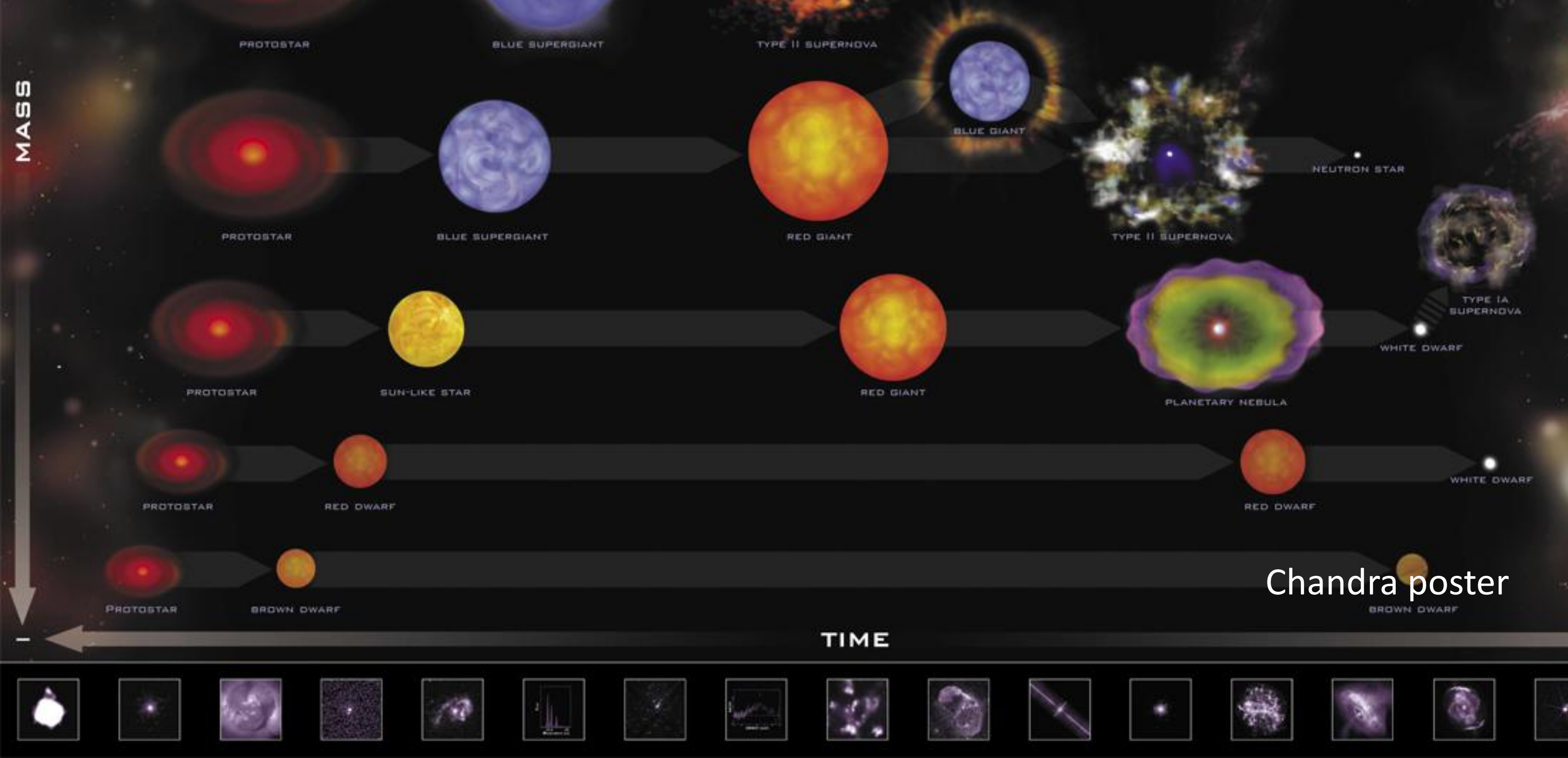
Stellar evolution in a nutshell

The main principles governing the evolution of stars



Chandra poster

STELLAR EVOLUTION IS NEEDED IN MANY TOPICAL PROBLEMS OF MODERN ASTROPHYSICS

- Supernovae and GRB progenitors, WD, NS and BH ptogenitors
- Physics of the objects used as standard candles
- The Starburst-AGN connection
- Chemical enrichment by the first generation of stars
- The evolution of the stellar populations in the high redshift galaxies
- Reionisation of the universe

The understanding of stellar evolution is required to understand large scale structure as the galaxies and structures as small as dust grains.

PHYSICAL INTERACTIONS IN STARS

GRAVITATION STRONG NUCL.	}	Energy production	contraction Nuclear energy
WEAK NUCL. ELECTRO.	}	Energy transport	radiation ν-emission

Intervene through numerous physical mechanisms

- equation of state
- Thermodynamic properties
- Opacities, atomic, molecular
- nuclear reaction rates
- neutrino emissions
- Hydrodynamics
- ……………

$$\frac{\partial P}{\partial M_P} = -\frac{GM_P}{4\pi r_P^4} f_P,$$

1D

$$\frac{\partial r_P}{\partial M_P} = \frac{1}{4\pi r_P^2 \bar{\rho}},$$

$$\frac{\partial L_P}{\partial M_P} = \epsilon_{nucl} - \epsilon_\nu + \epsilon_{grav},$$

$$\frac{\partial \ln T}{\partial M_P} = -\frac{GM_P}{4\pi r_P^4} f_P \min\left[\nabla_{ad}, \nabla_{rad}\frac{f_T}{f_P}\right],$$

