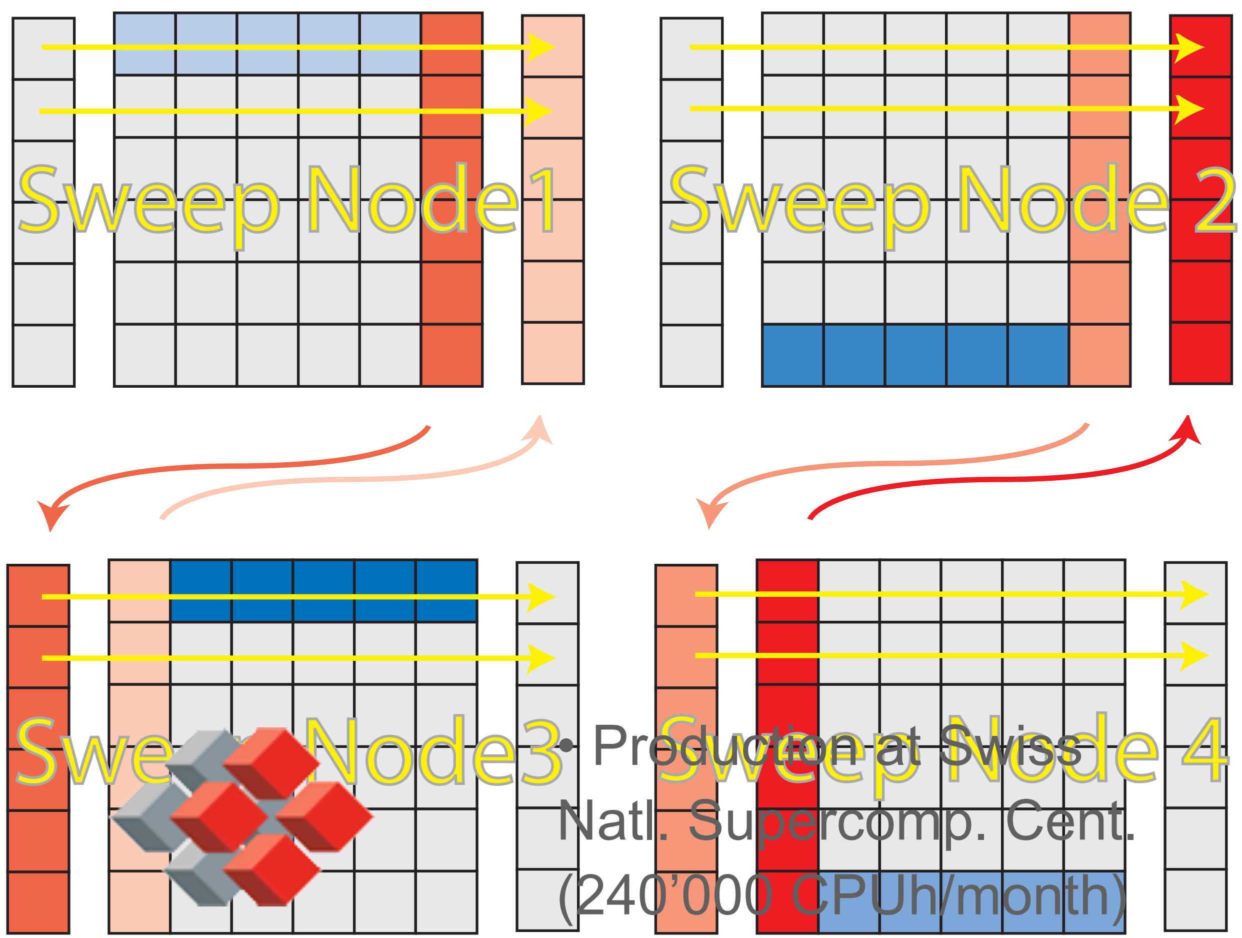
$$\frac{\Delta s}{\Delta t} = -\frac{\Delta Y_e}{\Delta t}\frac{\mu_e - \mu_n + \mu_p - E_\nu^{esc}}{T},$$

(~10MeV)

- Simple to implement (compared to neutrino transport...)
- Computationally very efficient!
- Performs well for collapse and bounce
- Not applicable in postbounce phase!

Liebendörfer 2005

Parallelisation of 3D MHD code



- Production at Swiss Natl. Supercomp. Cent. (240'000 CPUh/month)

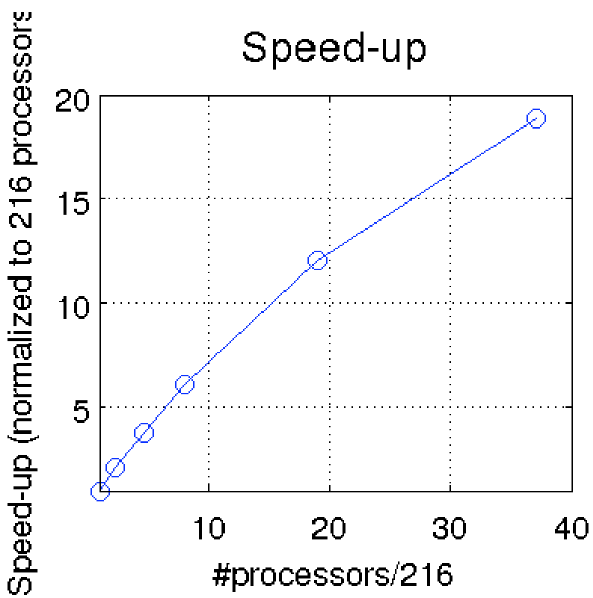
FISH code, Käppeli, Whitehouse, Pen, Liebendörfer, arXiv:0910.2854

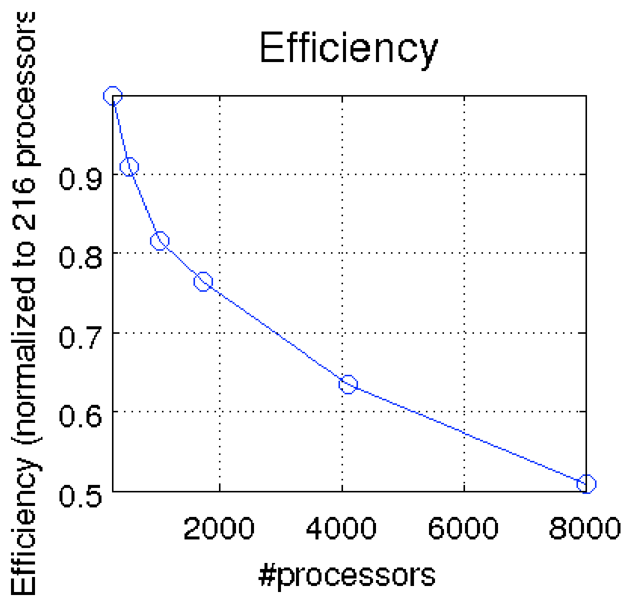
(Pen et al. 2003; Käppeli et al. 200X)

Communications for y-sweep

- 3D cubic domain decomposition (MPI)
- Directional operator-splitting
- Physical x-, y-, z-sweeps by data rotation
- Data consecutively loaded and stored into and from cache
- OpenMP applicable to parallelise sweeps on one node

