Transport processes on the main sequence

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Overview

- Introduction
- Atomic diffusion
- Rotation
- Magnetic fields and internal gravity waves
- Summary

Introduction

- Standard model of a star
  - Spherical systems in hydrostatic and radiative equilibrium
  - Mixing only occurs in convective zones
  - No transport mechanism in radiative zones
  - Basic equations of stellar evolution:
    - nuclear reactions rates
    - equation of state
    - opacities
    - prescription for the energy transport by convection
- Transport processes
  - Diffusion
  - Advection

Atomic diffusion

- Test atom approximation
  - Binary mixture with $\rho_i << \rho$
  - P, T and ρ are constant

$$F_{\mathrm{i}} = \frac{1}{\rho X_{\mathrm{i}}} \frac{\partial P_{\mathrm{i}}}{\partial r} - F_{\mathrm{ext_i}} = \frac{k_{\mathrm{B}} T}{X_{\mathrm{i}}} \frac{\partial X_{\mathrm{i}}}{\partial r} - F_{\mathrm{ext_i}}$$

$$V_{\mathrm{i}} = -D_{\mathrm{at}} \left[\frac{\partial \ln X_{\mathrm{i}}}{\partial r} - \frac{F_{\mathrm{ext_i}}}{k_{\mathrm{B}} T} \right] \quad \text{with } D_{\mathrm{at}} = \frac{1}{3} \ell_i v_{\mathrm{T},i}$$

- Gravitational settling

$$F_{\mathrm{ext_i}} = -(m_i - m) g$$

$$V_{\mathrm{i}} = -D_{\mathrm{at}} \left[\frac{\partial \ln X_{\mathrm{i}}}{\partial r} + (m_i - m) \frac{g}{k_{\mathrm{B}} T} \right]$$

Atomic diffusion

- Thermal gradients
  - Binary fluid + T gradient
  - Net flux of particles i through a plane

$$n_i V_i = \frac{1}{6} v_{\mathrm{T},i}(r+\ell_i) n_i(r+\ell_i) \frac{\sigma_i(r+\ell_i)}{\sigma_i(r)} - \frac{1}{6} v_{\mathrm{T},i}(r-\ell_i) n_i(r-\ell_i) \frac{\sigma_i(r-\ell_i)}{\sigma_i(r)}$$

$$V_{\mathrm{i}} = \frac{1}{3\sigma_i} \ell_i \frac{\partial (v_{\mathrm{T},i}\sigma_i)}{\partial r} = D_{\mathrm{at}} \frac{1}{v_{\mathrm{T},i}\sigma_i} \frac{\partial (v_{\mathrm{T},i}\sigma_i)}{\partial r} = D_{\mathrm{at}} \alpha_{T,i} \frac{\mathrm{d}\ln T}{\mathrm{d}r}$$

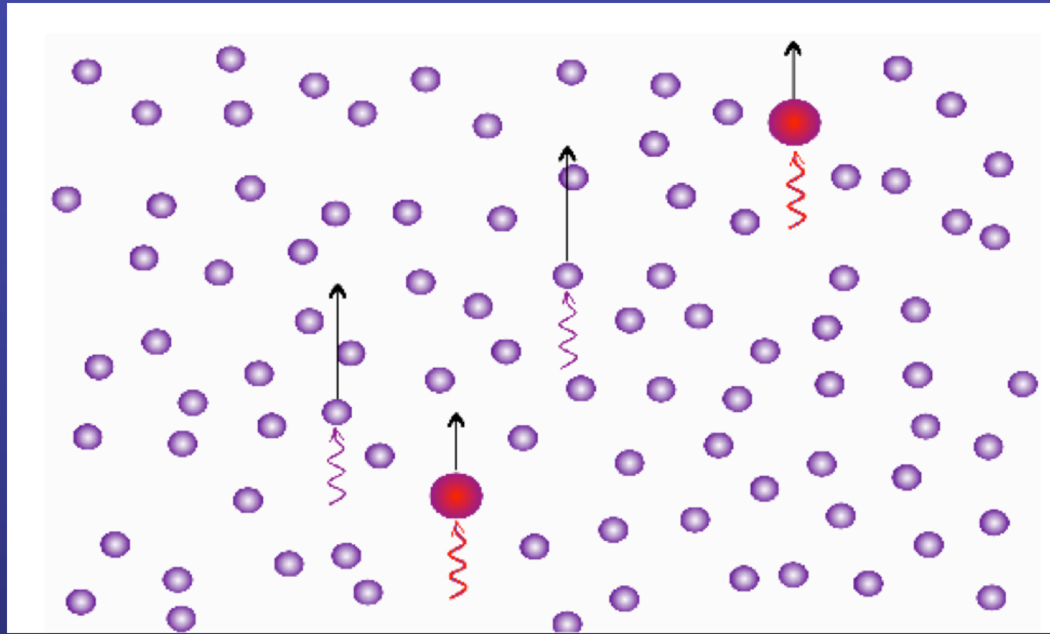
  - $v_T \sim T^{1/2}$; for ions $\sigma_i \sim T^{-2}$ and σ_i is constant with T for neutral atoms

$$V_{\mathrm{i}} \sim -\frac{3}{2} D_{\mathrm{at}} \frac{\mathrm{d}\ln T}{\mathrm{d}r} \quad \text{for ions}$$

$$V_{\mathrm{i}} \sim \frac{1}{2} D_{\mathrm{at}} \frac{\mathrm{d}\ln T}{\mathrm{d}r} \quad \text{for neutral atoms}$$

Atomic diffusion

- Radiative diffusion
  - Radiative acceleration



Talon 2007

$$g_{\mathrm{rad,i}}(\nu)\mathrm{d}\nu = \frac{\kappa_i(\nu)F_\nu}{c}\mathrm{d}\nu$$

$$g_{\mathrm{rad,i}} = -\frac{4\pi}{3c}\frac{1}{n_i m_i}\int_0^\infty \frac{\kappa_i(\nu)}{\kappa(\nu)}\frac{\partial B_\nu}{\partial T}\frac{\mathrm{d}T}{\mathrm{d}r}\mathrm{d}\nu$$

Atomic diffusion: modeling

- The Chapman-Enskog method (Chapman & Cowling 1970)
  - Boltzmann equation
  - Expansion of f in a series of decreasing order

$$\frac{\partial c_i}{\partial t} = D'_{1i}\frac{\partial^2 c_i}{\partial m_r^2} + \left(\frac{\partial D'_{1i}}{\partial m_r} - V'_{1i}\right)\frac{\partial c_i}{\partial m_r} - \left(\frac{\partial V'_{1i}}{\partial m_r} + \lambda_i\right) c_i$$

$$D'_{1i} = \left(4\pi\rho r^2\right)^2 \left(D_{\rm turb} + D_{1i}\right)$$

$$V'_{1i} = \left(4\pi\rho r^2\right) V_{1i}$$

$$V_{1i} = -D_{1i}\left[\left(A_i - \frac{Z_i}{2} - \frac{1}{2}\right)\left(\frac{m_H}{kT}\frac{Gm_r}{r^2}\right) - \alpha_{1i}\nabla \ln T\right]$$

Atomic diffusion: modeling

- Computation of the diffusion coefficients
  - Formalism of Paquette et al. (1986)

$$D_{st} = \frac{3E}{2nm(1-\Delta)} \quad \text{and} \quad \alpha_{st} = \frac{5C(x_s S_s - x_t S_t)}{x_s^2 Q_s + x_t^2 Q_t + x_s x_t Q_s t}$$

  - Collision integrals:

$$\Omega_{st}^{(ij)} = \left(\frac{kT}{2\pi m M_s M_t}\right)^{1/2} \int_0^\infty e^{-g^2} g^{2j+3} \phi_{st}^{(i)} \mathrm{d}g$$

$$\phi_{st}^{(i)} = 2\pi \int_0^\infty (1-\cos^i \chi_{st}) b\mathrm{d}b \quad \text{and} \quad \chi_{st} = \pi - 2\int_{r_{st}^{\min}}^\infty b\mathrm{d}r \left\{ r^2 \left[1 - \frac{b^2}{r^2} - \frac{V_{st}(r)}{g^2 kT}\right]^{1/2} \right\}^{-1}$$

with $r_{st}^{\min}$ defined by: $$1 - \frac{b^2}{\left(r_{st}^{\min}\right)^2} - \frac{V_{st}\left(r_{st}^{\min}\right)}{g^2 kT} = 0$$

Atomic diffusion: modeling

- Computation of the collision integrals
  - Static screened potential:

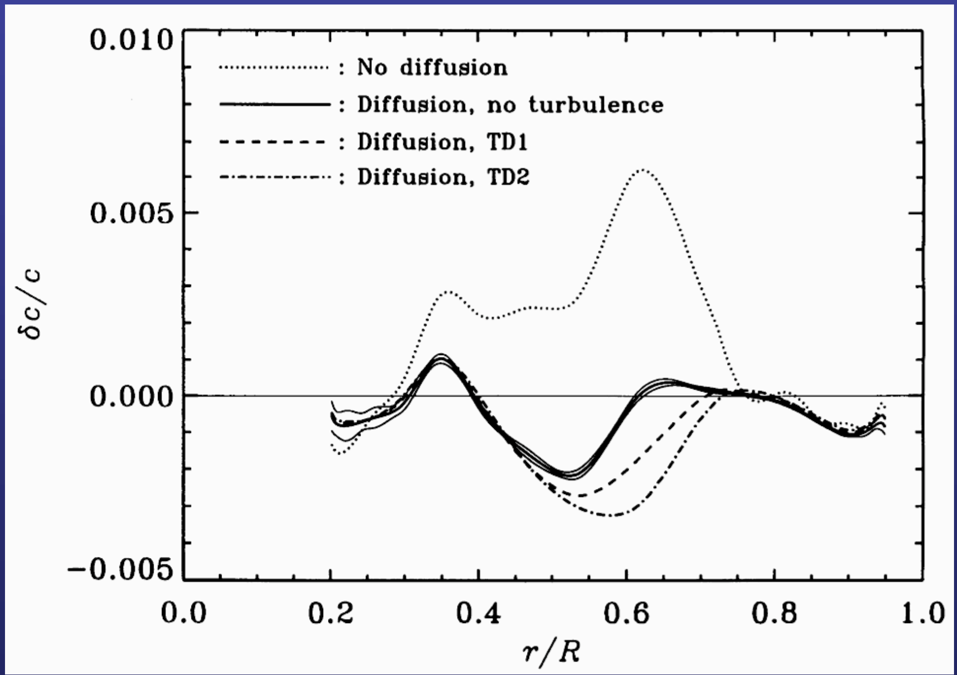
$$V_{st}(r) = Z_s Z_t e^2 \frac{e^{-r/\lambda}}{r} \quad \text{with} \quad \lambda_D = \left(\frac{kT}{4\pi e^2 \sum_i n_i Z_i^2} \right)^{1/2}, \quad \lambda_i = \left(\frac{3}{4\pi n_i} \right)^{1/3}$$