$$f_P = \frac{4\pi r_P^4}{GM_P S_P}\frac{1}{<g^{-1}>},$$

$$f_T = \left(\frac{4\pi r_P^2}{S_P}\right)^2 \frac{1}{<g><g^{-1}>}.$$

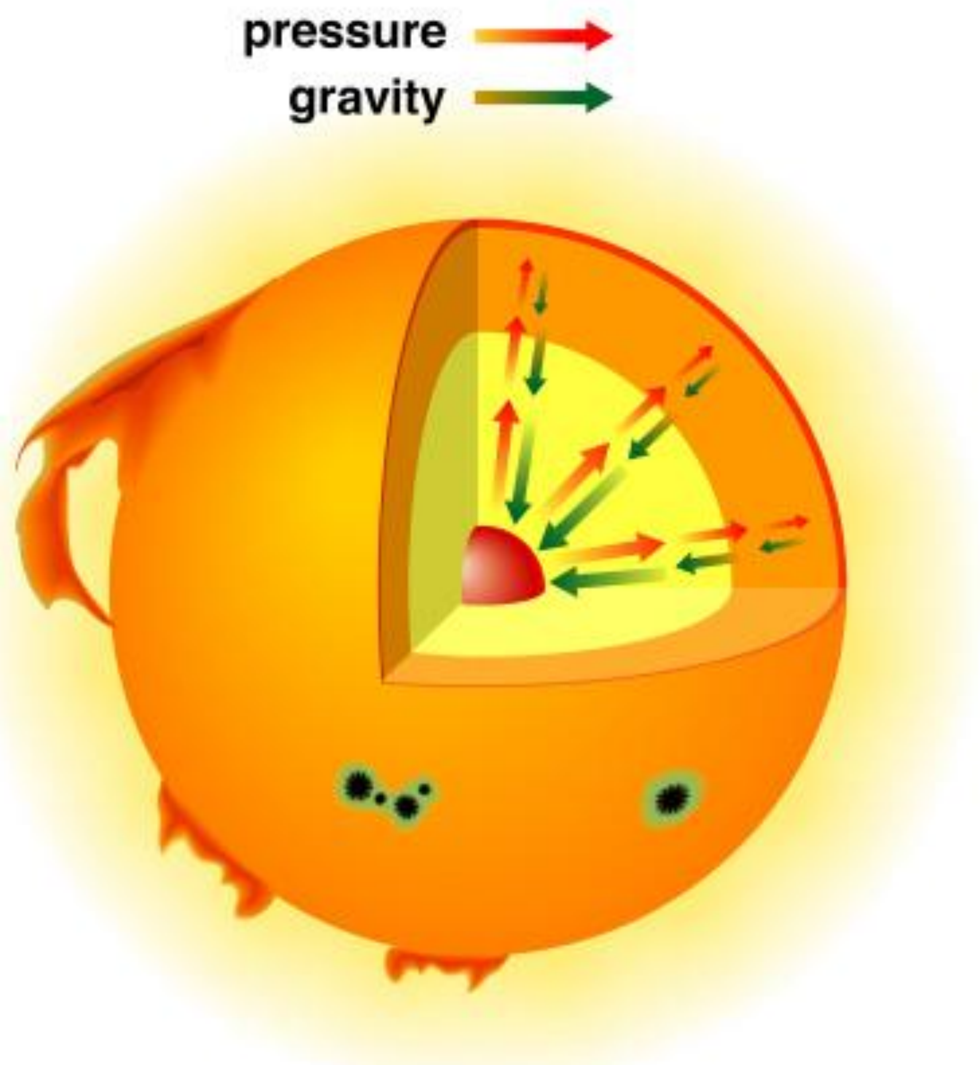
STRUCTURE

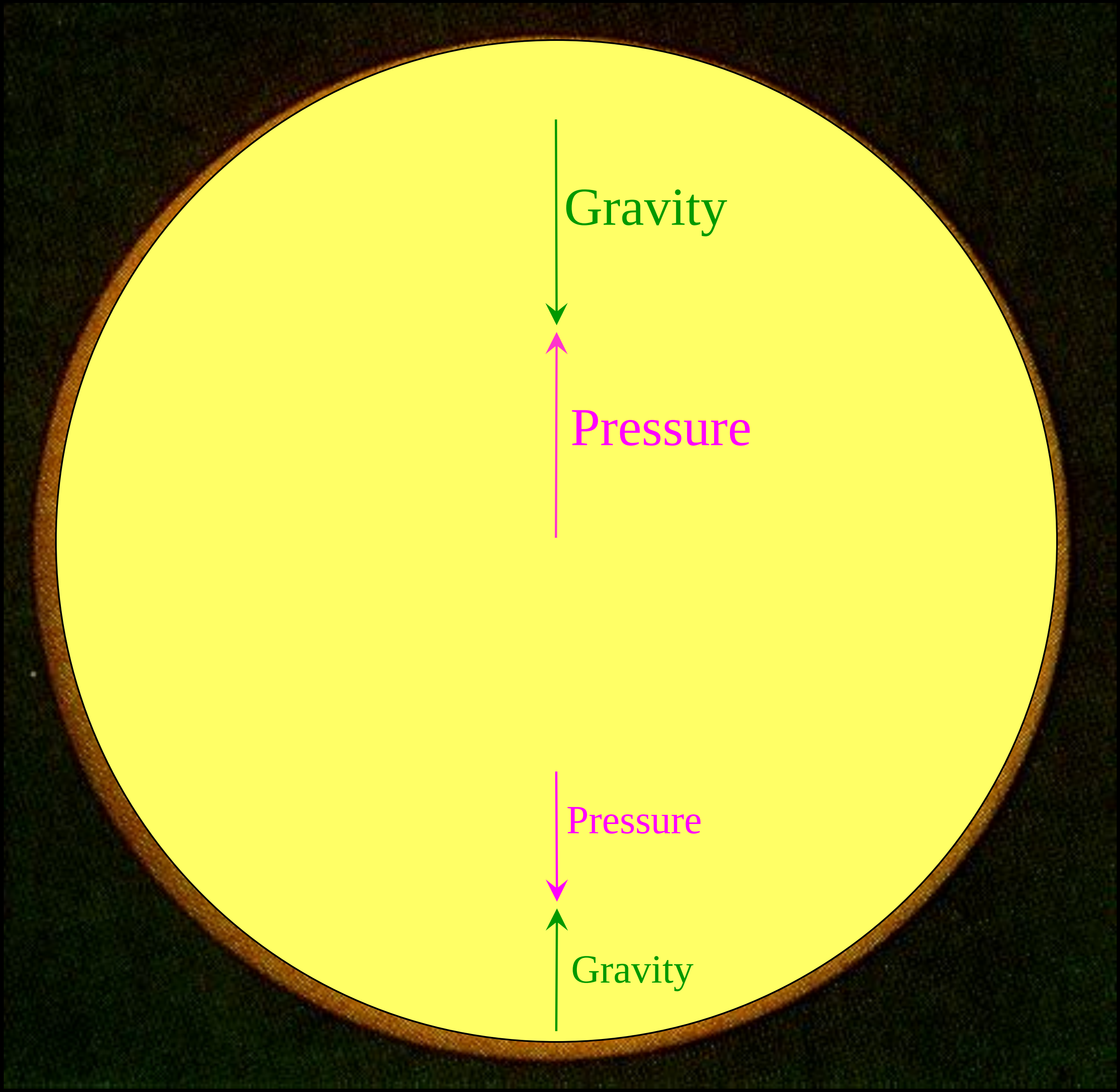
cf. Kippenhahn & Thomas '70

The equation scheme may be written with some modifications for

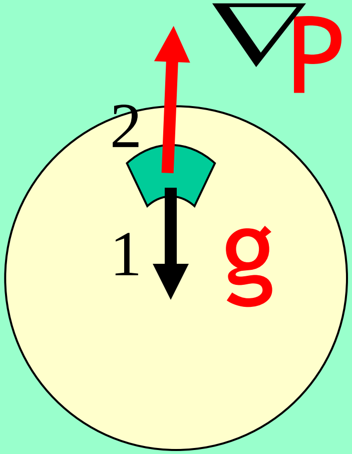
Meynet & Maeder '97

Maintain equilibrium has a price

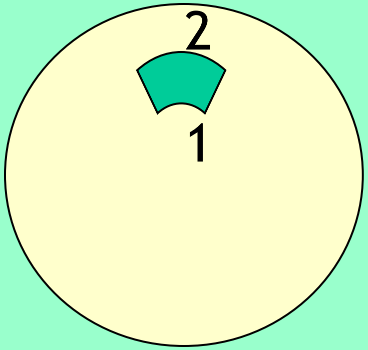




Gravity
Pressure
Pressure
Gravity

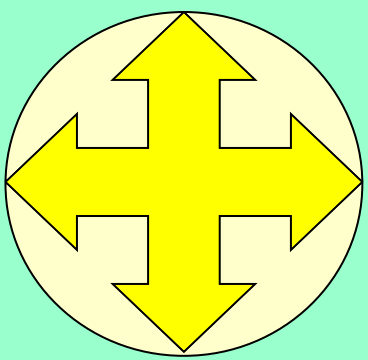


P1 > P2



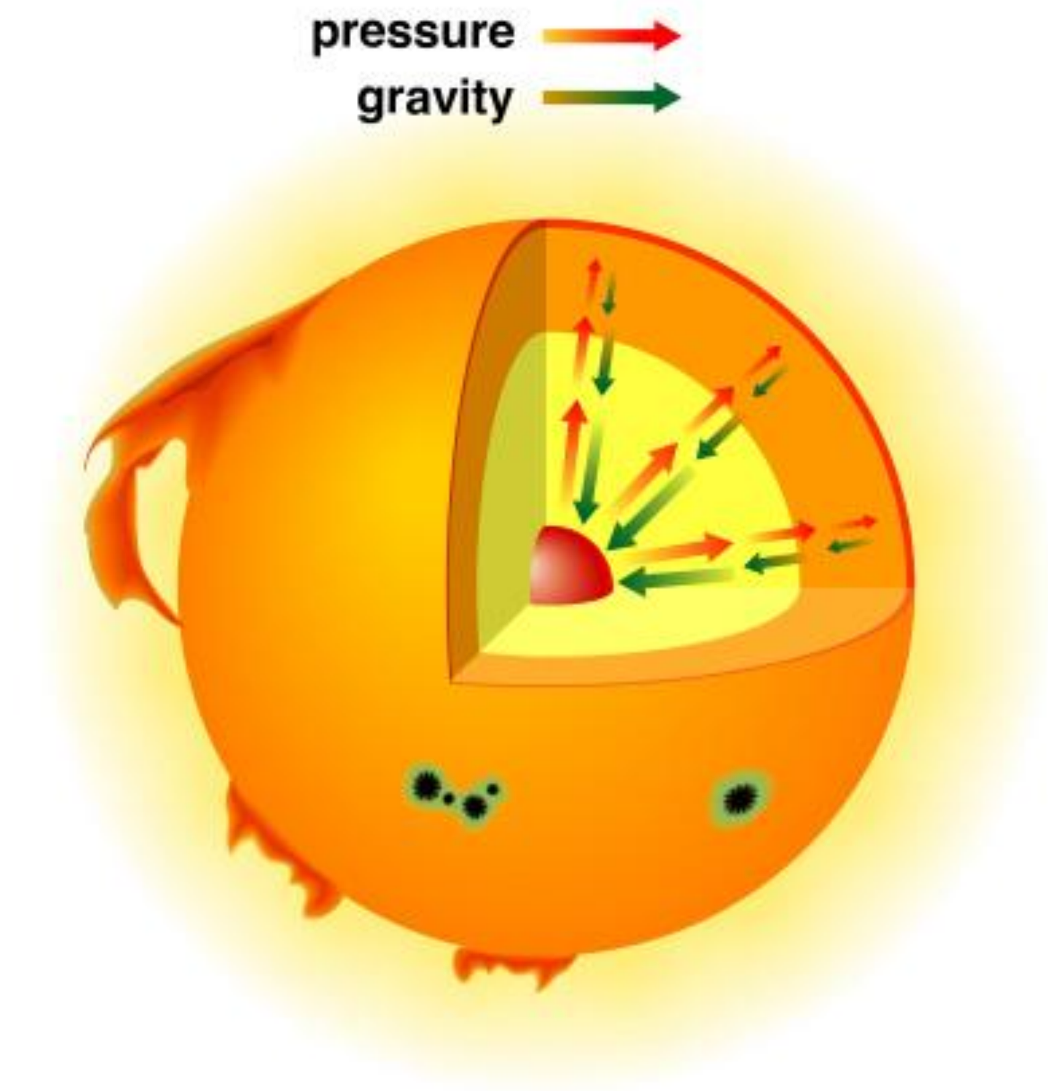
T1> T2

$U=aT^4$



U1 >>U2

Maintain equilibrium has a price



A few estimates

- Central pressure in the SUN
- Central temperature in the SUN

$$\frac{dP}{dr} = -\rho g$$

$$\frac{P_s - P_c}{R} \approx -\frac{M}{\frac{4}{3}\pi R^3}\frac{GM/2}{(R/2)^2}$$