Parallelisation of 3D MHD code



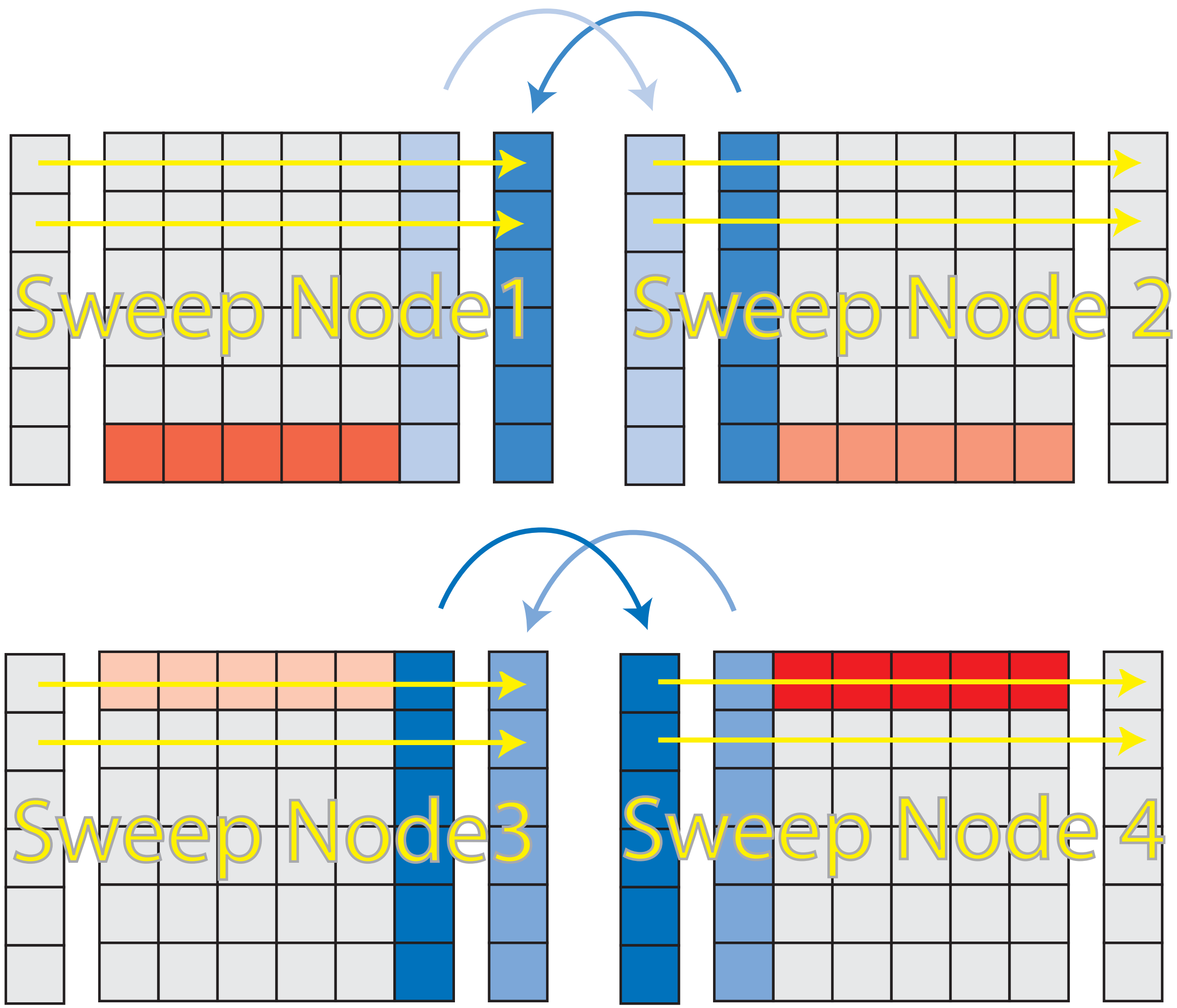


- 3D cubic domain decomposition (MPI)
- Directional operator-splitting
- Physical x-, y-, z-sweeps by data rotation
- Data consecutively loaded and stored into and from cache
- OpenMP applicable to parallelise sweeps on one node

- Production at Swiss Natl. Supercomp. Cent. (240’000 CPUh/month)

FISH code, Käppeli, Whitehouse, Pen, Liebendörfer, arXiv:0910.2854

Parallelisation of 3D MHD code



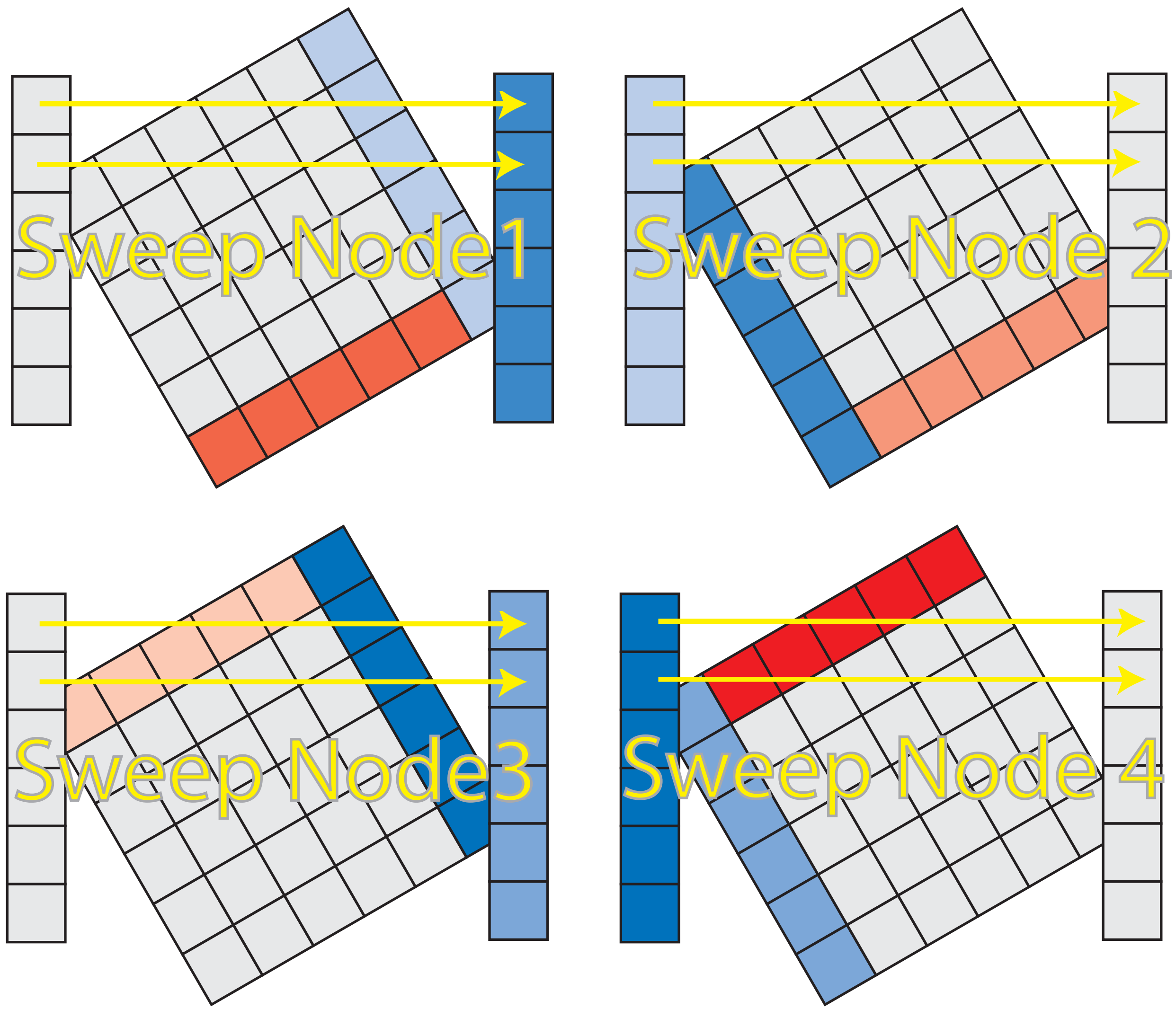
(Pen et al. 2003, Käppeli et al. 200X)