- Summary
  - Computation of the collision integrals
  - Determination of the diffusion coefficients D_{1i} and α_{1i}, as well as the diffusion velocity V_{1i}
  - The diffusion equation is then solved

Atomic diffusion

- Some applications

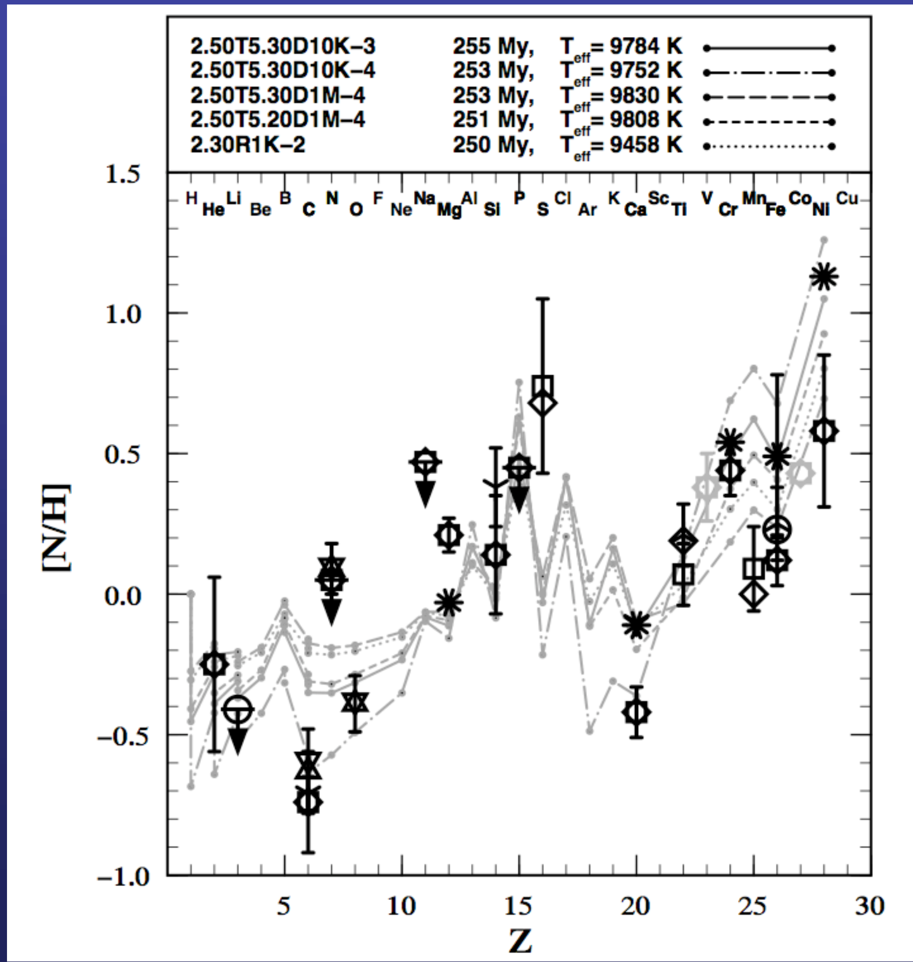
Solar models



Christensen-Dalsgaard et al. 1993

AmFm stars

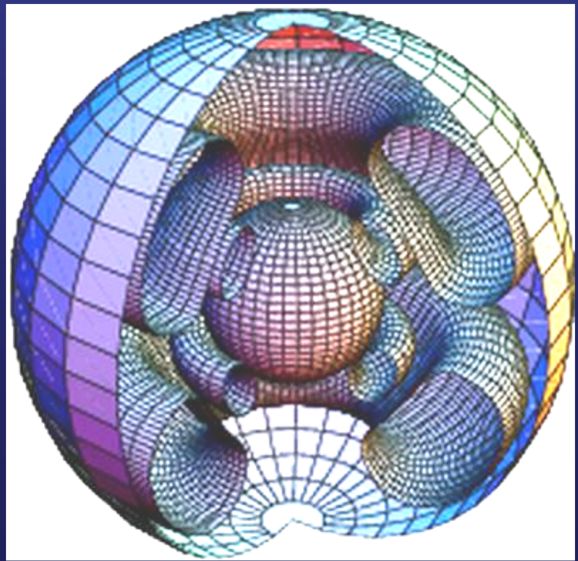
Richer et al. 2000



Rotation

- Physical description
  - Shellular rotation (Zahn 1992)
    - breaking of the spherical symmetry
    - meridional circulation
    - differential rotation and shear instabilities (D_{shear})
    - horizontal turbulence (D_h)

→ Ω is approximately constant on an isobar



Rotation: modeling

- Transport of angular momentum
  - Advection-diffusion equation:

$$\rho \frac{\mathrm{d}}{\mathrm{d}t}\left(r^2\Omega\right)_{M_r} = \frac{1}{5r^2}\frac{\partial}{\partial r}\left(\rho r^4 \Omega U(r)\right) + \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho D r^4 \frac{\partial \Omega}{\partial r}\right)$$

  - Meridional circulation:

$$U(r) = \frac{P}{\rho g C_P T[\nabla_{\mathrm{ad}} - \nabla + (\varphi/\delta)\nabla_\mu]} \times \left\{\frac{L}{M}(E_\Omega + E_\mu)\right\}$$

$$\text{with} \quad \nabla = \frac{\partial \ln T}{\partial \ln P}\,;\ \delta = -\left(\frac{\partial \ln \rho}{\partial \ln T}\right)_{P,\mu}\,;\ \varphi = \left(\frac{\partial \ln \rho}{\partial \ln \mu}\right)_{P,T}$$

  - Shear mixing:

$$D_{\mathrm{shear}} = \frac{4(K_T + D_{\mathrm{h}})}{\left[\frac{\varphi}{\delta}\nabla_\mu(1 + \frac{K}{D_{\mathrm{h}}}) + (\nabla_{\mathrm{ad}} - \nabla_{\mathrm{rad}})\right]} \times \frac{H_{\mathrm{p}}}{g\delta}\,\frac{\alpha}{4}\left(0.8836\,\Omega\,\frac{\mathrm{d}\ln\Omega}{\mathrm{d}\ln r}\right)^2$$

Rotation: modeling

- Transport of angular momentum
  - Horizontal turbulence:

$$D_{\rm h} = Ar\left(r\bar{\Omega}(r)V[2V - \alpha U]\right)^{\frac{1}{3}} \quad \text{with} \quad \alpha = \frac{1}{2}\frac{{\rm d}\ln r^2\bar{\Omega}}{{\rm d}\ln r}$$

  - Boundary conditions

momentum conservation and absence of differential rotation at convective boundaries

$$\frac{\partial}{\partial t}\left[\Omega\int_{r_{\rm t}}^{R} r^4\rho\,{\rm d}r\right] = -\frac{1}{5}r^4\rho\Omega U + \mathcal{F}_{\Omega} \qquad \text{for } r = r_{\rm t}$$

$$\frac{\partial}{\partial t}\left[\Omega\int_{0}^{r_{\rm b}} r^4\rho\,{\rm d}r\right] = \frac{1}{5}r^4\rho\Omega U \qquad \text{for } r = r_{\rm b}$$

$$\frac{\partial\Omega}{\partial r} = 0 \text{ for } r = r_{\rm t},\, r_{\rm b}$$

Rotation: modeling

- Transport of angular momentum
  - Magnetic braking:

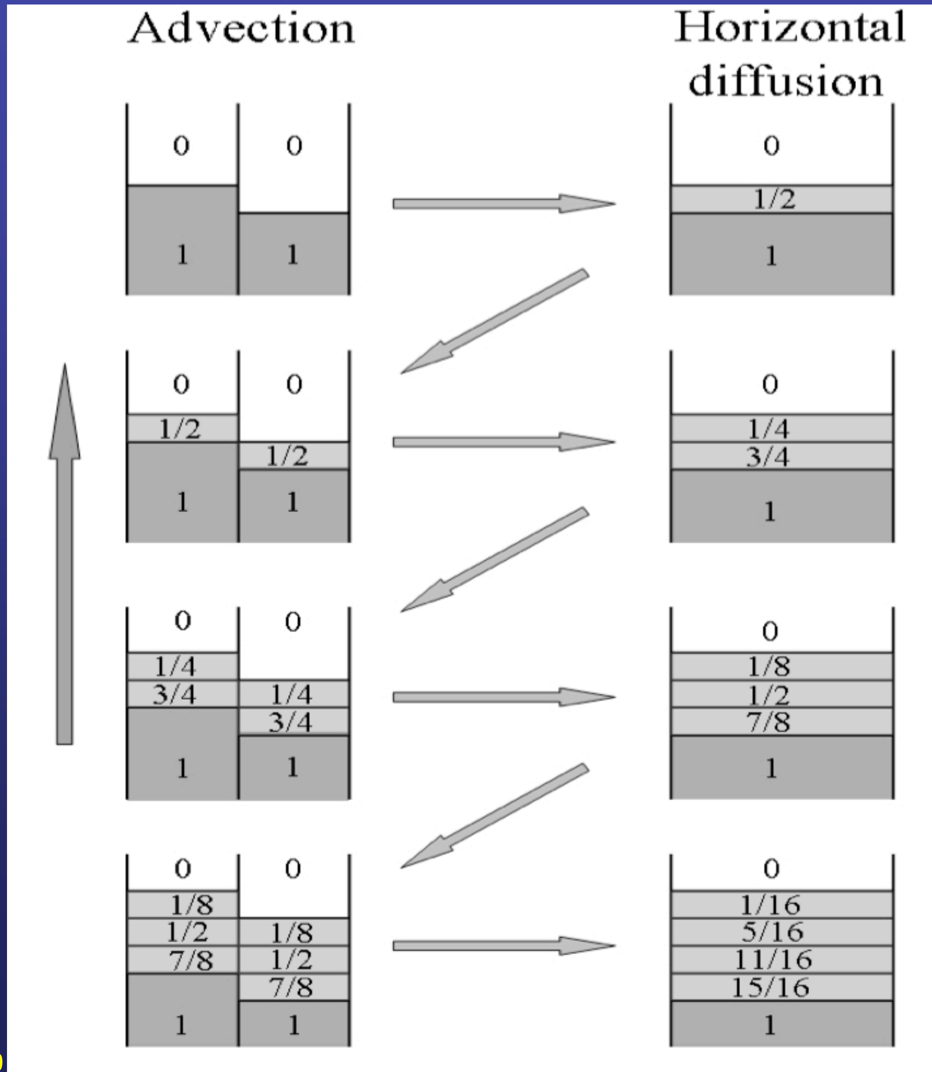
$$\frac{\mathrm{d}J}{\mathrm{d}t} = \begin{cases} -K\Omega^3 \left(\frac{R}{R_\odot}\right)^{1/2} \left(\frac{M}{M_\odot}\right)^{-1/2} & (\Omega \leq \Omega_{\mathrm{sat}}) \\ -K\Omega\,\Omega^2{}_{\mathrm{sat}} \left(\frac{R}{R_\odot}\right)^{1/2} \left(\frac{M}{M_\odot}\right)^{-1/2} & (\Omega > \Omega_{\mathrm{sat}}) \end{cases}$$