$$P_c \approx \frac{1}{2} G \frac{M^2}{R^4}$$

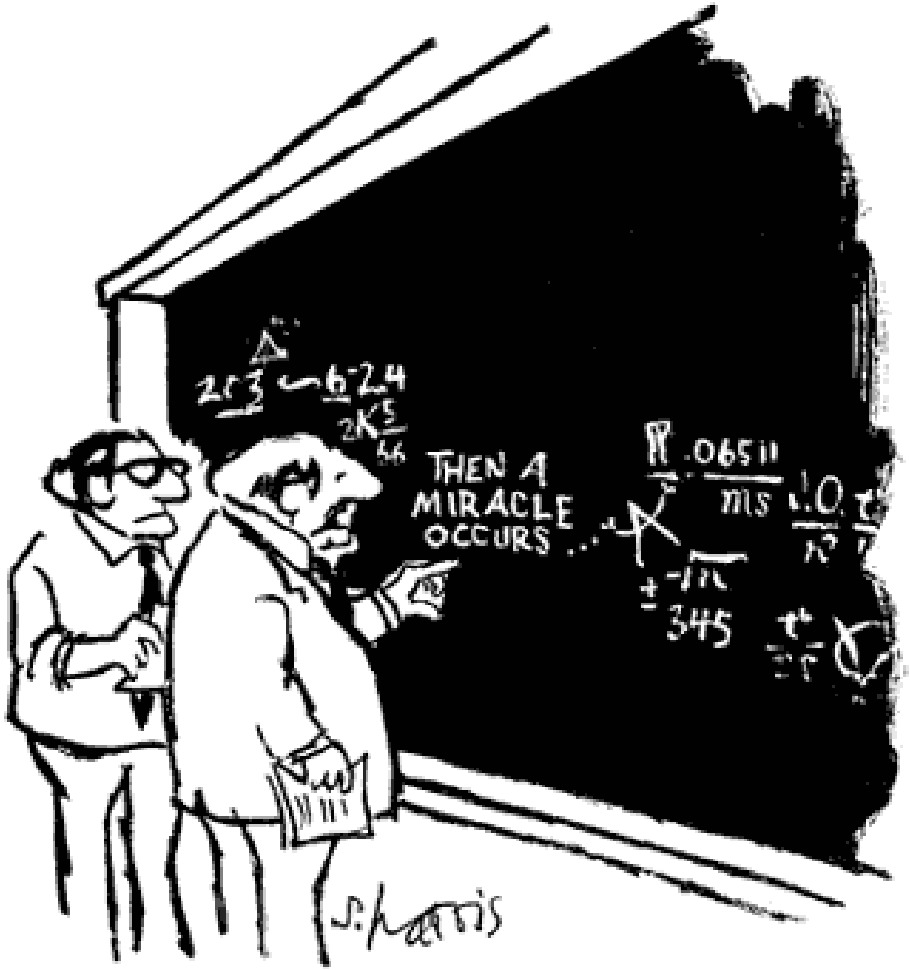
$$P_c \approx \frac{1}{2} G \frac{M^2}{R^4}$$

$$\boldsymbol{P = \frac{k}{\mu m_H} \rho T}$$

$$\rho_c = f\rho$$

$$T_c = \frac{2Gm_H}{k} \frac{\mu_c M}{fR}$$

A PROBLEM WITH THESE ESTIMATES…



"I think you should be more explicit here in step two."

http://www.sciencecartoonsplus.com/gallery/

J. Homer Lane 1870

Temperature ➤ 10^7 K

Density ➤ 150 000 kg/m^3

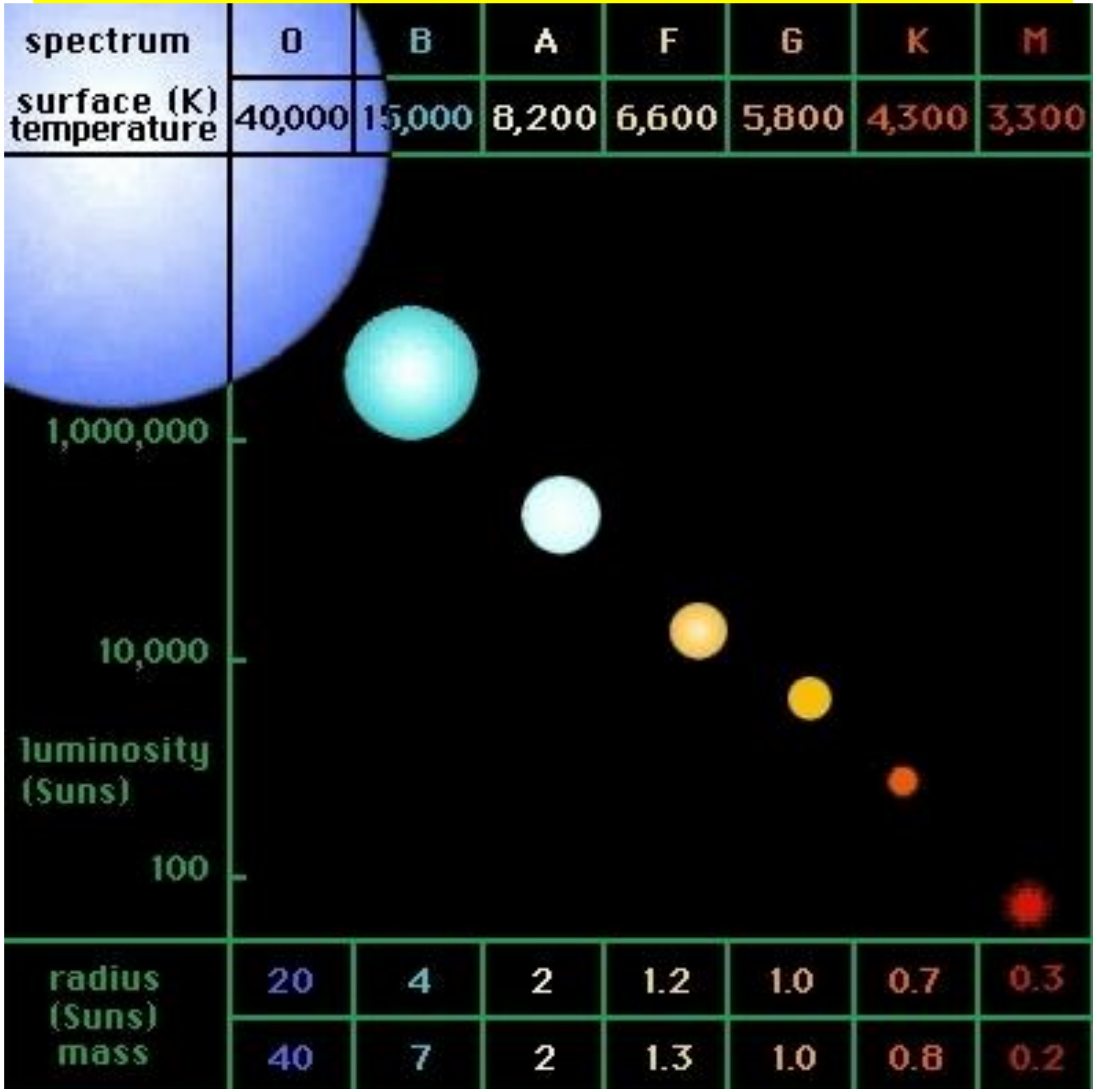
PROBLEM !!!

Do we still have a gaz at such a high density?

Density obtained in matter composed of H atomes adjacent to each other

13 600 kg/m^3

A NATURAL SCALE FOR THE MASS OF STARS…



https://ca-science7.wikispaces.com/Alli_Astro

IMPORTANCE OF THE RADIATION PRESSURE

Eddington

The parable of the physicist on a cloud Bound planet Eddington 1926

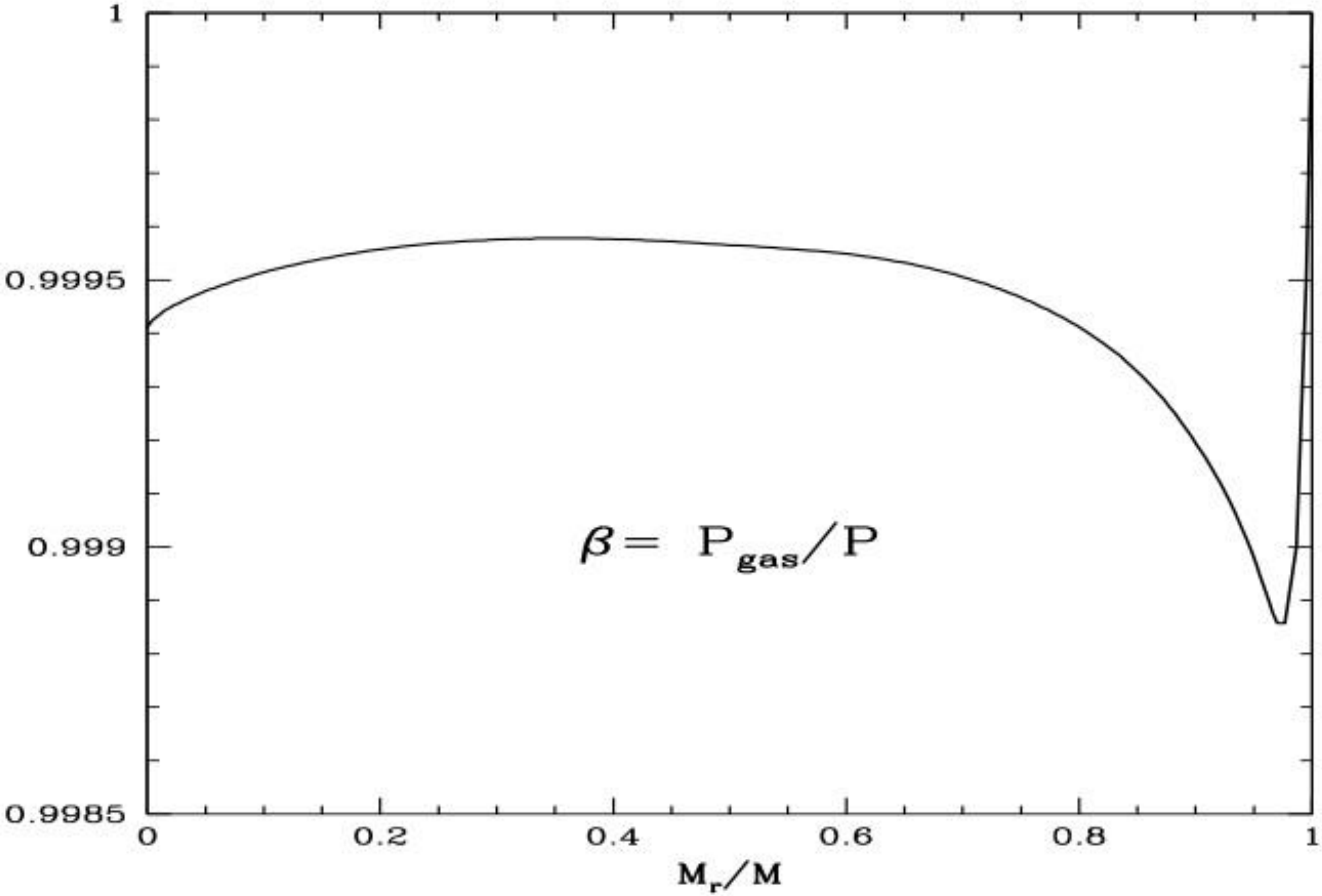
Reported by Srinivasan, Saas-Fee Advanced Course 25 (1995)