Communications for x-sweep

- 3D cubic domain decomposition (MPI)
- Directional operator-splitting
- Physical x-, y-, z-sweeps by data rotation
- Data consecutively loaded and stored into and from cache
- OpenMP applicable to parallelise sweeps on one node

Parallelisation of 3D MHD code

UNI BASEL

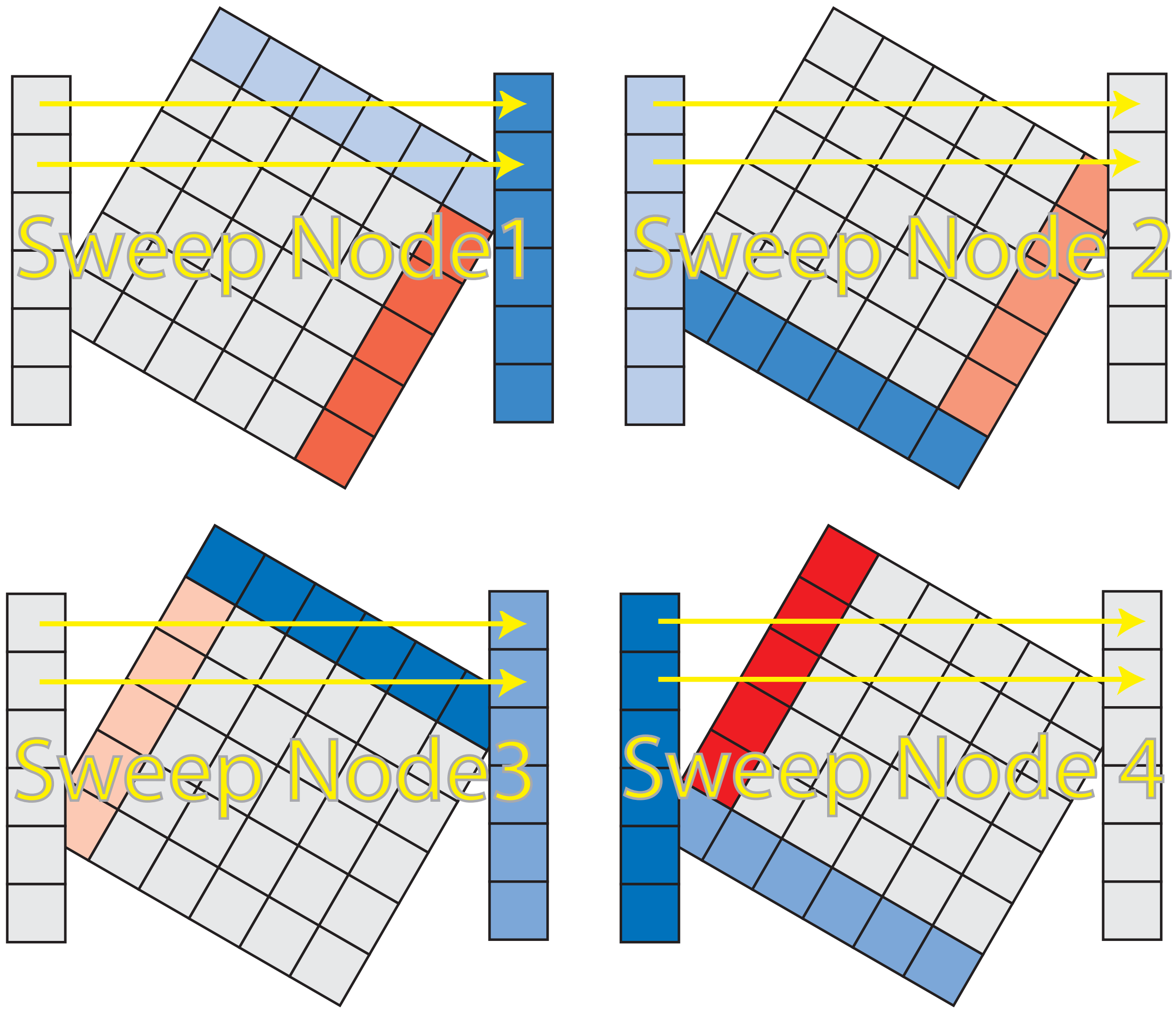


(Pen et al. 2003, Käppeli et al. 200X)

Communications for x-sweep?

- 3D cubic domain decomposition (MPI)
- Directional operator-splitting
- Physical x-, y-, z-sweeps by data rotation
- Data consecutively loaded and stored into and from cache
- OpenMP applicable to parallelise sweeps on one node