Rotation: modeling

- Transport of chemical elements
  - Diffusion equation:

$$\rho \frac{\mathrm{d}c_i}{\mathrm{d}t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \rho (D_{\mathrm{eff}} + D_{\mathrm{shear}})\frac{\partial c_i}{\partial r}\right] + \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \rho V_{\mathrm{at}} c_i\right] + \rho \dot{c}_i$$

$$D_{\mathrm{eff}} = \frac{|rU(r)|^2}{30 D_{\mathrm{h}}}$$



Maeder 2009

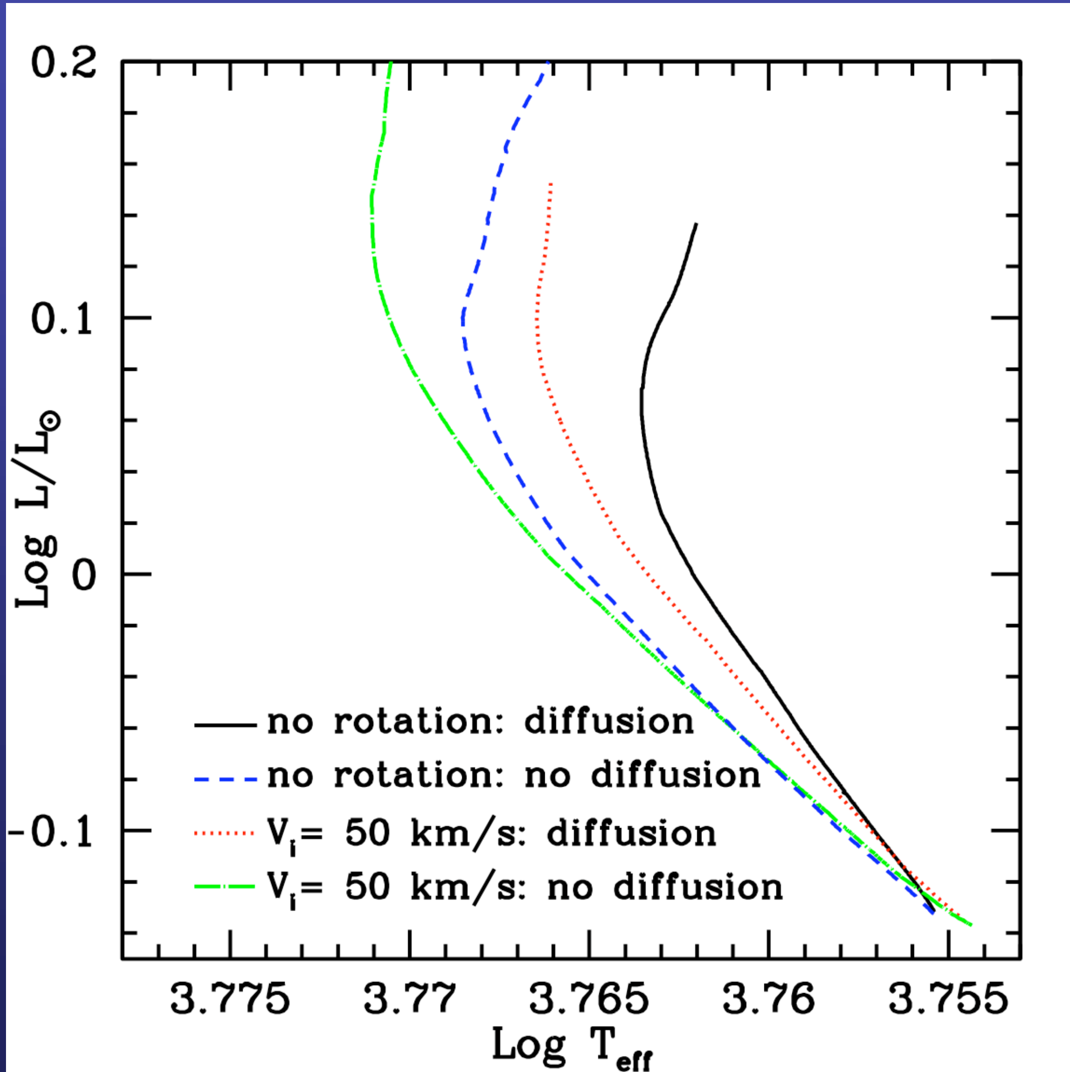
Effects of rotation and atomic diffusion

- 1 $M_\odot$ models
  - Rotation:

    shift to the blue due to rotational mixing

  - Atomic diffusion:

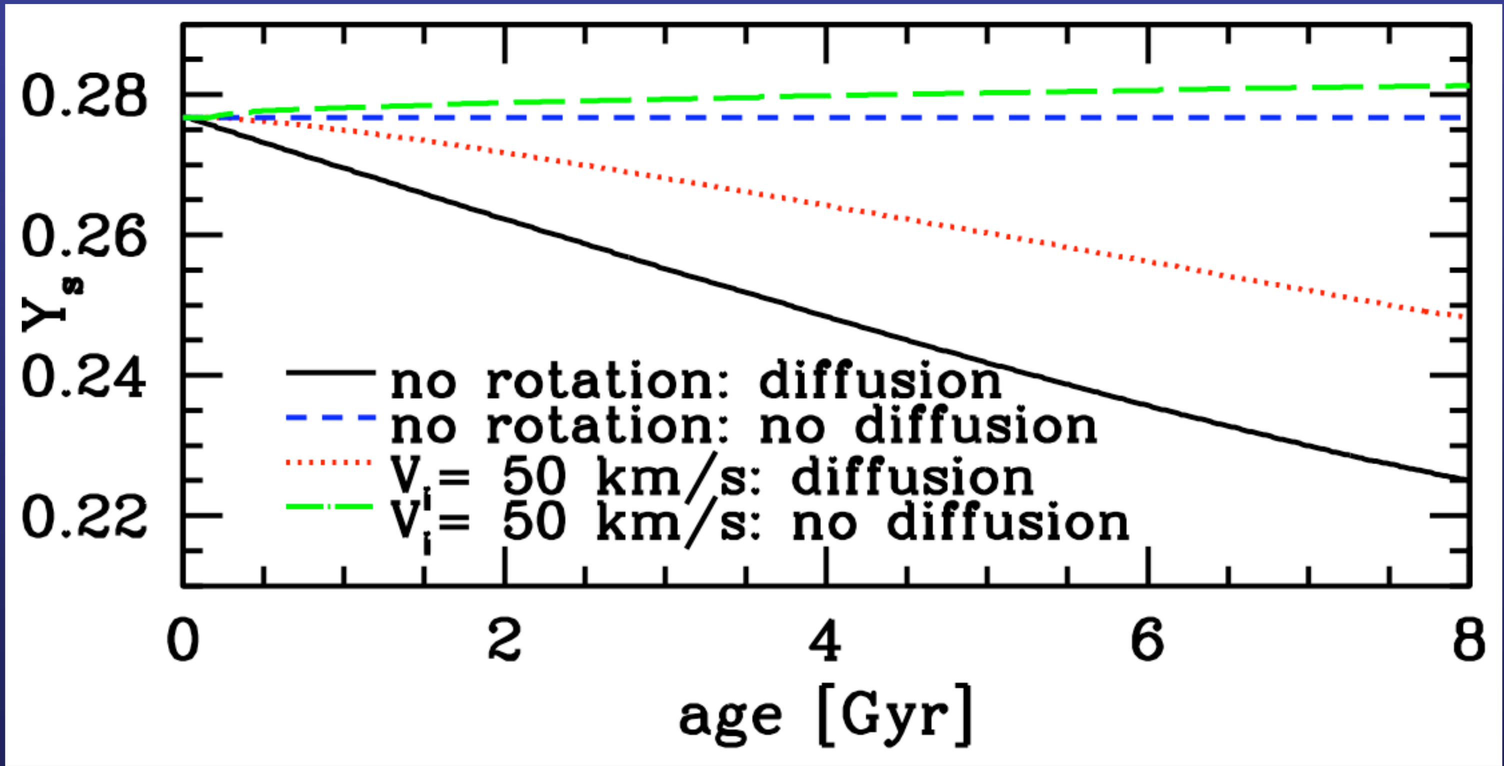
    decrease of L and T_{eff}



Eggenberger et al. 2010a

Effects of rotation and atomic diffusion

- Surface abundances
  - Rotation counteracts the effects of atomic diffusion in the external layers