``The outward flowing radiation may be compared to a wind blowing through the star and helping to distend it against gravity.''

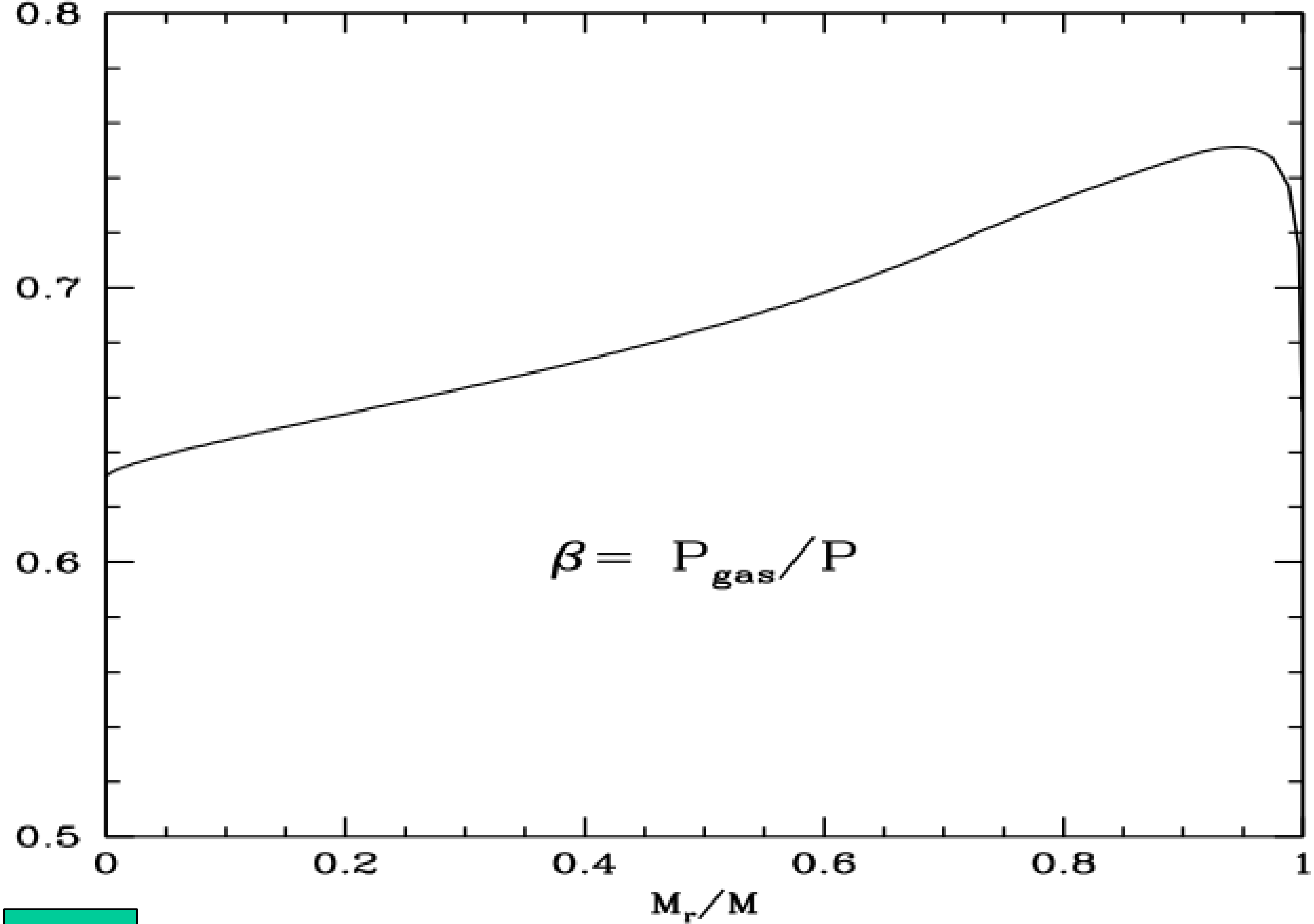
- Possible to compute what proportion of the weight is borne by radiation the rest being supported by the gaz
- To a first approximation, this proportion is the same at all parts of the star

``We can imagine a physicist on a cloud-bound planet who has never heard tell of the star, calculating the ratio of Radiation pressure to gas pressure for a series of globes of gas of various sizes, starting, say, with a globe of mass 10 g, then 100g, 1000g and so on, so that his n^{th} globe contains 10^ng.''

The results



1
0.9995
0.999
0.9985
0
0.2
0.4
0.6
0.8
1
M_r/M
$\beta = P_{gas}/P$


0.8
0.7
0.6
0.5
0
0.2
0.4
0.6
0.8
1
M_r/M
$\beta = P_{gas}/P$

For Low Mass ⇒ P_{gas}

For High Mass ⇒ P_{rad}

Only for a relatively narrow mass interval the table becomes interesting (with numbers different from 0 or 1

n	Prad/P	Pgas/P
…	….	…
32	0.0016	0.9984
33 1Ms	0.106	0.894
34 10Ms	0.570	0.430
35 100Ms	0.850	0.150
36	0.951	0.049
37	0.984	0.016
38	0.9951	0.0049
39	0.9984	0.0016
….	…	…

We may Expect Something To happen

What happens is the stars !

The observed masses of the stars are in majority between 10^{33} - 10^{34} g where the serious challenge of radiation pressure to compete with gaz pressure is beginning


AIP Niels Bohr Library

NATURAL SCALES OF THE STELLAR MASSES

Chandrasekhar 1936

In any equilibrium configuration in which the mean density inside decreases outwards we have the inequality

$$\frac{1}{2}G\left(\frac{4}{3}\pi\right)^{1/3}\bar{\rho}^{4/3}(r)M^{2/3}(r)\leq P_c - P \leq \frac{1}{2}G\left(\frac{4}{3}\pi\right)^{1/3}\rho_c^{4/3}M^{2/3}(r)$$

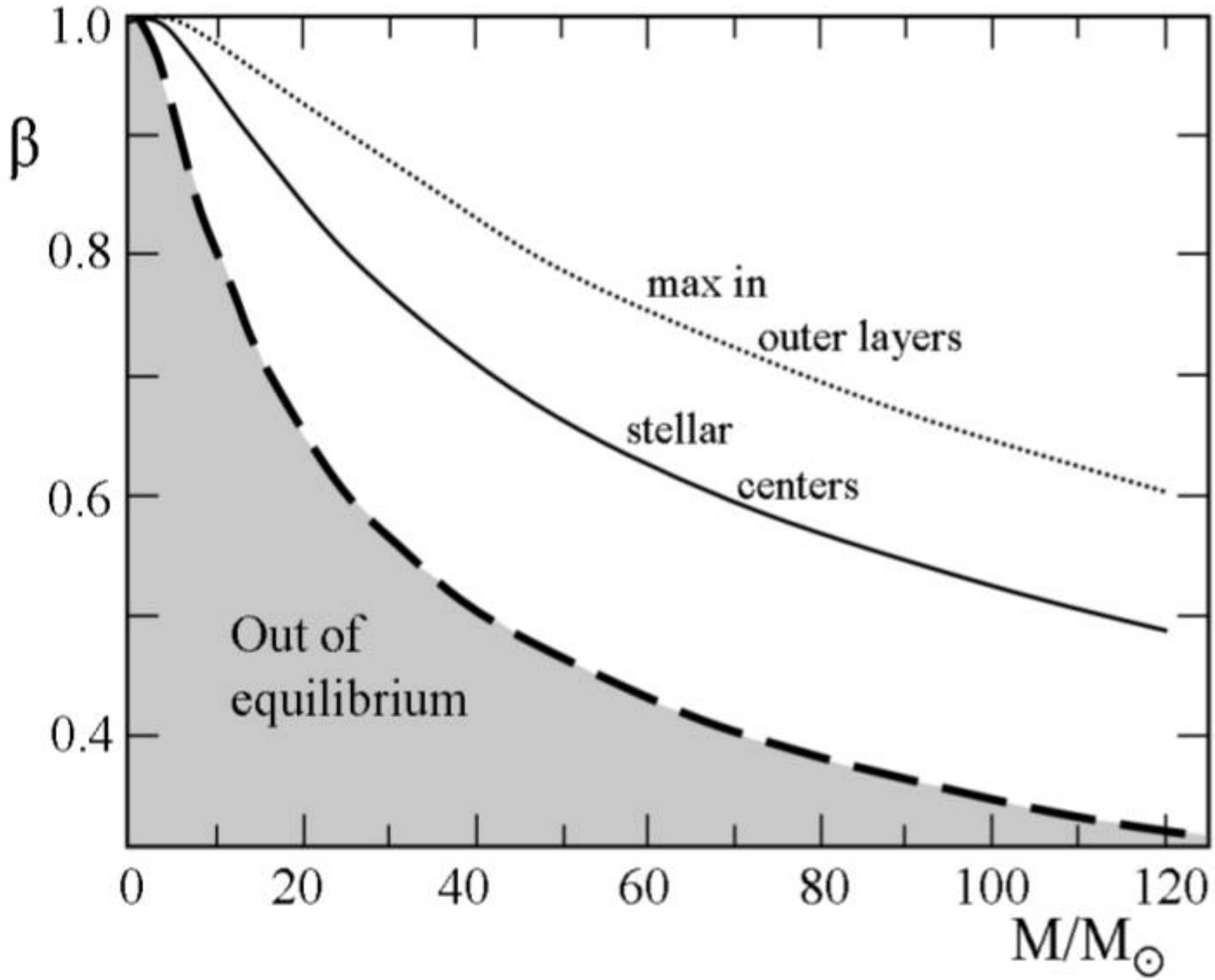
where $\bar{\rho}(r)$ denotes the mean density inside r, and ρ_c the central density and P_c the central pressure