Parallelisation of 3D MHD code

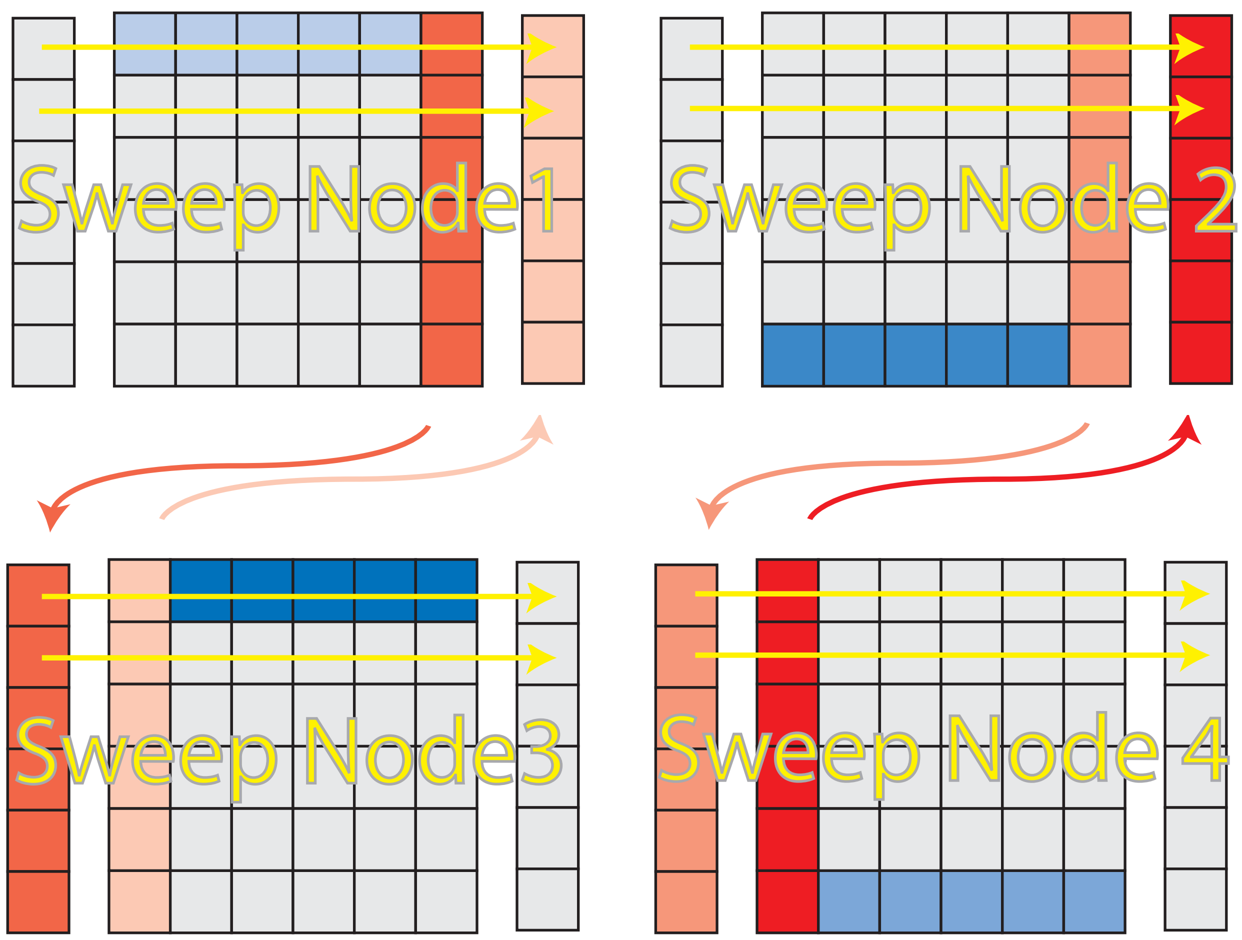


(Pen et al. 2003, Käppeli et al. 200X)

Communications for y-sweep?

- 3D cubic domain decomposition (MPI)
- Directional operator-splitting
- Physical x-, y-, z-sweeps by data rotation
- Data consecutively loaded and stored into and from cache
- OpenMP applicable to parallelise sweeps on one node

Parallelisation of 3D MHD code



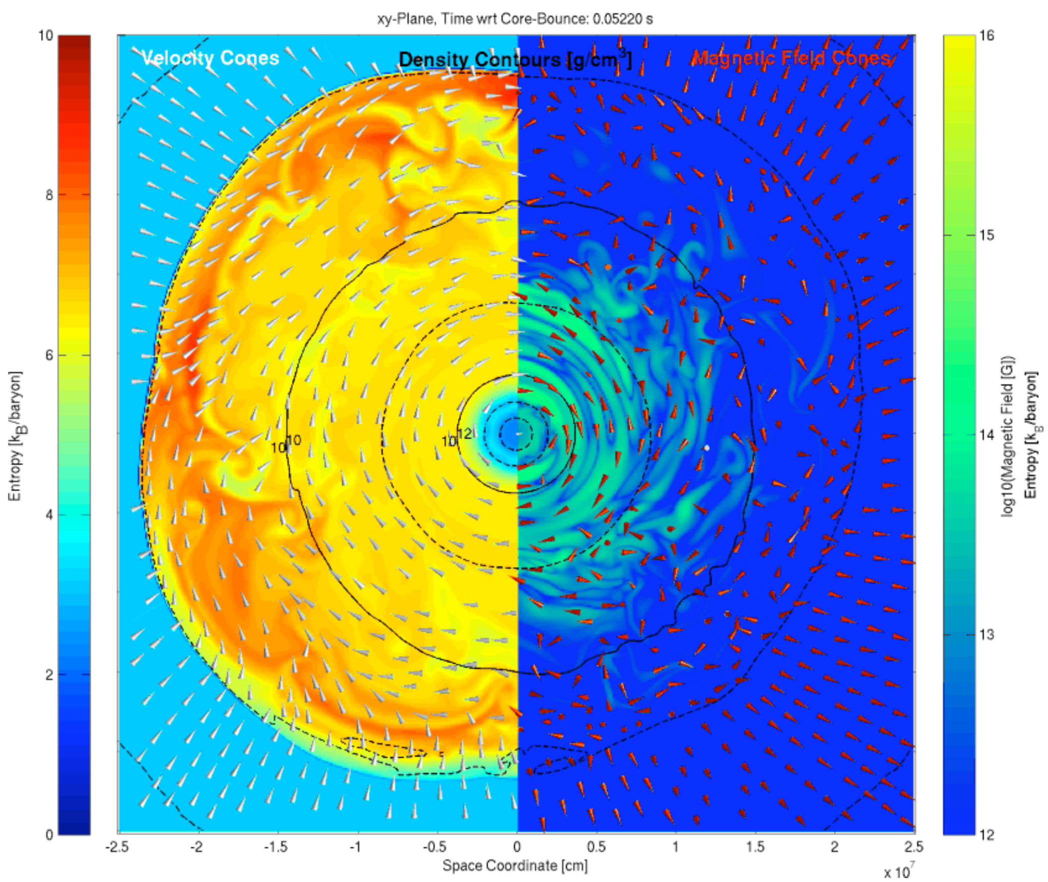
(Pen et al. 2003, Käppeli et al. 200X)

Communications for y-sweep

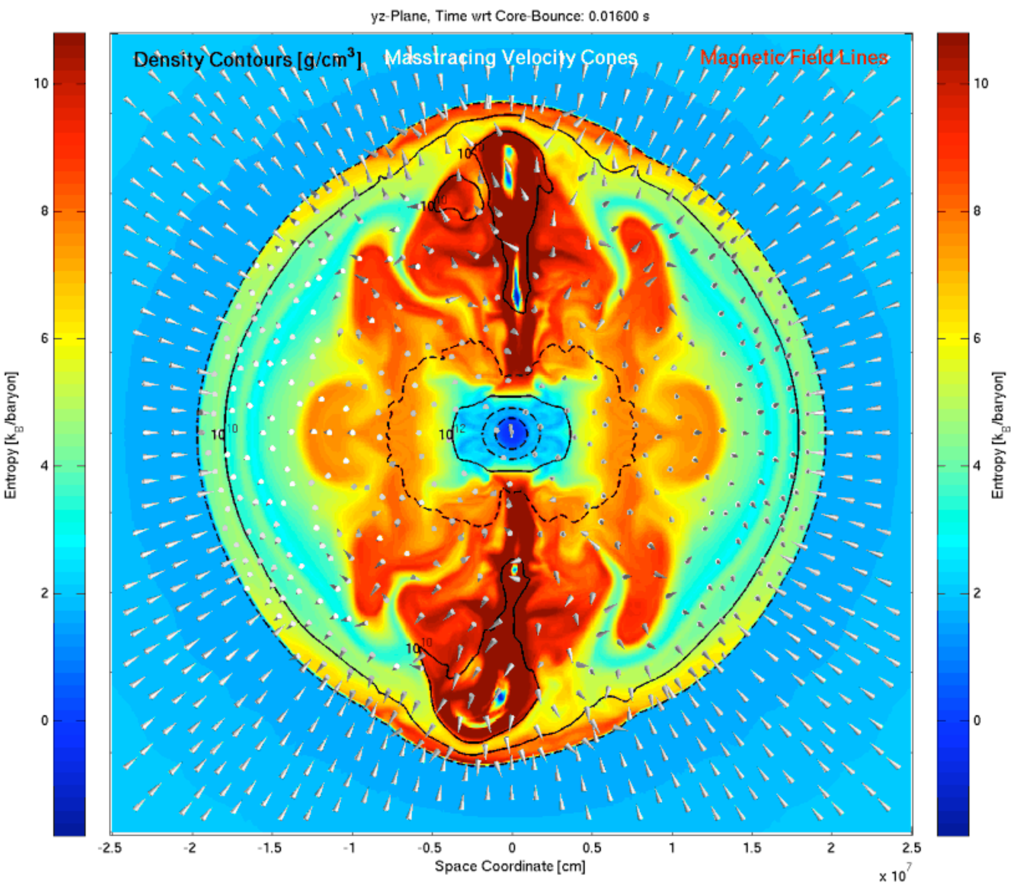
- 3D cubic domain decomposition (MPI)
- Directional operator-splitting
- Physical x-, y-, z-sweeps by data rotation
- Data consecutively loaded and stored into and from cache
- OpenMP applicable to parallelise sweeps on one node