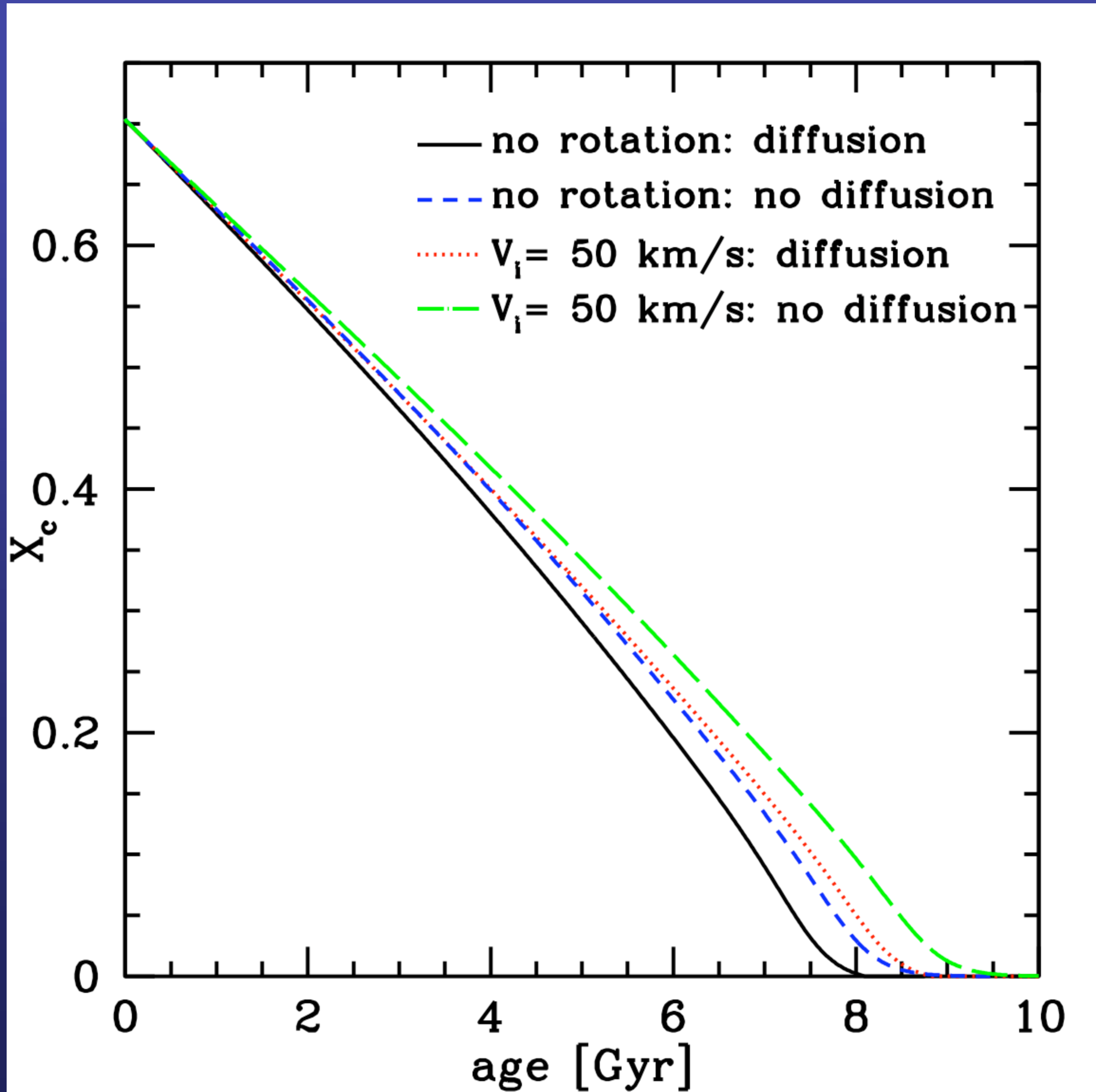


Effects of rotation and atomic diffusion

- Central layers
  - Rotation:

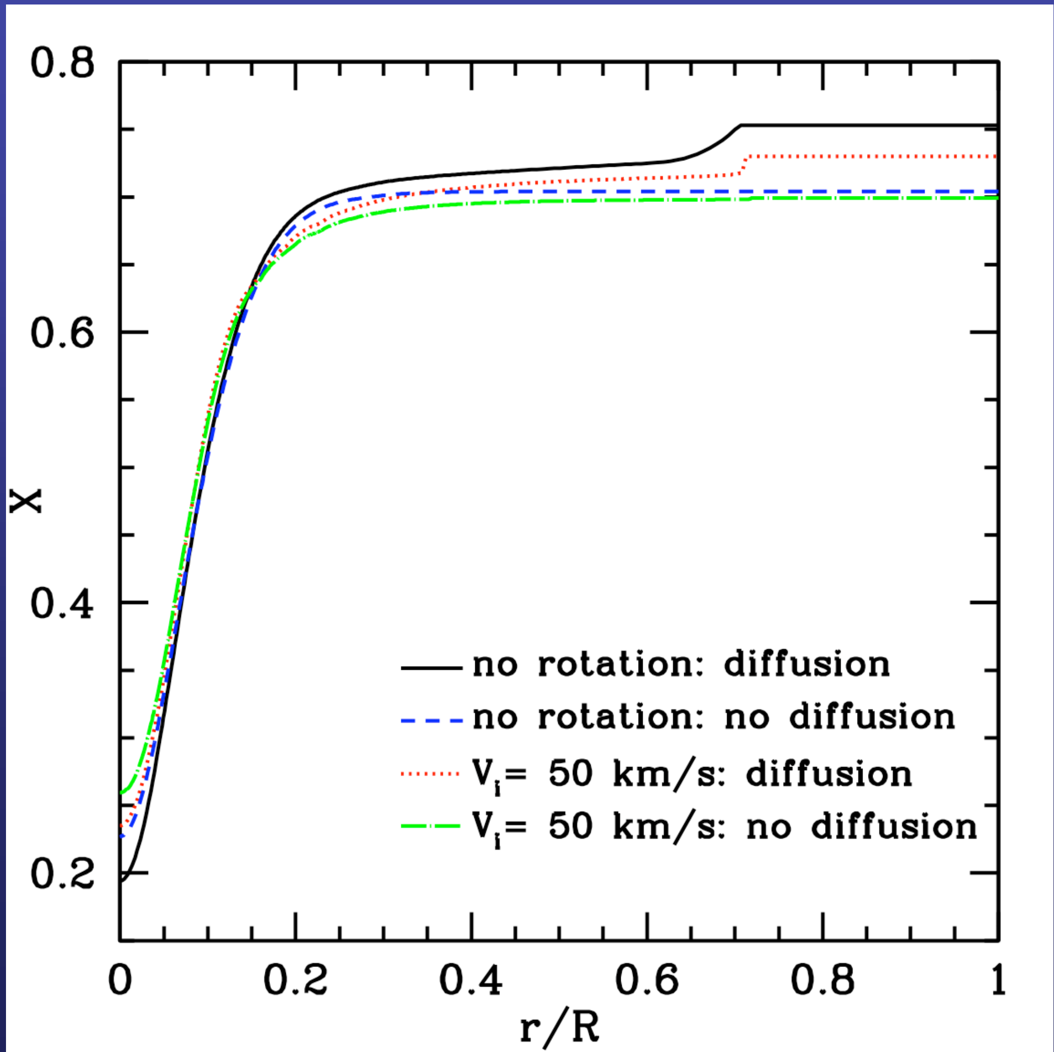
X_c increases
→ the MS lifetime is enhanced

Larger efficiency of rotational mixing in the central layers



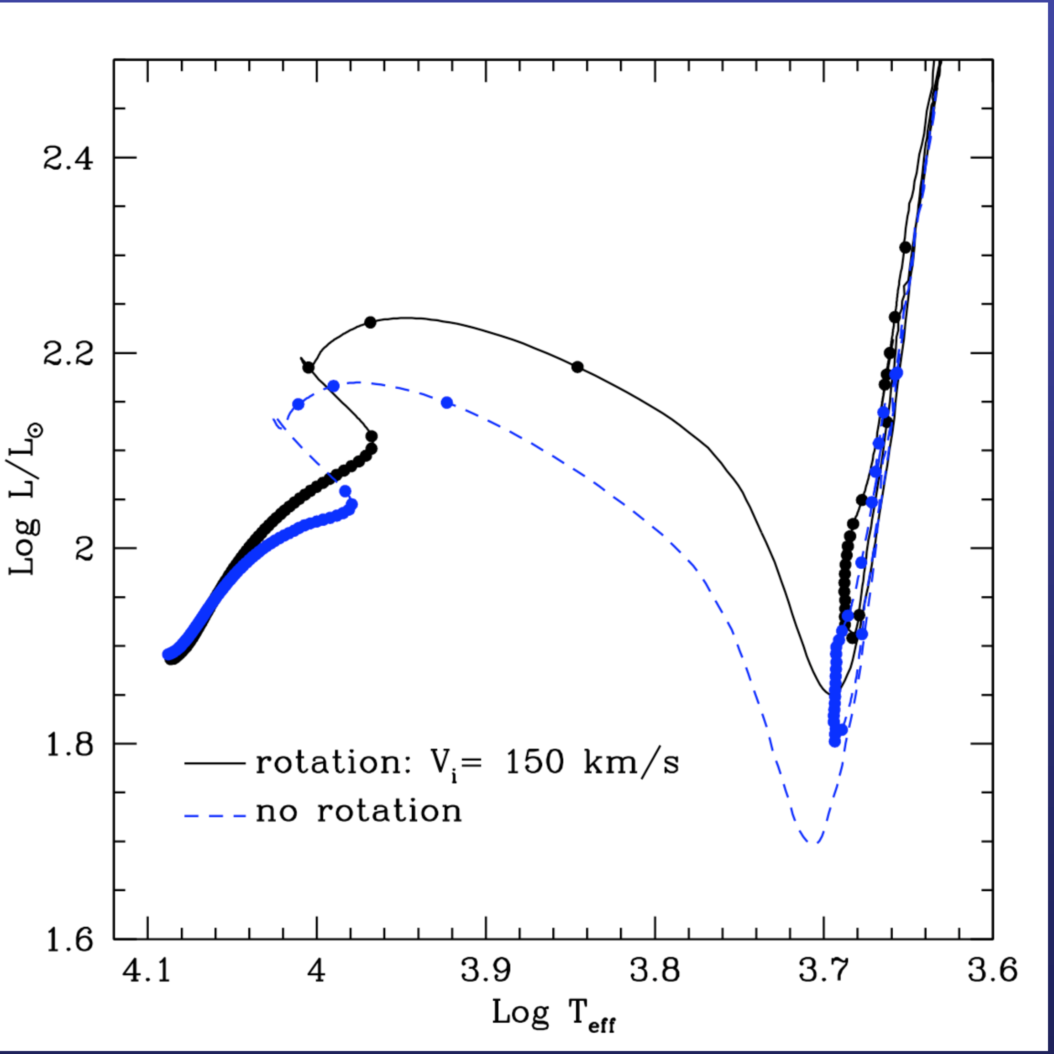
Effects of rotation and atomic diffusion