$$P = \underbrace{\frac{k}{\mu m_H}\rho T}_{P_{gaz}} + \frac{1}{3}aT^4, \beta = P_{gaz}/P, a = \frac{8\pi^5 k^4}{15h^3c^3}$$

$$\mu^2 M\left(\frac{\beta_c^4}{1-\beta_c}\right)^{1/2} \geq \underbrace{0.1873\left(\frac{hc}{G}\right)^{3/2}\frac{1}{m_H^2}}_{5.48 M_{sol}}$$

$$\mu^2 M\left(\frac{\beta_c^4}{1-\beta_c}\right)^{1/2} \geq \underbrace{0.18733\left(\frac{hc}{G}\right)^{3/2}\frac{1}{m_H^2}}_{5.48 M_{sol}}$$

Supposing the mechanical equilibrium is maintained by both the gaz and radiation pressure, one obtains a combination of physical constants providing a natural scale for the masses of the stars.



1.0
β
0.8
0.6
0.4
max in
outer layers
stellar
centers
Out of
equilibrium
0
20
40
60
80
100
120
M/M⊙

Estimate for the luminosity

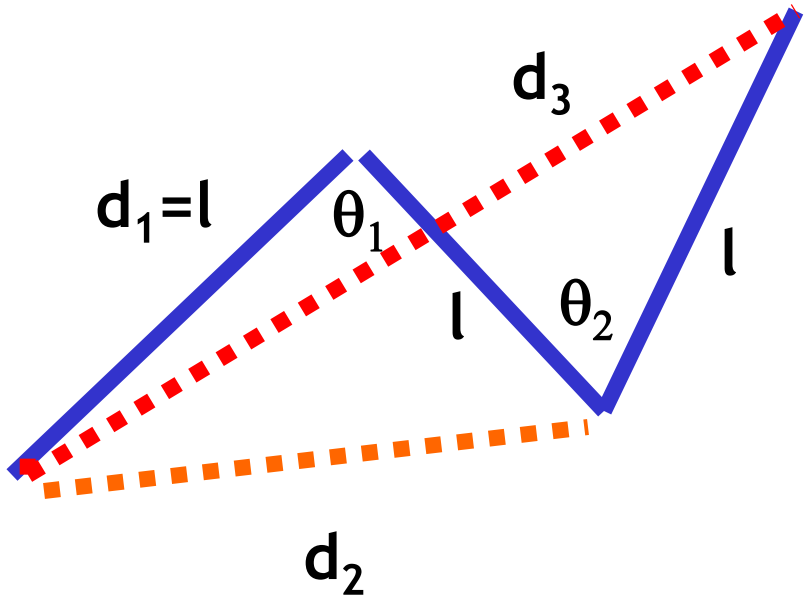
L=A/B

A=Quantity of energy under the form of radiation

$$a < T^4 > \frac{4}{3}\pi R^3$$

B=Time for a photon at the centre to reach the surface

$$\frac{l}{c} N_{diff}$$



$$N_{diff} = \frac{R^2}{l^2}$$

$$l = \frac{1}{\kappa\rho}$$

$$d_1^2 = l^2$$

$$d_2^2 = l^2 + l^2 - 2l^2 \cos\theta_1$$

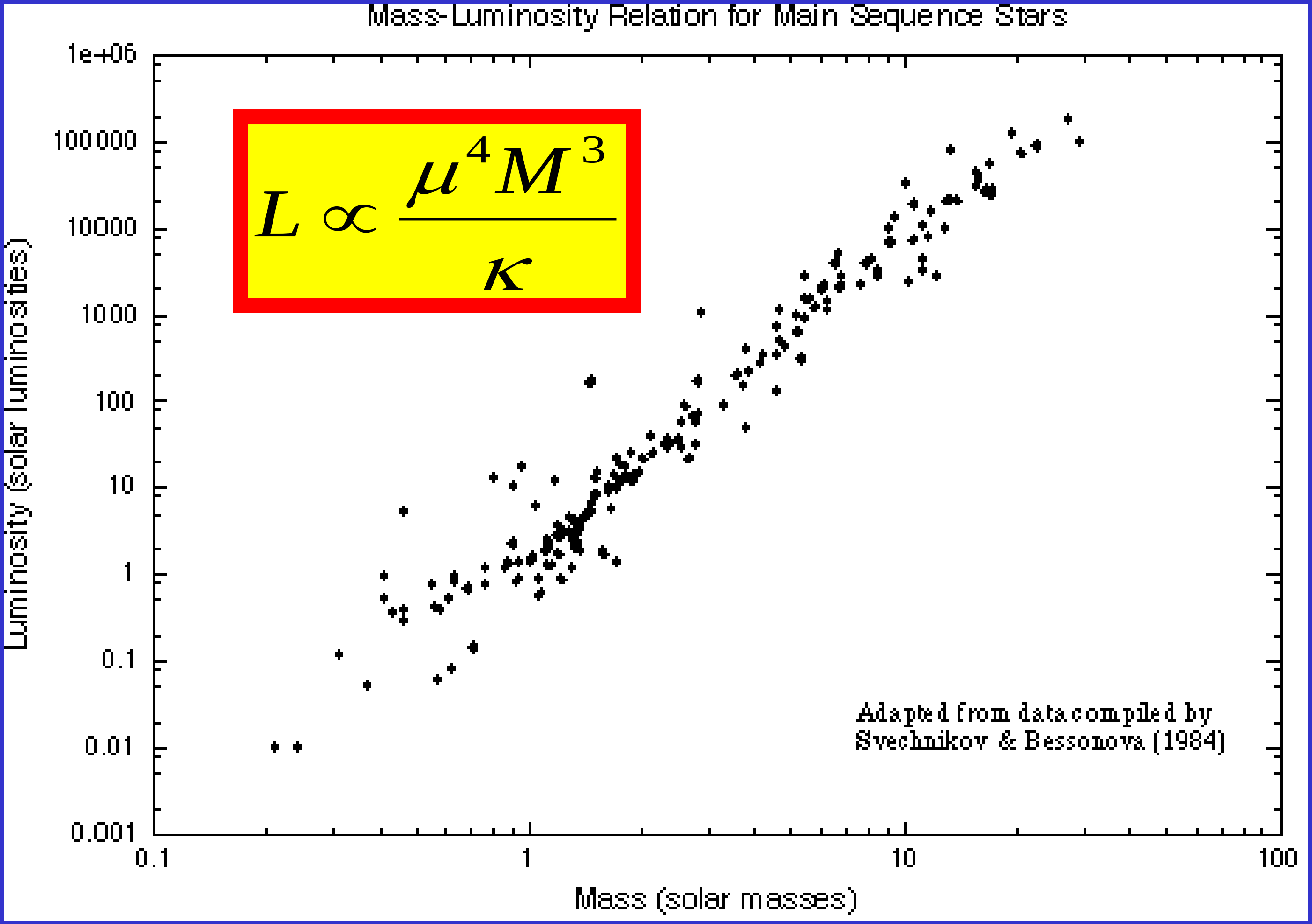
$$d_n^2 = nl^2 - 2l(\sum_{i=2}^{n} d_{i-1} \cos\theta_{i-1}) \approx nl^2$$

$$L = \frac{a < T^4 > \frac{4}{3}\pi R^3}{\frac{l}{c} N_{diff}}$$

$$L \sim \frac{T^4 R^3}{\kappa R^2 \frac{M}{R^3}}$$

$$T \, \alpha \, \mu \frac{M}{R}$$

The mass-luminosity relation for 192 stars in double-lined spectroscopic binary systems.



$$L \propto \frac{\mu^4 M^3}{\kappa}$$

Luminosity is a consequence of equilibrium

More a star massive is, more luminous it is

Higher the averaged opacity, lower the luminosity

A Helium star is more luminous than a hydrogen stars (given a mass and opacity)

Where does this energy come from ?