(-->movie)

Experimental 3D magneto-rotational runs



Setup with weak toroidal field
--> winding
Liebendörfer et al. 2006

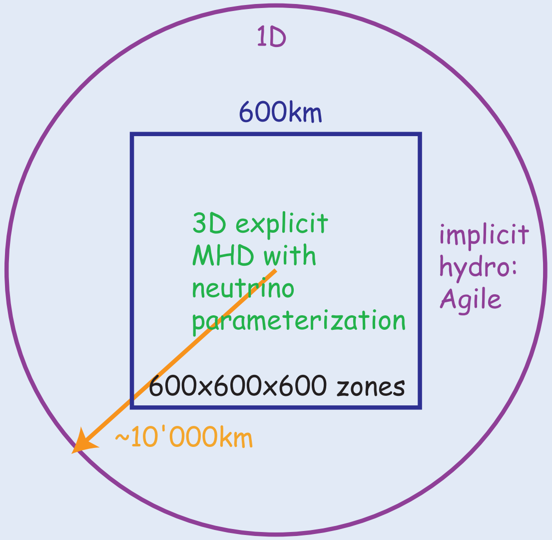


Setup with poloidal field as in Burrows et al. 2007,
--> jet
Käppeli, Scheidegger et al.

3D Magneto-Hydrodynamics

Elegant parallel hydrodynamics with approximate neutrino transport

- Lattimer-Swesty EoS
- Effective GR potential
- constrained transport
- 2nd order TVD
- e-flavour neutrinos



(Liebendörfer, Pen, Thompson 2006)

Pitfalls of multi-D Boltzmann ν-transport

Boltzmann transport:

- One fluid elemement contains
  4 ν types x 20 energies x 100 angles = 8000 variables
- At a resolution of 1000^3 zones
  --> 64TB per time step

Hydrodynamics:

- One fluid element contains ~10 variables
- At a resolution of 1000^3 zones
  --> 80GB per step

Pitfalls of multi-D Boltzmann ν-transport

Boltzmann transport:

- One fluid elemement contains
  4 ν types x 20 energies x 100 angles = 8000 variables
- At a resolution of 1000^3 zones
  --> 64TB per time step

Compression of Fermi-gas:

$$\frac{dF}{dt} - \frac{1}{3E^2}\frac{\partial}{\partial E}\left(E^3 \rho F\right)\frac{d}{dt}\left(\frac{1}{\rho}\right) - \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{c\lambda}{3}\frac{\partial F}{\partial r}\right) = \left(\frac{dF}{dt}\right)_{collision}$$

de pdV diffusion = interactions

Hydrodynamics:

- One fluid element contains ~10 variables
- At a resolution of 1000^3 zones
  --> 80GB per step

difficult energy-terms must not be neglected!

Pitfalls of multi-D Boltzmann ν-transport

UNI BASEL

Boltzmann transport:

- One fluid elemement contains
  4 ν types x 20 energies x 100 angles = 8000 variables
- At a resolution of 1000^3 zones
  --> 64TB per time step

Compression of Fermi-gas:

$$\frac{dF}{dt} - \frac{1}{3E^2}\frac{\partial}{\partial E}\left(E^3 \rho F\right)\frac{d}{dt}\left(\frac{1}{\rho}\right) - \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{c\lambda}{3}\frac{\partial F}{\partial r}\right) = \left(\frac{dF}{dt}\right)_{collision}$$

de pdV diffusion = interactions

Diffusion limit:

$$\frac{\lambda}{3}\frac{\partial F}{\partial r} \ll F, \qquad \frac{H}{cJ} \sim 10^{-4}, \qquad H = \int_{-1}^{+1} F(\mu)\,\mu d\mu$$

Hydrodynamics:

- One fluid element contains ~10 variables
- At a resolution of 1000^3 zones
  --> 80GB per step

difficult energy-terms must not be neglected!

Inaccurate fluxes in diffusion-regime due to large cancellations in angle integral!

There is no perfect transport algorithm...

	Diffusive regime	Semi-transparent	Transparent regime
Boltzmann solver	Truncation errors in flux		Inefficient ang. resol.
Flux-limited diffusion		Flux-factor estimated	Flux-factor unknown
Ray-tracing	Short mean free path	Limited by reaction rates	

- Variable Eddington Factor method
  successful in 2D but very computationally expensive!
  (Rampp & Janka, Buras et al. 2002-5)

- Grey diffusion in one regime and grey transparent elsewhere
  successful in 3D but not accurate enough!
  (e.g. Fryer & Warren 2004)

- Multi-Group Flux-Limited diffusion
  difficulty of local flux limiters & multi-D
  (e.g. Arnett 1966, Bruenn 1985,...)

There is no perfect transport algorithm...

UNI BASEL

	Diffusive regime	Semi-transparent	Transparent regime
Boltzmann solver	Truncation errors in flux		Inefficient ang. resol.
Flux-limited diffusion		Flux-factor estimated	Flux-factor unknown
Ray-tracing	Short mean free path	Limited by reaction rates	

New three-dimensional simulations based on the
Isotropic Diffusion Source Approximation (IDSA)
Liebendörfer, Whitehouse, Fischer (2009)