- Chemical profiles



Effects of rotation and atomic diffusion

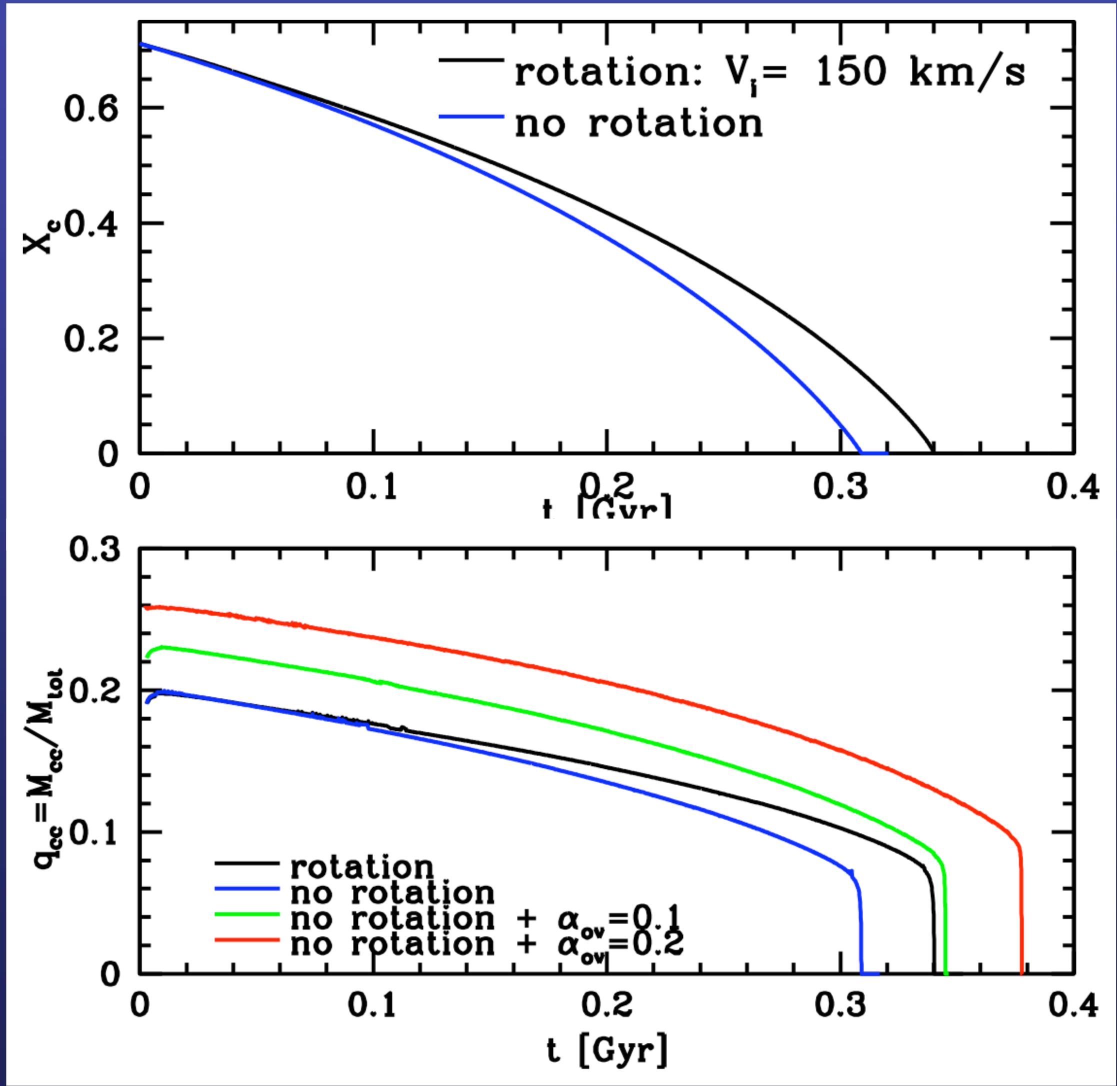
- 3 $M_\odot$ models



Eggenberger et al. 2010b

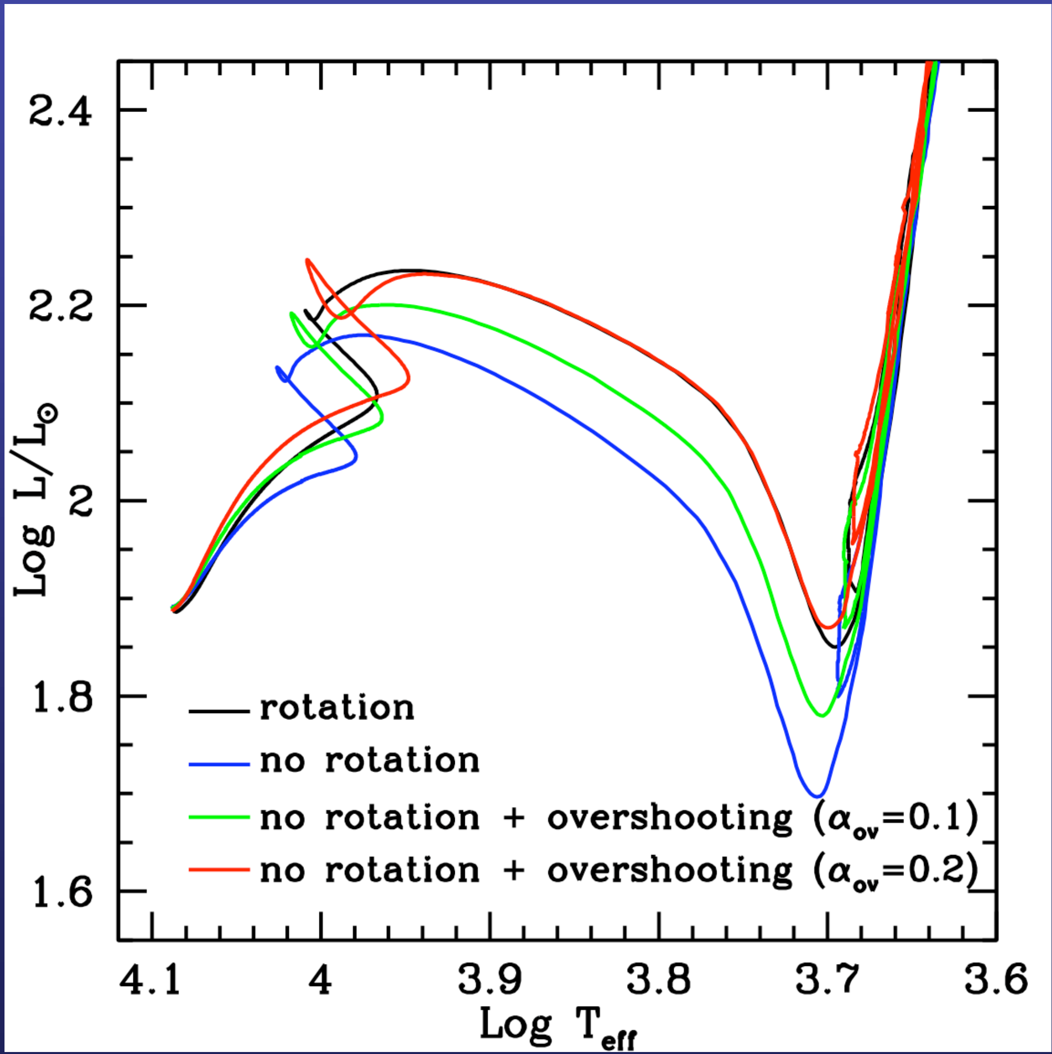
Effects of rotation and atomic diffusion

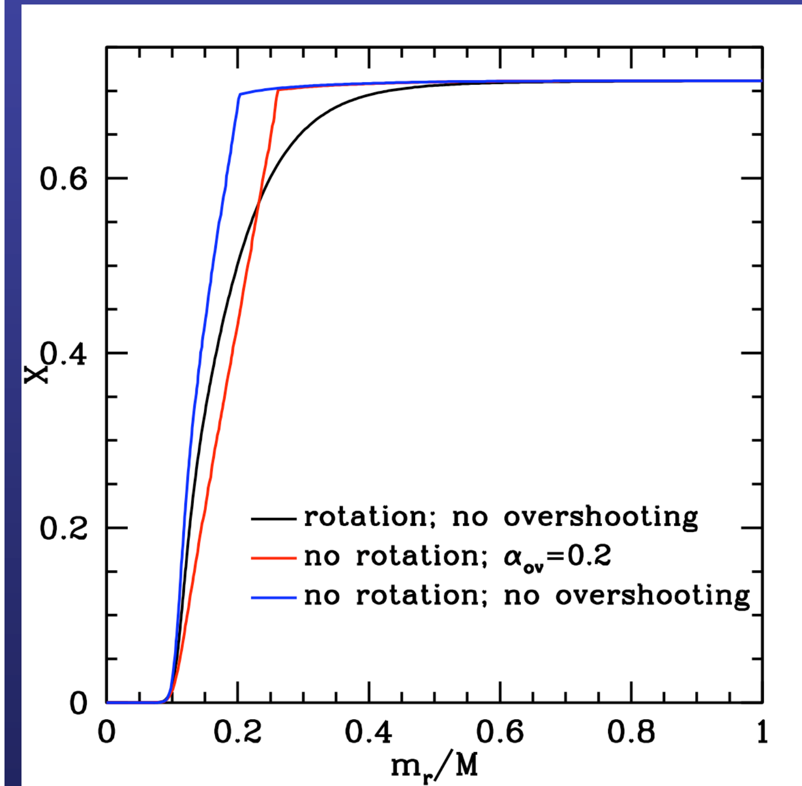
- 3 $M_\odot$ models: convective cores



Effects of rotation and atomic diffusion

- 3 $M_\odot$ models: overshooting and rotational mixing





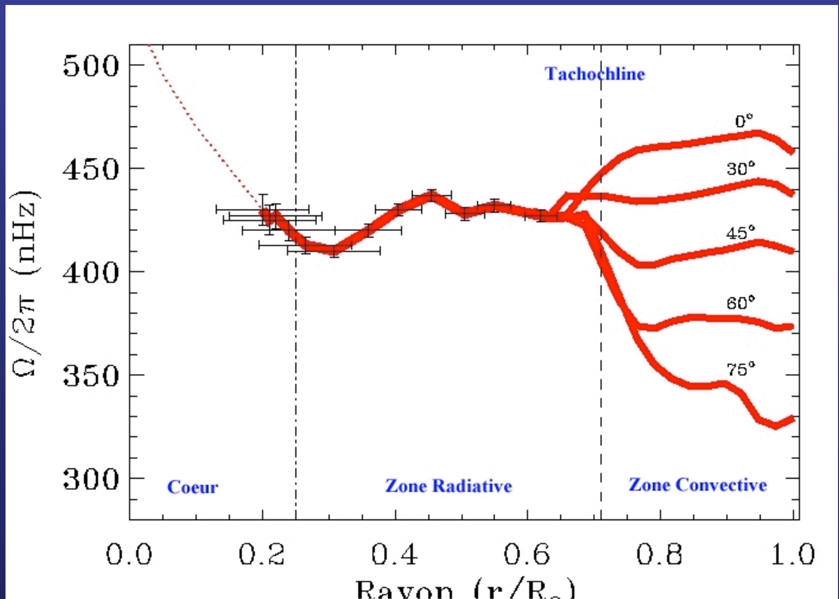
Magnetic fields and Internal Gravity Waves