Let us first consider a uniform contraction of a mass M. In that case a variation in radius ΔR corresponds to a variation in pressure ΔP and to a variation in density $\Delta\rho$ so that we have the following relations:

$$\frac{\Delta P}{P} = -4\frac{\Delta R}{R}, \quad \text{and} \quad \frac{\Delta\rho}{\rho} = -3\frac{\Delta R}{R}.$$

The first equality is deduced from the hydrostatic equilibrium equation and the second from the continuity equation. From these two relations, we can write

$$\Delta \ln P = \frac{4}{3}\Delta \ln \rho.$$

Let us now write the equation of state as follows

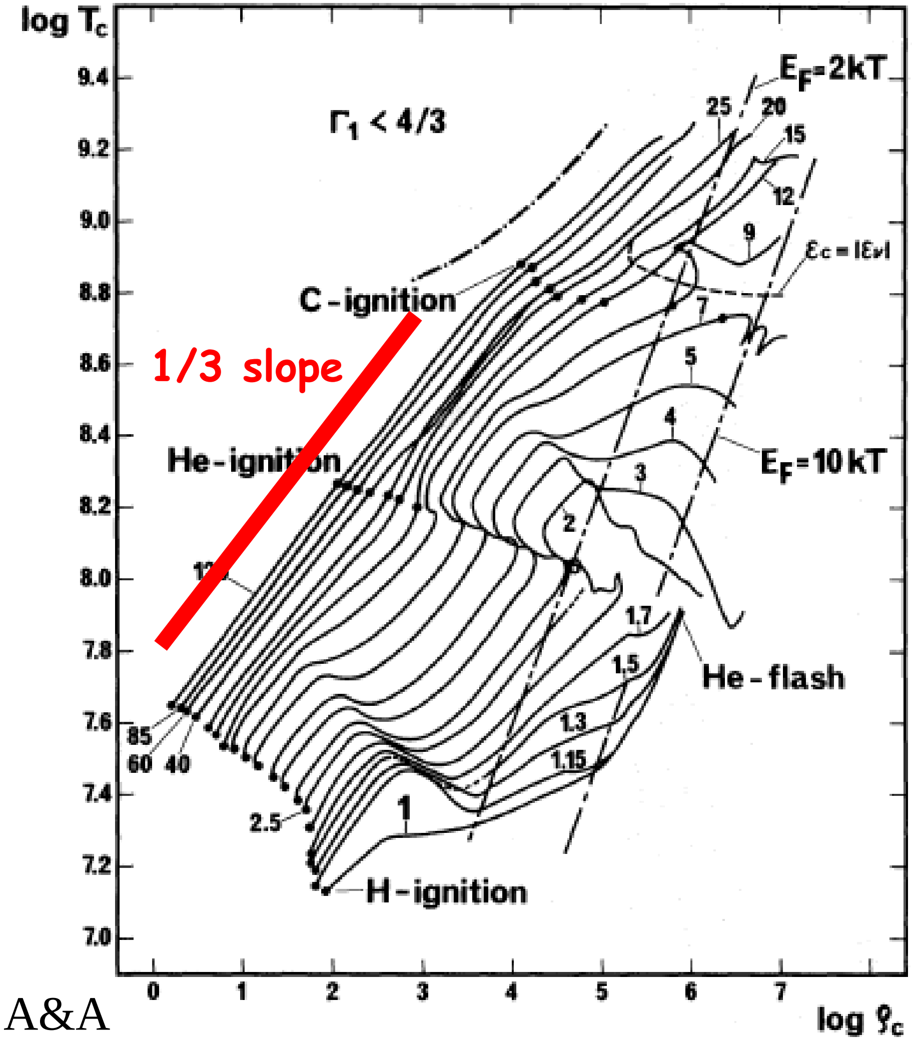
$$\Delta \ln \rho = \alpha \Delta \ln P - \delta \Delta \ln T,$$

where α and δ are defined by $\alpha = \left(\frac{\partial \ln \rho}{\partial \ln P}\right)_{T,\mu}$ and $\delta = -\left(\frac{\partial \ln \rho}{\partial \ln T}\right)_{P,\mu}$, and where μ, the mean molecular weight, is supposed to remain constant. From these two relations one obtains, by eliminating ΔP the two following relations between a variation in log T and log ρ:

$$\Delta \ln T = \left(\frac{4\alpha - 3}{3\delta}\right)\Delta \ln \rho. \quad (1)$$

For a perfect gas law we have $\alpha = \delta = 1$. Therefore an increase of, for instance, 30% in density implies an increases of 10% in temperature.

Stars=system with a negative specific heat!

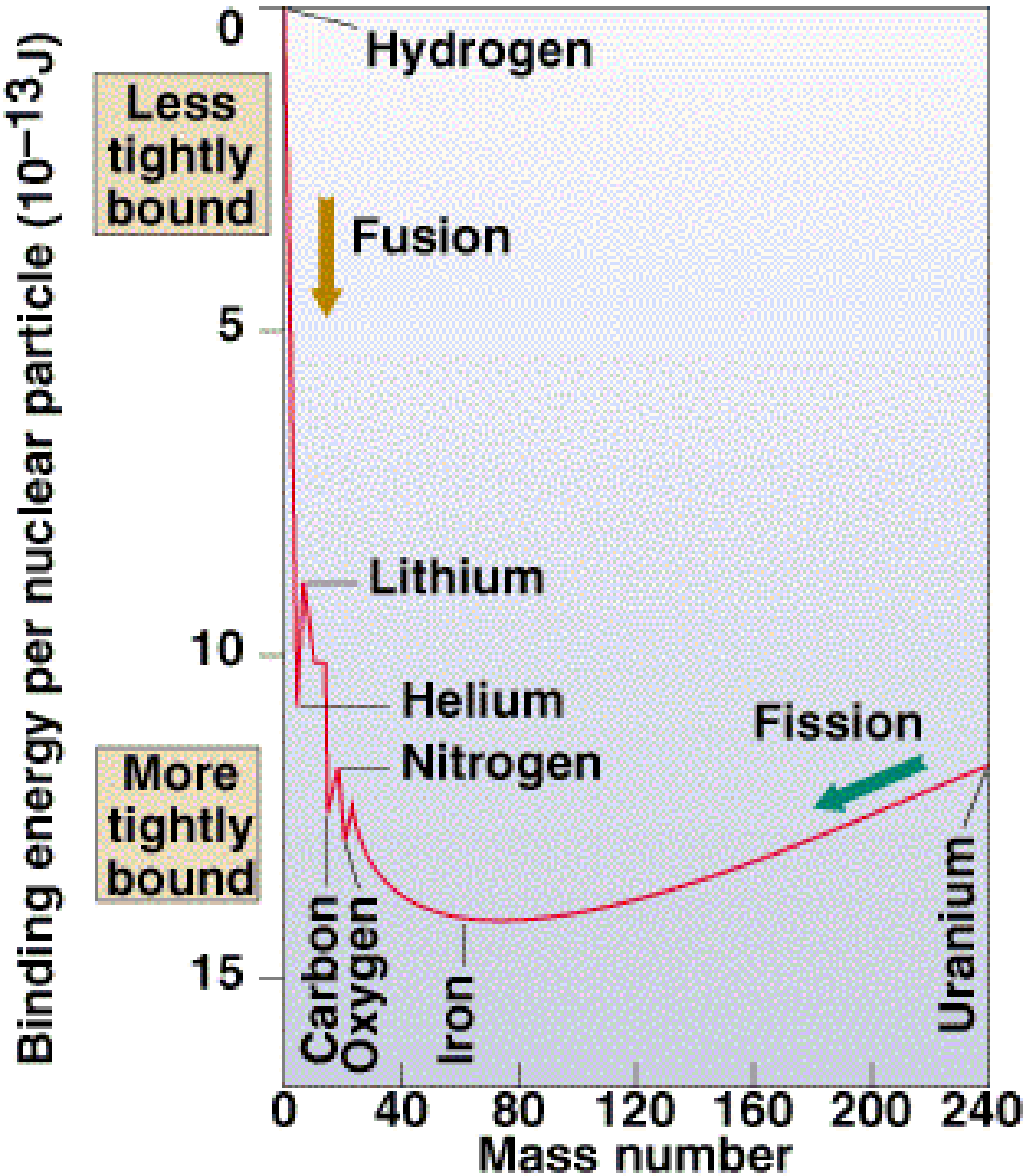


Maeder & Meynet 1989, A&A

. GRAVITATIONAL ENERGY

HOW LONG CAN IT LAST?

$$\tau_{KH} = \frac{GM^2}{RL} \rightarrow \tau_{KH} = 10^7\ years$$



Binding energy per nuclear particle (10−13J)
0
5
10
15
Less tightly bound
More tightly bound
Hydrogen
Fusion
Lithium
Helium
Nitrogen
Fission
Carbon
Oxygen
Iron
Uranium
0
40
80
120
160
200
240
Mass number

• NUCLEAR ENERGY

HOW LONG CAN IT LAST?

$$\tau_{nucl} \approx \frac{MqX0.007c^2}{L} \rightarrow \tau_{nucl} \approx 10^{10}\, years$$

THE RESERVOIRS OF ENERGY

- **GRAVITATIONAL ENERGY**

$$\tau_{KH} = \frac{GM^2}{RL} \rightarrow \tau_{KH} = 10^7\ years$$

- **NUCLEAR ENERGY**

$$\tau_{nucl} \approx \frac{MqX0.007c^2}{L} \rightarrow \tau_{nucl} \approx 10^{10}\ years$$

The mechanisms of extraction of the energy from these reservoirs are responsible for the evolution of the stars.