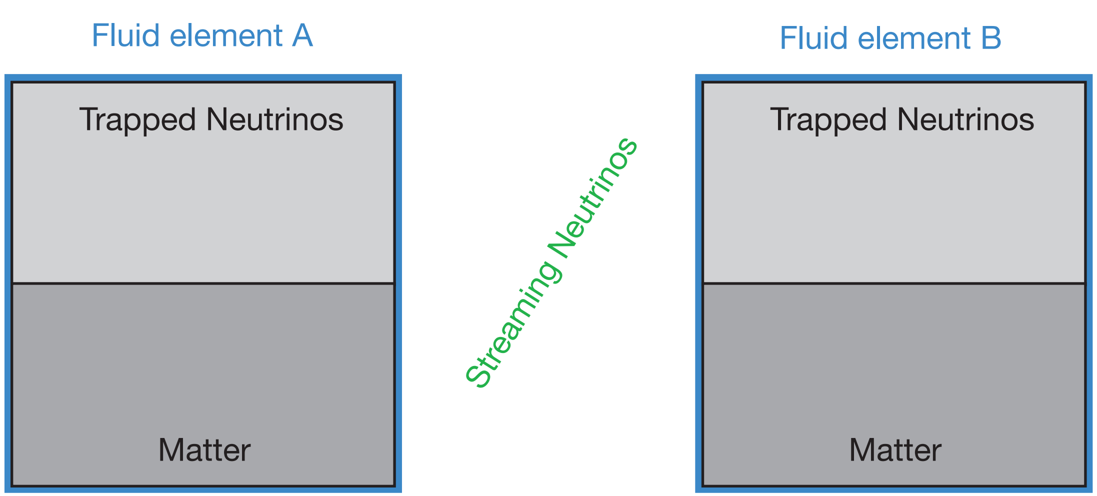
- Variable Eddington Factor method
  successful in 2D but very computationally expensive!
  (Rampp & Janka, Buras et al. 2002-5)
- Grey diffusion in one regime and grey transparent elsewhere
  successful in 3D but not accurate enough!
  (e.g. Fryer & Warren 2004)
- Multi-Group Flux-Limited diffusion
  difficulty of local flux limiters & multi-D
  (e.g. Arnett 1966, Bruenn 1985,...)

Spectral neutrino transport after bounce

$D(f) = j - \chi^* f$

$f = f(\text{trapped}) + f(\text{streaming}) = ft + fs$

Different approx.
for trapped & streaming
neutrino components!



I sotropic
D iffusion
S ource
A pproximation

(Liebendörfer, Whitehouse, Fischer 2007)

Spectral neutrino transport after bounce

$D(f) = j - \chi^* f$

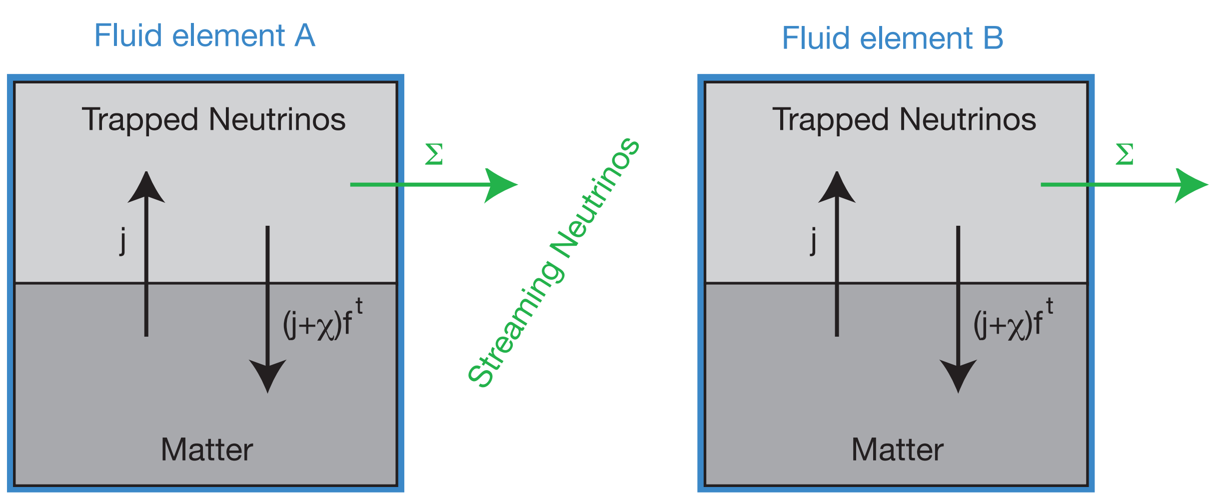
$f = f(\text{trapped}) + f(\text{streaming}) = ft + fs$

$D(ft) = j - \chi^* ft - \Sigma$ (1)

$D(fs) = - \chi^* fs + \Sigma$ (2)

Different approx.
for trapped & streaming
neutrino components!

Σ determined by diffusion limit of (1)



I sotropic
D iffusion
S ource
A pproximation

(Liebendörfer, Whitehouse, Fischer 2007)

Spectral neutrino transport after bounce

$D(f) = j - \chi^* f$

f = f(trapped) + f(streaming) = ft + fs

$D(ft) = j - \chi^* ft - \Sigma$ (1)

$D(fs) = - \chi^* fs + \Sigma$ (2)

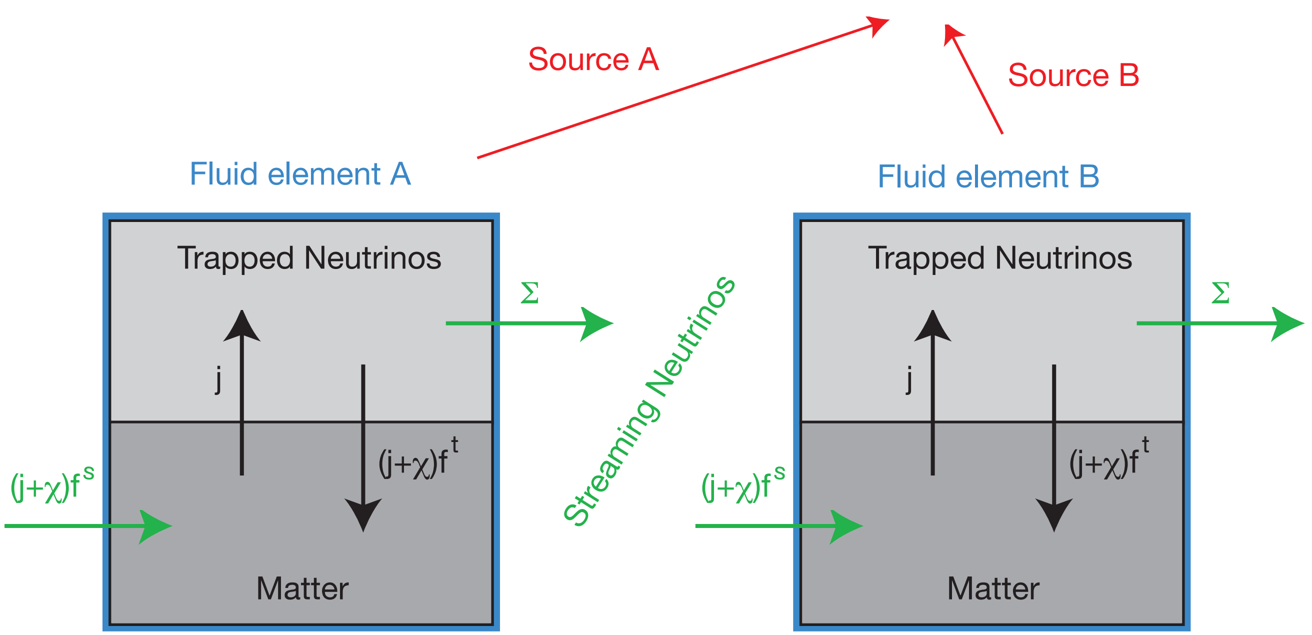
Different approx. for trapped & streaming neutrino components!