- The solar rotation profile
  - Problem with shellular rotation

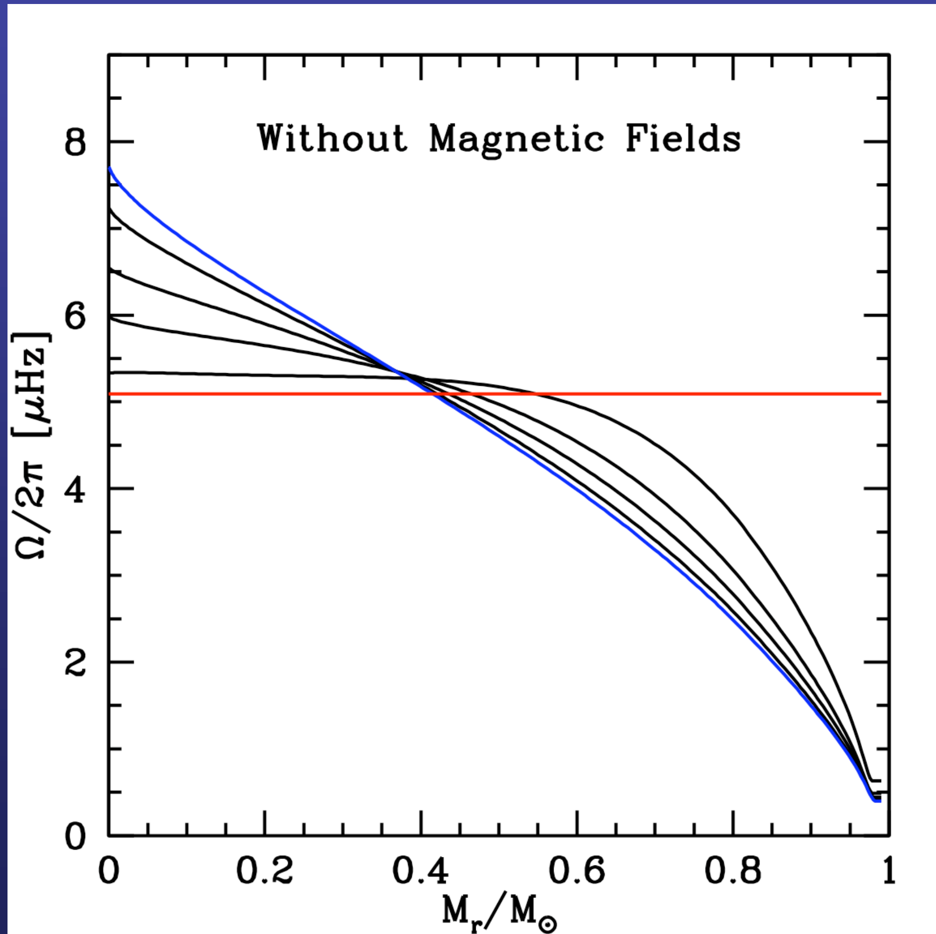
Pinsonneault et al. 1989

Chaboyer, Demarque & Pinsonneault 1995

Talon et al. 1997

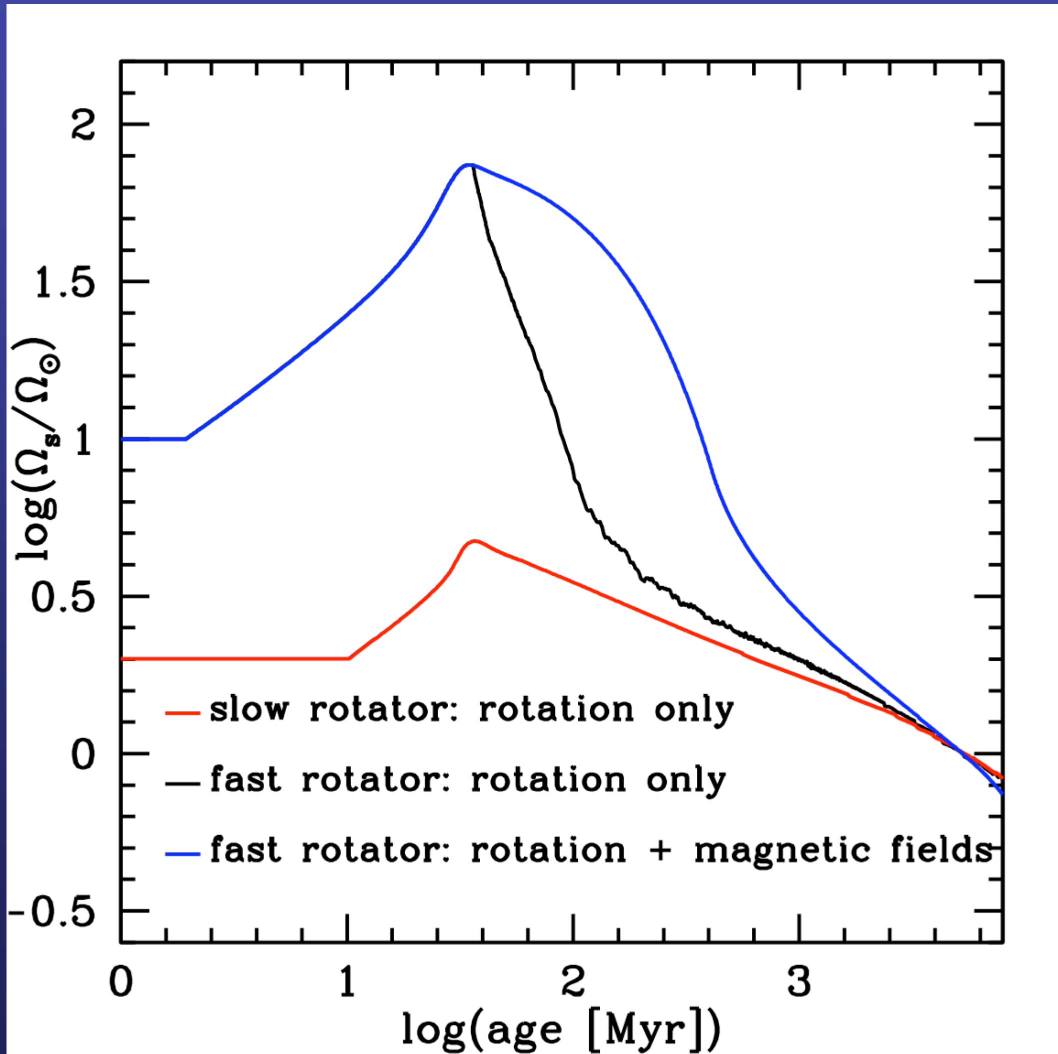


Garcia et al. 2007



Magnetic fields and Internal Gravity Waves

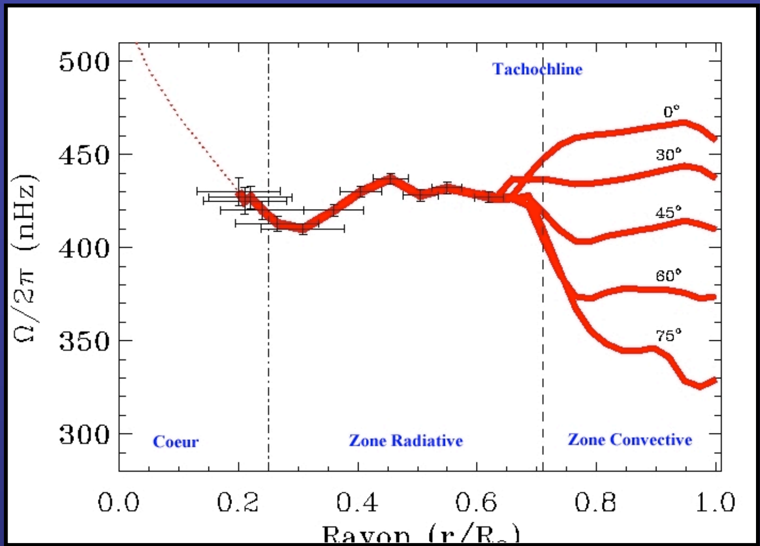
- Importance of Magnetic Fields
  - Disk-locking during the PMS
  - Magnetic braking during the MS
  - Internal magnetic fields



Eggenberger, Maeder & Meynet 2010

Magnetic fields and Internal Gravity Waves

- Modeling of Internal Magnetic fields



Garcia et al. 2007

  - Transport of angular momentum:

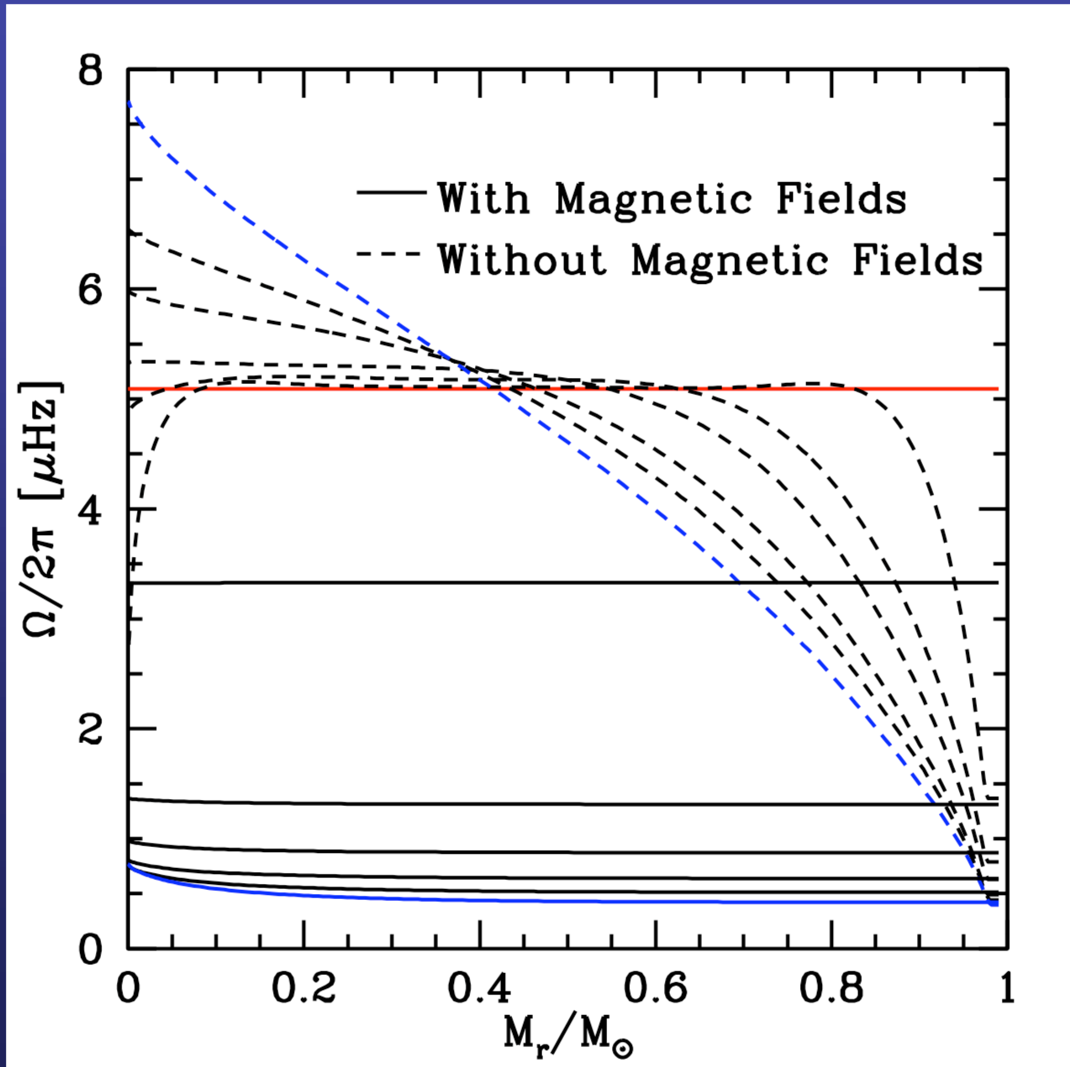
$$\rho \frac{\mathrm{d}}{\mathrm{d}t}\left[r^2\Omega\right] = \frac{1}{5r^2}\frac{\partial}{\partial r}\left[\rho r^4 \Omega U\right] + \frac{1}{r^2}\frac{\partial}{\partial r}\left[\rho (D_{\mathrm{shear}} + \nu_{\mathrm{magn}}) r^4 \frac{\partial \Omega}{\partial r}\right]$$