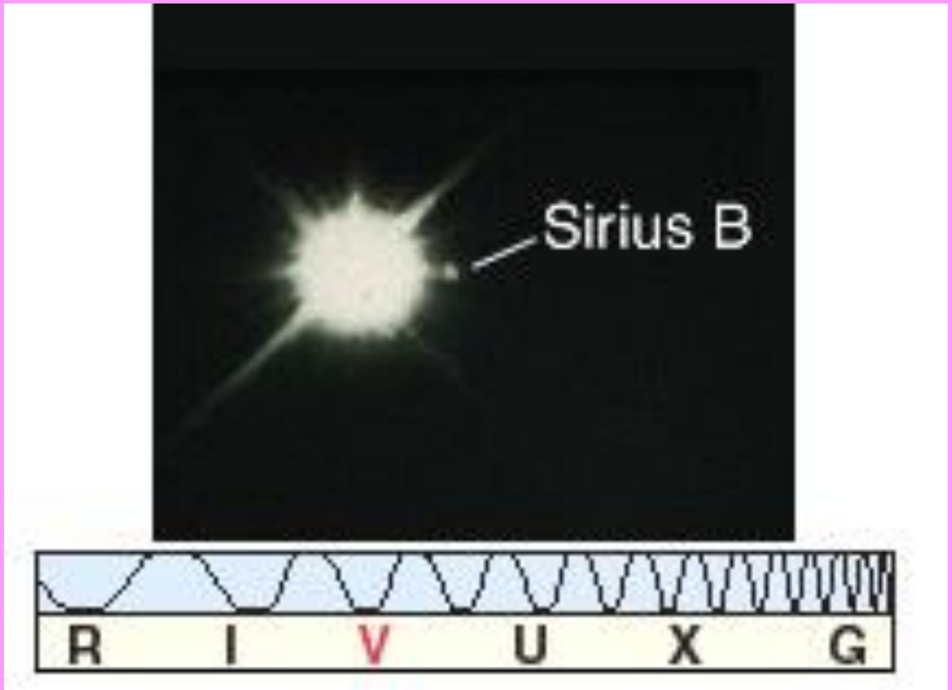
EQUILIBRIUM→EVOLUTION

- Hyrostatic equilibrium→loss of energy
- Energy produced by contraction

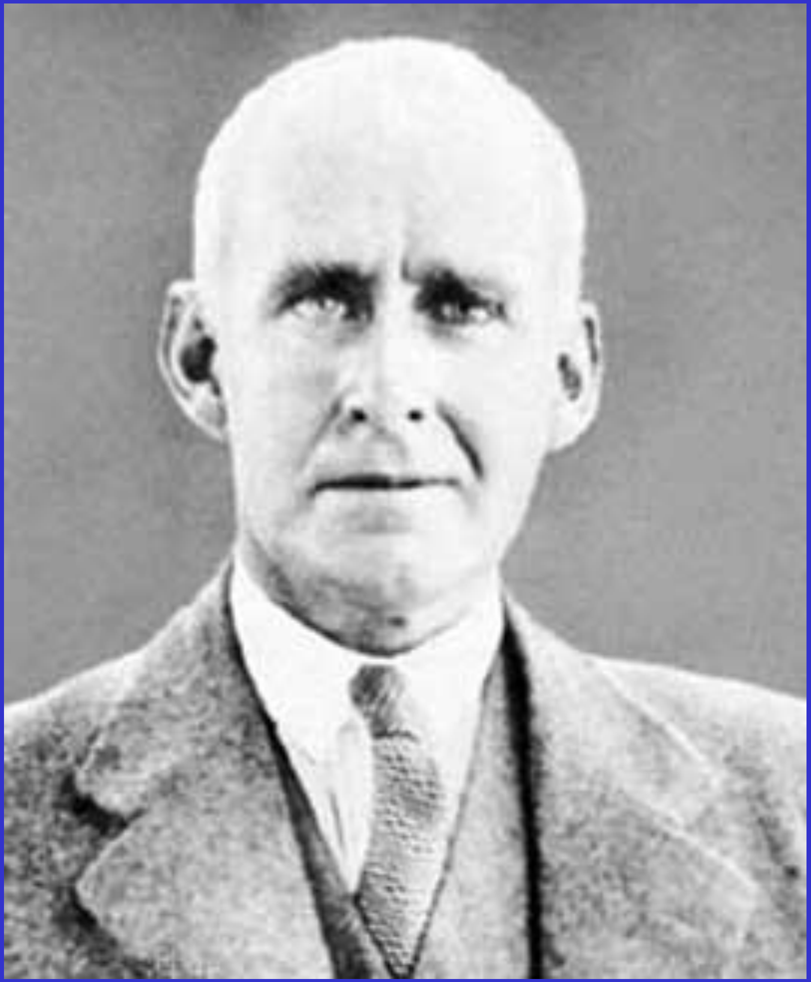
  nuclear reactions
- The state of the stars evolve.
- **Can it continue for ever?**

Sirius B

Mass	1.1 solar masses
Radius	0.008 solar radii (5500 km)
Luminosity (total)	0.04 solar luminosities (1.6 x10^{25} W)
Surface temperature	24,000 K
Average density	3x10^{9} kg/m^{3}



TOO LITTLE ENERGY TO COOL!!!



Eddington

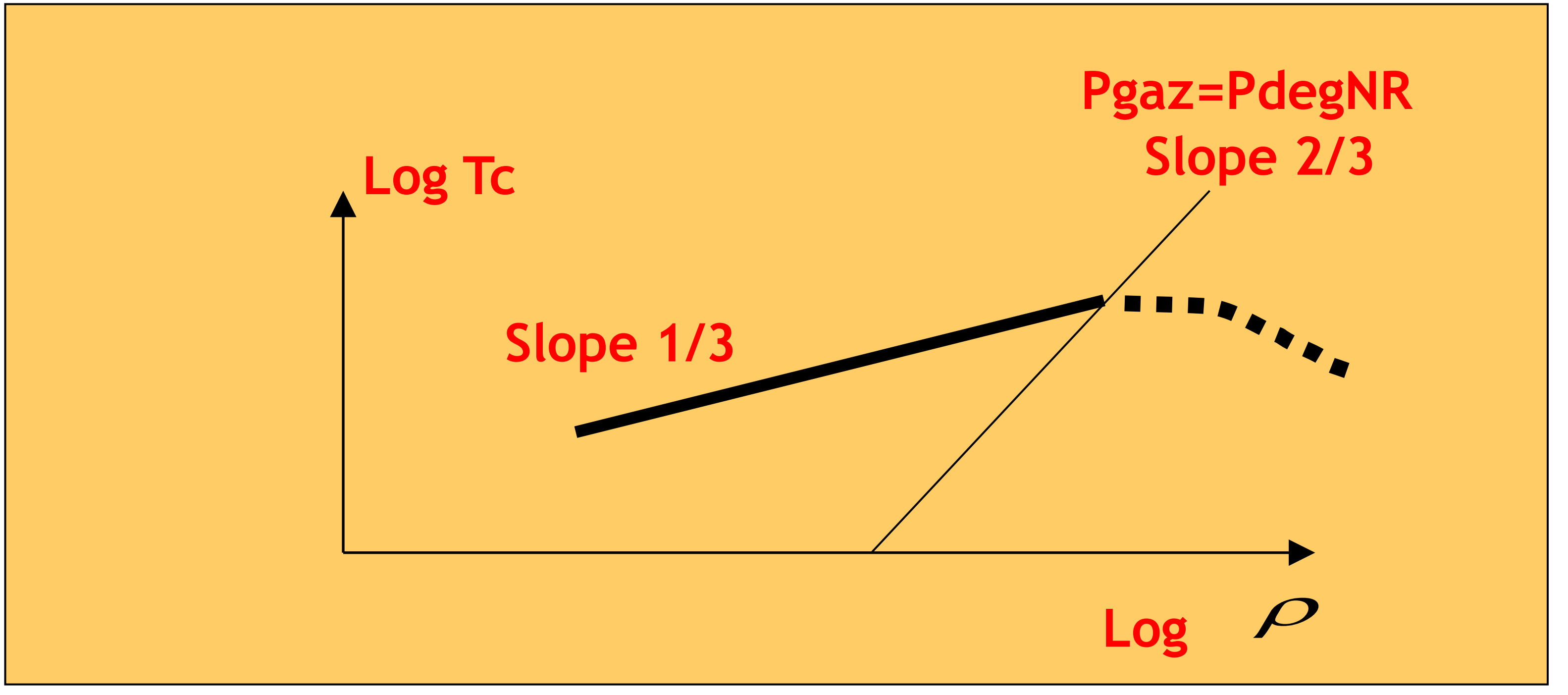
$$\Delta \ln T = \left(\frac{4\alpha - 3}{3\delta} \right) \Delta \ln \rho.$$

$$P \propto \rho^{5/3}$$

$$\alpha = 3/5 \text{ and } \delta = 0$$

no longer valid, but if during the course of evolution, when the central conditions pass from the non-degenerate region to the degenerate one, α becomes inferior to three quarters before δ is equal to zero, then a contraction can produce a cooling! This can be understood as due to the fact that, in order to allow electrons to occupy still higher energy state, some energy has to be extracted from the non degenerate nuclei which, as a consequence, cool down.