I sotropic
D iffusion
S ource
A pproximation

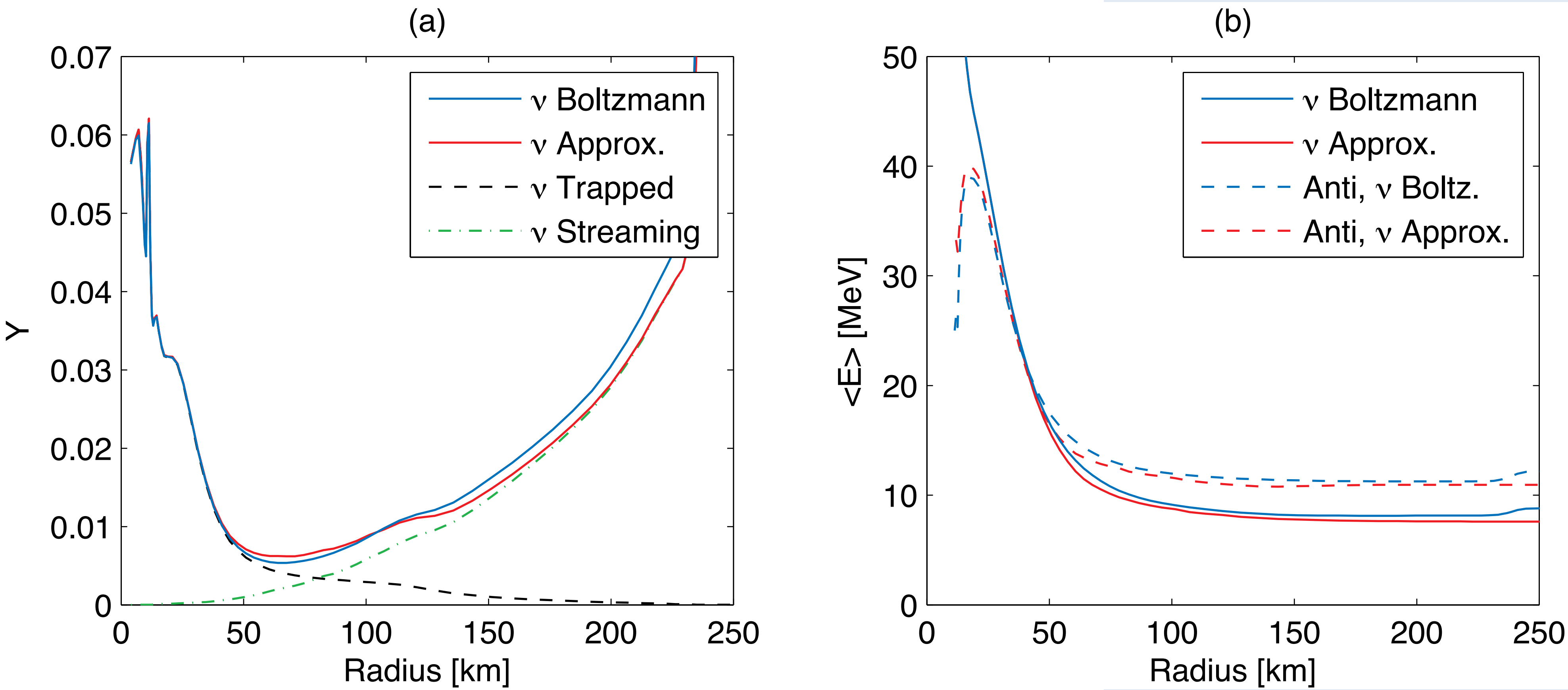
(Liebendörfer, Whitehouse, Fischer 2007)

Σ determined by diffusion limit of (1)

Stationary state approx. for (2) --> Poisson Eq.



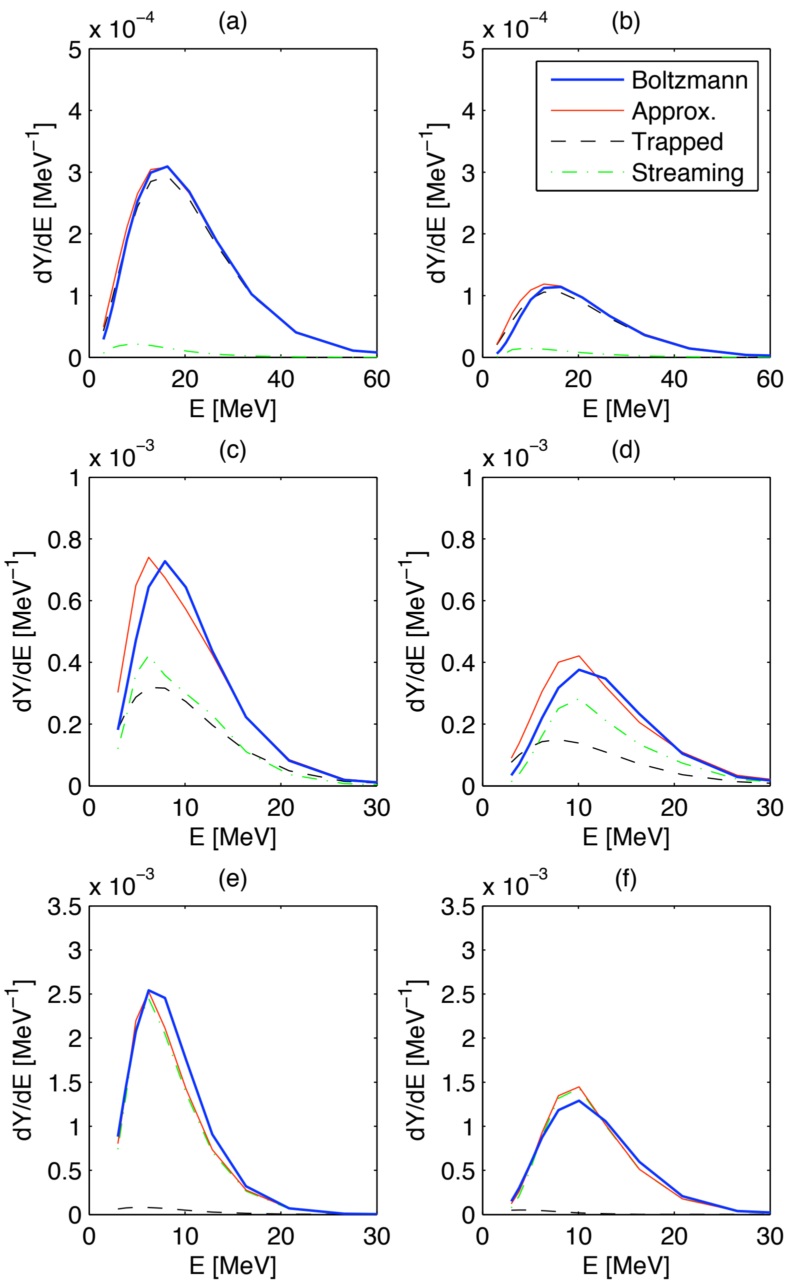
IDSA <--> Boltzmann



- *trapped neutrinos at center*
- *transition to streaming neutrinos toward surface*
- *sum of both compared to Boltzmann simulation*

Net neutrino abundance and mean energy

Comparison of IDSA Spectra



at 40 km radius
(trapped regime)