    - Tayler-Spruit dynamo (Spruit 1999, 2002; Maeder & Meynet 2004, 2005)

$$\nu_{\mathrm{magn}} = \frac{\Omega\, r^2}{q}\left(\frac{\omega_{\mathrm{A}}}{\Omega}\right)^3 \left(\frac{\Omega}{N}\right) \quad \text{with } q = -\frac{\partial \ln \Omega}{\partial \ln r}\ ;\ \omega_{\mathrm{A}} = \frac{B}{(4\,\pi\rho)^{\frac{1}{2}}\ r}$$

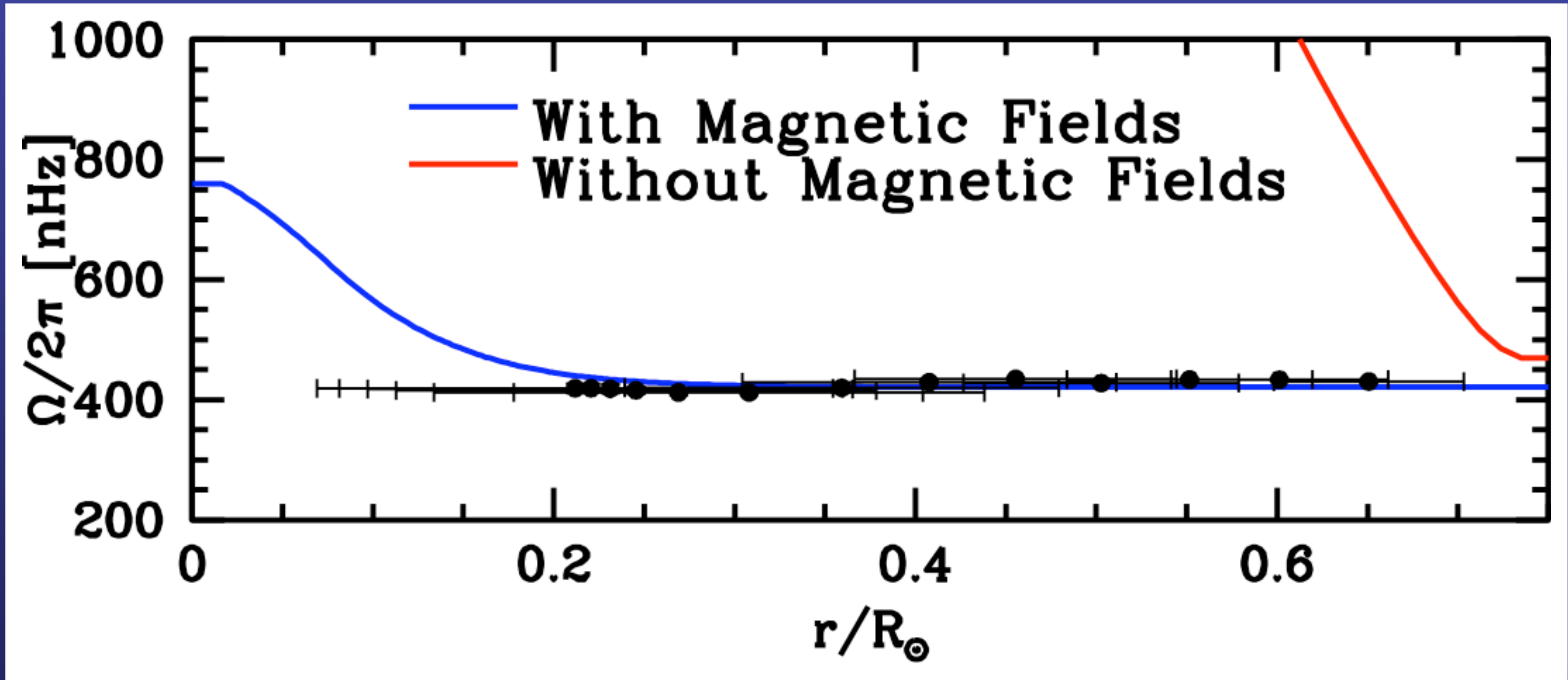
Magnetic fields and Internal Gravity Waves

- Effects of Magnetic fields



Magnetic fields and Internal Gravity Waves

- Effects of Magnetic fields
  - The solar rotation profile



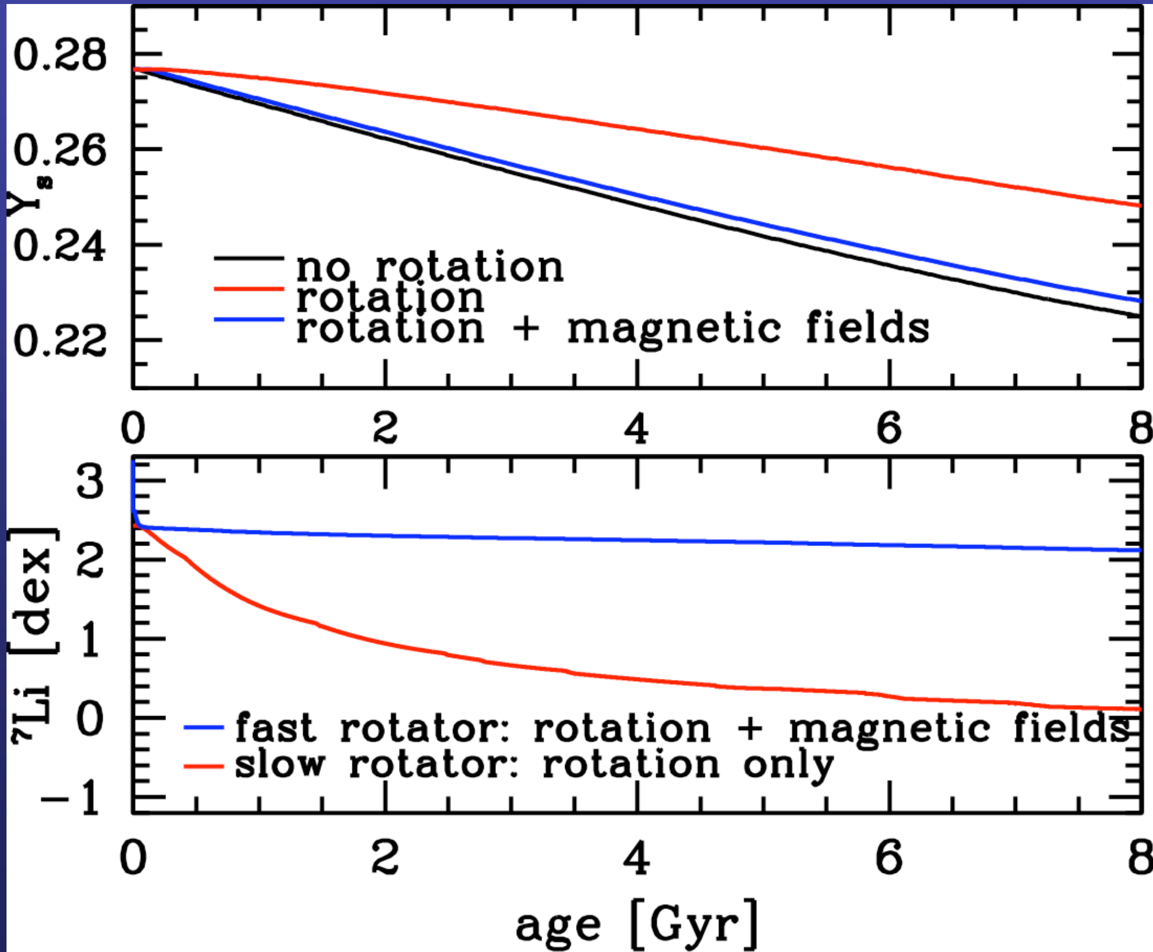
Eggenberger, Maeder & Meynet 2005

Magnetic fields and Internal Gravity Waves

- Effects of Magnetic fields
  - Transport of chemical elements with magnetic fields:

mixing is strongly reduced

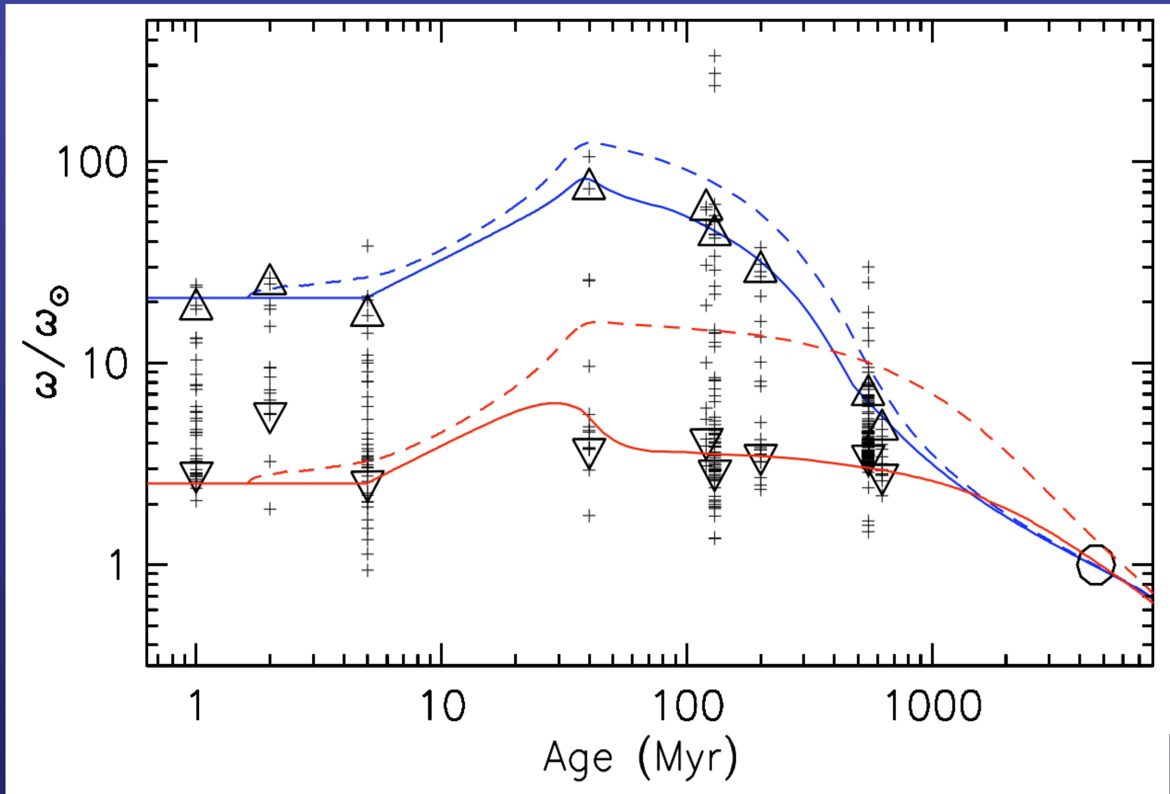
fast rotators with high Li abundances



Eggenberger, Maeder & Meynet 2010

Magnetic fields and Internal Gravity Waves

- Rotational history of exoplanet-host stars
  - Rotation rates of solar-type stars:



Bouvier 2008

fast rotators : little differential rotation

slow rotators : strong differential rotation

Magnetic fields and Internal Gravity Waves

- Rotational history of exoplanet-host stars
  - Disk lifetime: if longer then …
    - Lower rotation on the ZAMS
      disk locking
    - Higher Li depletion
      efficient mixing associated to differential rotation in slow rotating solar-type stars
    - Giant exoplanets formation and/or migration
    - Other correlations?
      asteroseismic observables

Observations in the Pleiades (Soderblom et al. 1993)

Observations by Israelian et al. (2009) (see also Baumann et al. 2010)

Magnetic fields and Internal Gravity Waves

- Modeling of Internal Gravity Waves
  - Transport of angular momentum

$$\rho \frac{\mathrm{d}}{\mathrm{d}t}\left[r^2\Omega\right] = \frac{1}{5r^2}\frac{\partial}{\partial r}\left[\rho r^4 \Omega U\right] + \frac{1}{r^2}\frac{\partial}{\partial r}\left[\rho (D_{\mathrm{shear}} + \nu_{\mathrm{waves}}) r^4 \frac{\partial \Omega}{\partial r}\right] - \frac{3}{8\pi}\frac{1}{r^2}\frac{\partial}{\partial r}\mathcal{L}_J(r)$$

  - Net momentum deposition

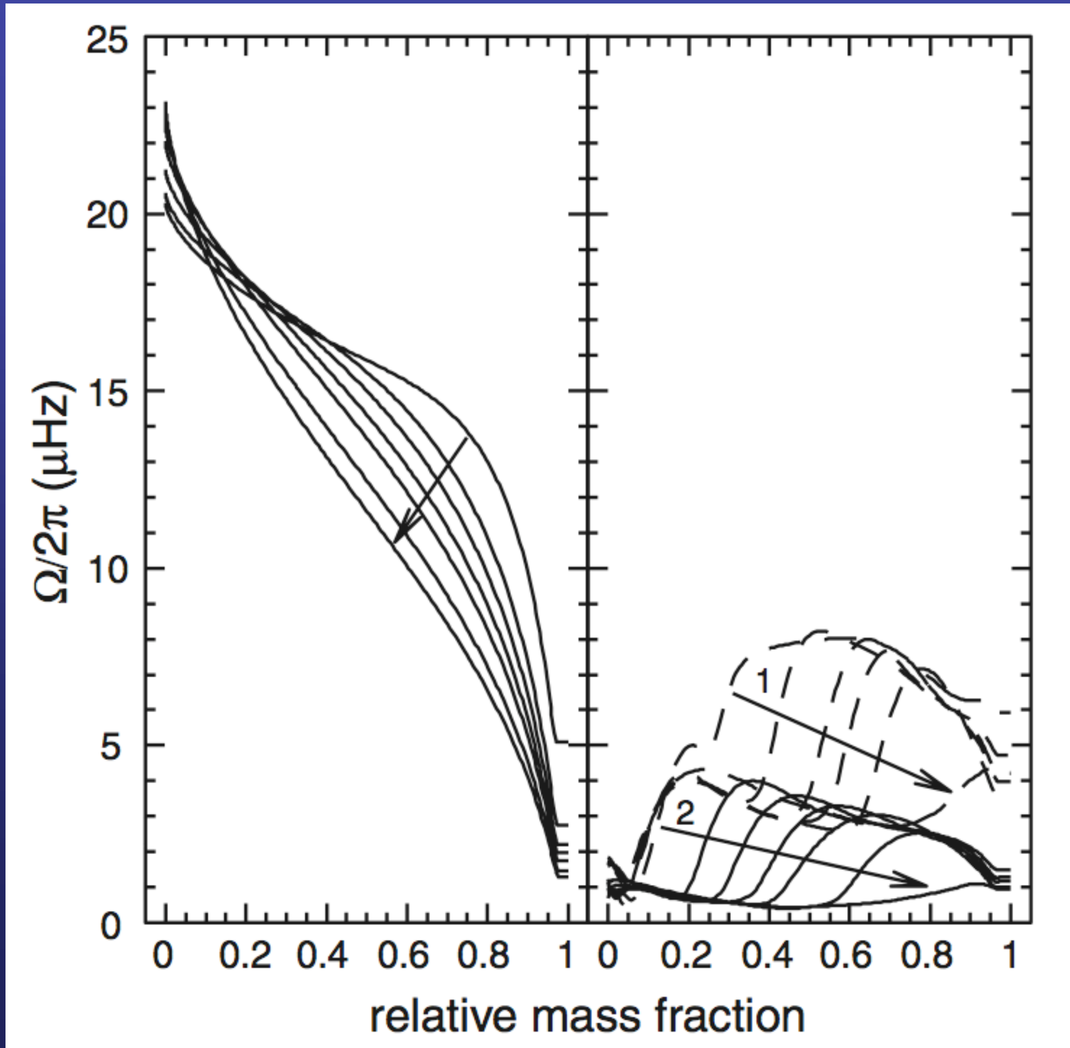
$$\mathcal{L}_J(r) = \sum_{\sigma,\ell,m} \mathcal{L}_{J\ell,m}\left(r_{\mathrm{shear\ \ layer}}\right) \exp\left[-\tau(r,\sigma,\ell)\right]$$

$$\tau = \left[\ell(\ell+1)\right]^{\frac{3}{2}} \int_r^{r_c} (K + \nu_t)\ \frac{N N_T^2}{\sigma^4} \left(\frac{N^2}{N^2 - \sigma^2}\right)^{\frac{1}{2}} \frac{\mathrm{d}r}{r^3}$$

$$\sigma(r) = \omega - m\left(\Omega(r) - \Omega_{\mathrm{cz}}\right)$$

Magnetic fields and Internal Gravity Waves

- Effects of Internal Gravity Waves



Charbonnel & Talon 2005

Magnetic fields and Internal Gravity Waves

- Effects of Internal Gravity Waves
  - The Lithium dip

T_{eff} ≥ 6900 K:

no magnetic braking,

no shear: no Li destruction

6900 K ≥ T_{eff} ≥ 6650 K:

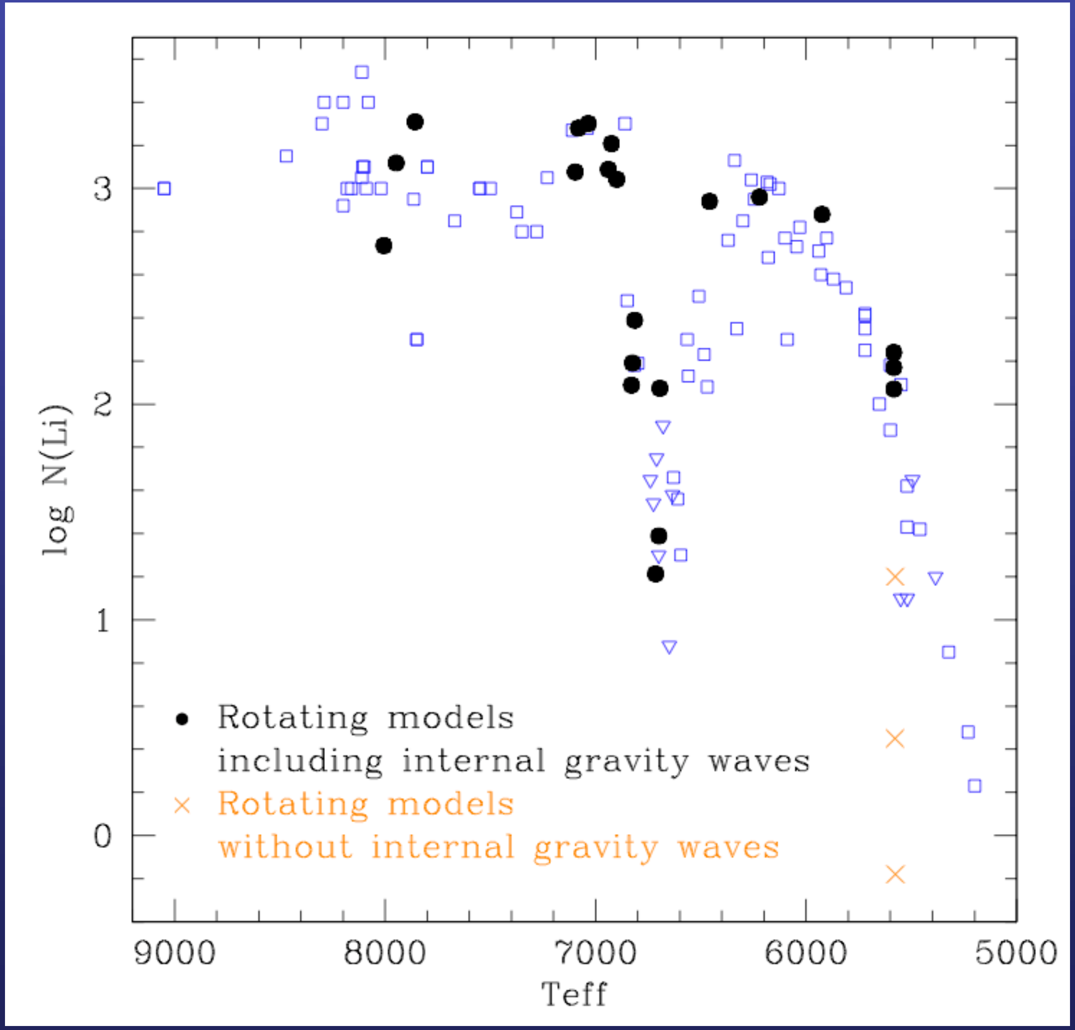
magnetic braking, shear

mixing: Li destruction

6650 K ≥ T_{eff} ≥ 6200 K:

IGW, no shear mixing:

no Li destruction



Charbonnel & Talon 2005

Summary

- Atomic diffusion
  - gravitational settling, thermal and radiative diffusion
  - application to solar models and abundances of AmFm stars

- Rotation
  - increase of T_{eff}, Y_s, X_c and of the main-sequence lifetime
  - counteracts the effects of atomic diffusion

- Effects of magnetic fields and internal gravity waves
  - rotation profile of the Sun
  - mixing is strongly reduced (solar-type stars ≠ massive stars)
  - exoplanet-host stars: Li abundances and presence of planets can be related to the rotational history of the star
  - internal gravity waves as a possible explanation for the Li dip