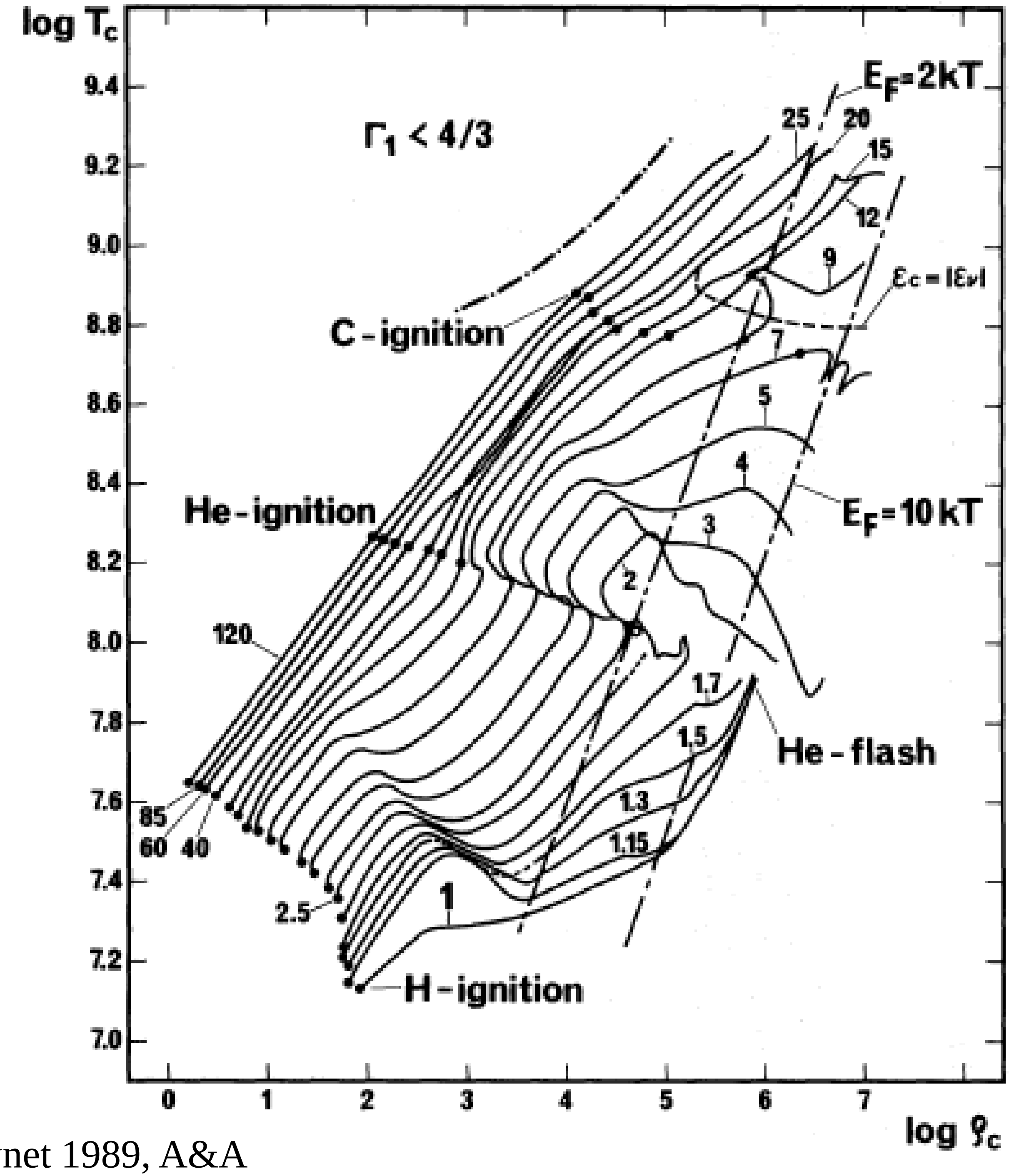
Evolution of the temperature and density at the centre



Pgaz=PdegNR

$$\frac{k}{\mu m_H}\rho T = K_1\left(\frac{\rho}{\mu e}\right)^{5/3}$$

$$\rightarrow T = K_1 \frac{\mu m_H}{k}\frac{1}{\mu_e^{5/3}}\rho^{2/3}$$



Maeder & Meynet 1989, A&A

Nuclear reaction in PG conditions

Let us now study the nuclear source in degenerate conditions. Let us imagine that for whatever reason an excess of energy is produced at the center of the star. This will produce a heating of the matter. When the perfect gas law prevails, an increase of temperature will produce an increase of pressure and therefore an expansion. This implies an increase of the potential energy and through the Virial theorem a decrease of the internal energy, therefore the temperature decreases as well as the nuclear reaction rates. We see that in perfect gas conditions, there is a negative feedback which stops the runaway. The nuclear reactions are stable when the perfect gas law prevails.

Nuclear reactions in DG conditions

When the matter is degenerate, the behavior is quite different. The excess of energy produced at the center, which implies an increase of the temperature does no long provoke an expansion, since there is no long a coupling between pressure and temperature. The nuclear reaction rates increase, new excesses of energy are produced, a flash or an explosion occurs. The nuclear reactions are unstable in degenerate matter. This process is responsible for the explosion of type Ia supernovae. It triggers also what is called the helium flash at the tip of the Red Giant Branch for stars with masses below about 1.8 $M_{\odot}$ at solar metallicity. These

Stellar evolution in a nutshell

When perfect gaz prevails→ hydrostatic equilibrium implies continuous loss of energy

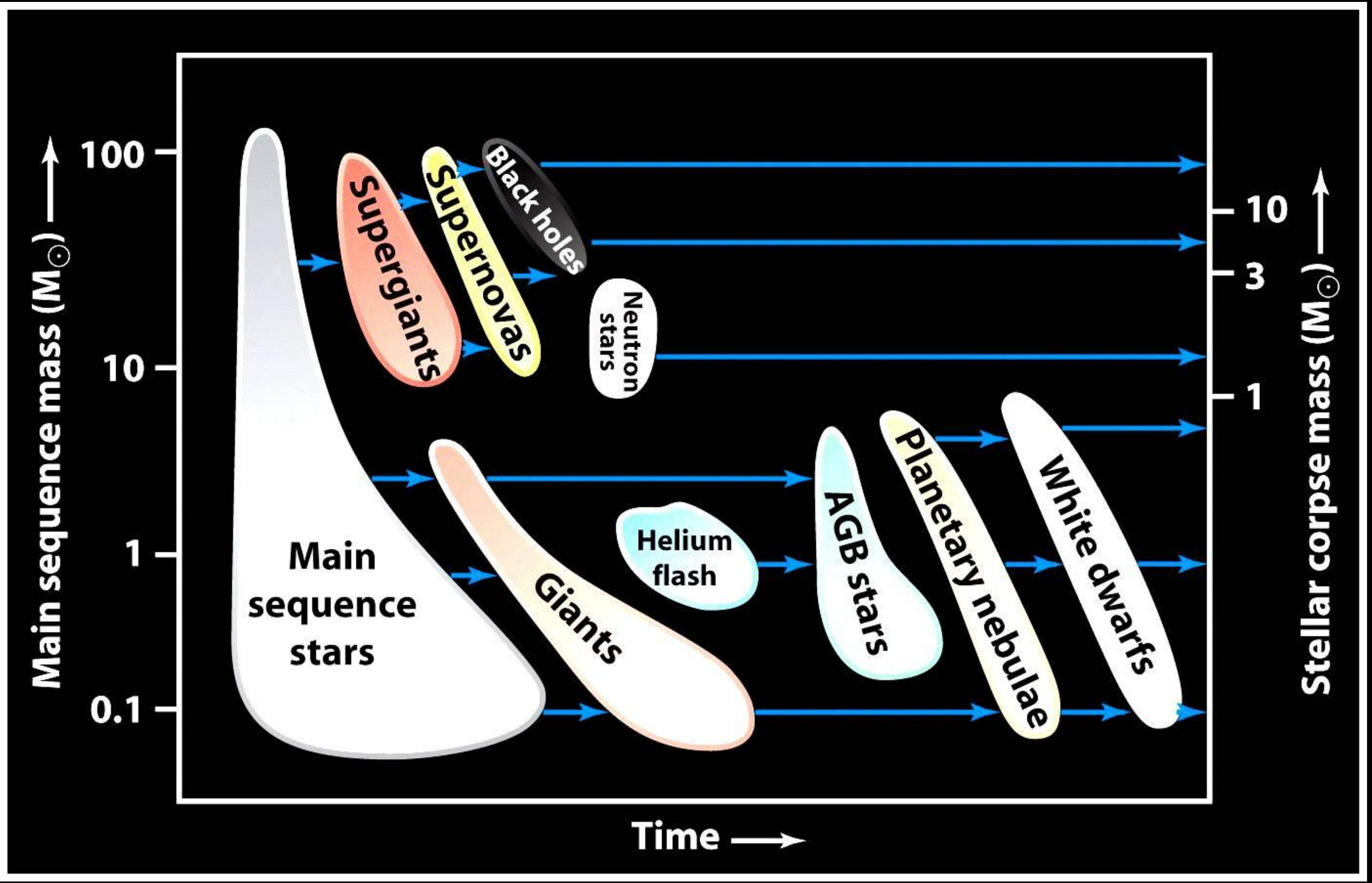
Star compensate for this loss either by macroscopic contraction or microscopic ones

This changes the structures and the composition of the star

These processes drive the central regions in degenerate regimes

In degenerste regime: nuclear reaction unstable, contraction may lead to cooling

Hydrostatique equilibrium is for free! No long evolution



Main sequence mass (M⊙)
100
10
1
0.1
Main sequence stars
Supergiants
Supernovas
Black holes
Neutron stars
Giants
Helium flash
AGB stars
Planetary nebulae
White dwarfs
Stellar corpse mass (M⊙)
10
3
1
Time

Bibliography

TWO RECENT BOOKS

Physics, Formation and Evolution of Rotating Stars, Maeder, A&A Library, Springer 2009

Stars and Stellar Evolution, K.S. de Boer & W. Seggewiss, EDP Sciences, 2008

PART OF THIS LESSON TAKEN FROM

From stars to nuclei, Meynet 2008, The European Physical Journal Special Topics, Volume 156, Issue 1, pp.257-263