Trapped neutrinos dominate spectrum

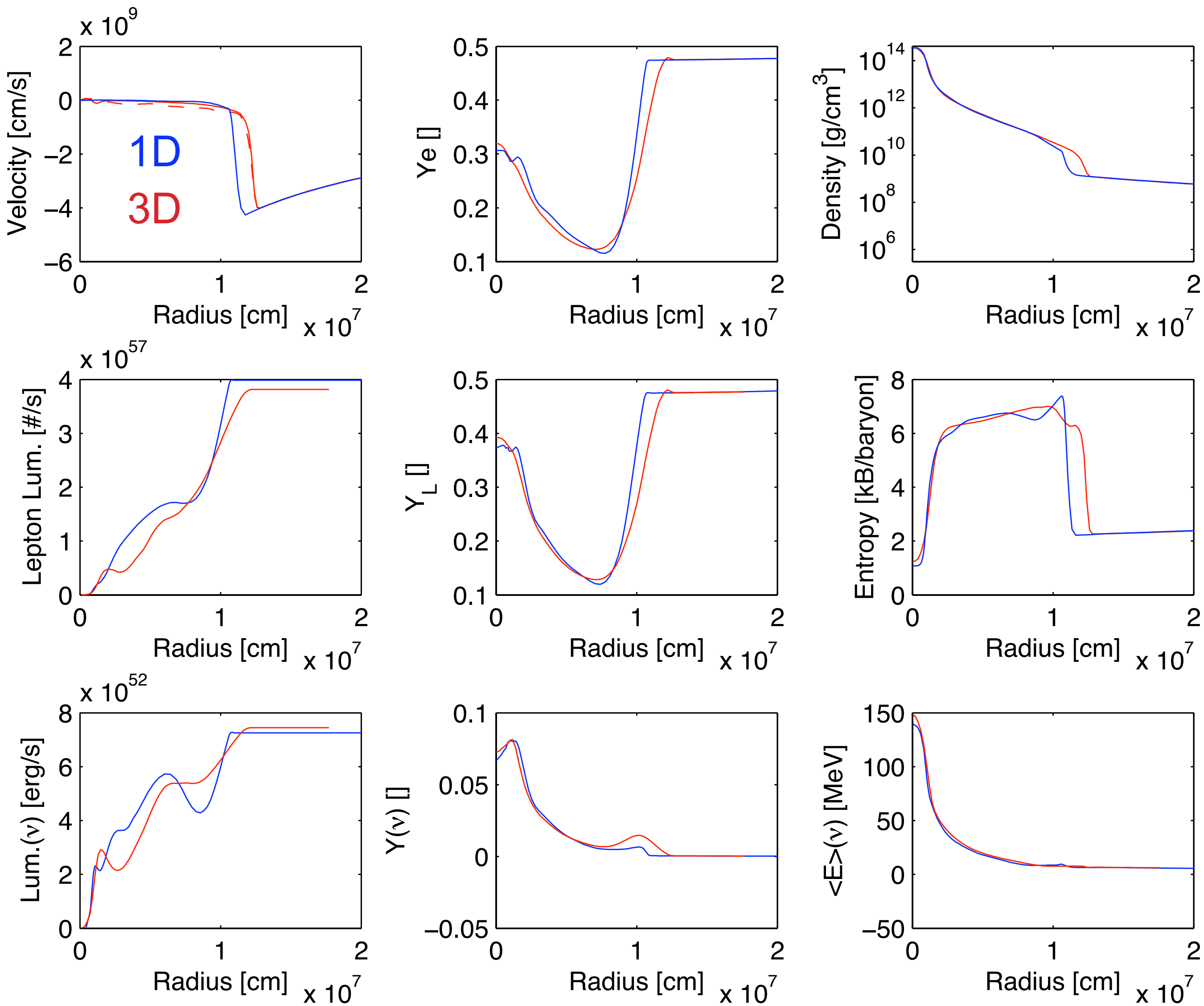
at 80 km radius
(semi-transparent)

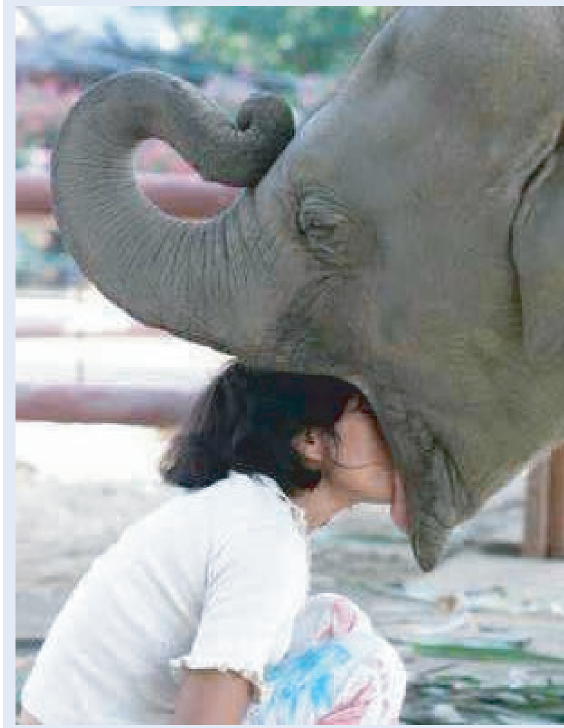
Trapped *and* streaming neutrinos form spectrum

at 160 km radius
(free streaming)

Streaming neutrinos dominate spectrum

Checking the 3D Elephant code...





20 ms postbounce, same input physics

Physics <--> Model <--> Observation

Physics <--> Model <--> Observation

UNI BASEL

Spherical Symmetry:

- Excellent ν-transport with detailed input physics
- 5 different codes give consistent results!

  Bruenn et al. 2001, Liebendörfer et al. 2001-5, Rampp & Janka 2000-2, Thompson et al. 2003, Sumiyoshi et al. 2005-7

- No explosions obtained for most progenitors, exploring neutrino interactions & nuclear physics

Physics <--> Model <--> Observation

Spherical Symmetry:

- Excellent ν-transport with detailed input physics
- 5 different codes give consistent results!

Bruenn et al. 2001, Liebendörfer et al. 2001-5, Rampp & Janka 2000-2, Thompson et al. 2003, Sumiyoshi et al. 2005-7

Axisymmetry:

- ray-by-ray or MGFLD ν-transport
- computationally very expensive

Buras et al. 2003/5, Walder et al. 2004, Bruenn et al. 2006, Marek & Janka 2007, Ott et al. 2008

- No explosions obtained for most progenitors, exploring neutrino interactions & nuclear physics

- Some explosions obtained, results not yet converged --> likely and less likely explosion mechanisms

Physics <--> Model <--> Observation

UNI BASEL

Spherical Symmetry:

- Excellent ν-transport with detailed input physics
- 5 different codes give consistent results!

Bruenn et al. 2001, Liebendörfer et al. 2001-5, Rampp & Janka 2000-2, Thompson et al. 2003, Sumiyoshi et al. 2005-7

Axisymmetry:

- ray-by-ray or MGFLD ν-transport
- computationally very expensive

Buras et al. 2003/5, Walder et al. 2004, Bruenn et al. 2006, Marek & Janka 2007, Ott et al. 2008

Three-dimensional:

- ν-transport approximations
- enable 3D flow pattern & magnetic fields

Fryer & Warren 2002/4, Scheck et al. 2003, Ott et al. 2007, Scheidegger et al. 2008, Iwakami et al. 2008

- No explosions obtained for most progenitors, exploring neutrino interactions & nuclear physics
- Some explosions obtained, results not yet converged --> likely and less likely explosion mechanisms
- Phenomenology --> predictive power, coming up...