Types of Equilibria

- Steady Flow of Reactions
- Chemical Equilibrium of Reactions
- Complete Chemical Equilibrium (NSE)
- Clusters of Chemical Equilbrium (QSE)
- QSE Clusters linked by Steady Flow

CNO(I)-Cycle in Steady Flow

The CNO-Cycles in Hydrogen Burning

cycle	reaction sequence
CNOI	$^{12}C(p,\gamma)^{13}N(e^+\nu)^{13}C(p,\gamma)^{14}N\ (p,\gamma)^{15}O(e^+\nu)^{15}N(p,\alpha)^{12}C$
CNOII	$^{15}N(p,\gamma)^{16}O(p,\gamma)^{17}F\ (e^+\nu)^{17}O(p,\alpha)^{14}N$
CNOIII	$^{17}O(p,\gamma)^{18}F(e^+\nu)^{18}O(p,\alpha)^{15}N$
CNOIV	$^{18}O(p,\gamma)^{19}F(p,\alpha)^{16}O$

$$\dot{Y}_1 = -\rho N_A \langle 12,1\rangle Y_{12}Y_1 - \rho N_A \langle 13,1\rangle Y_{13}Y_1 - \rho N_A \langle 14,1\rangle Y_{14}Y_1 - \rho N_A \langle 15,1\rangle Y_{15}Y_1$$
$$= -4C_{CNO} = -4\rho N_A \langle 14,1\rangle Y_{14}Y_1 = -\frac{1}{\tau_{1,14}}Y_1$$

$$\dot{Y}_4 = \rho N_A \langle 15,1\rangle Y_{15}Y_1 = C_{CNO}$$

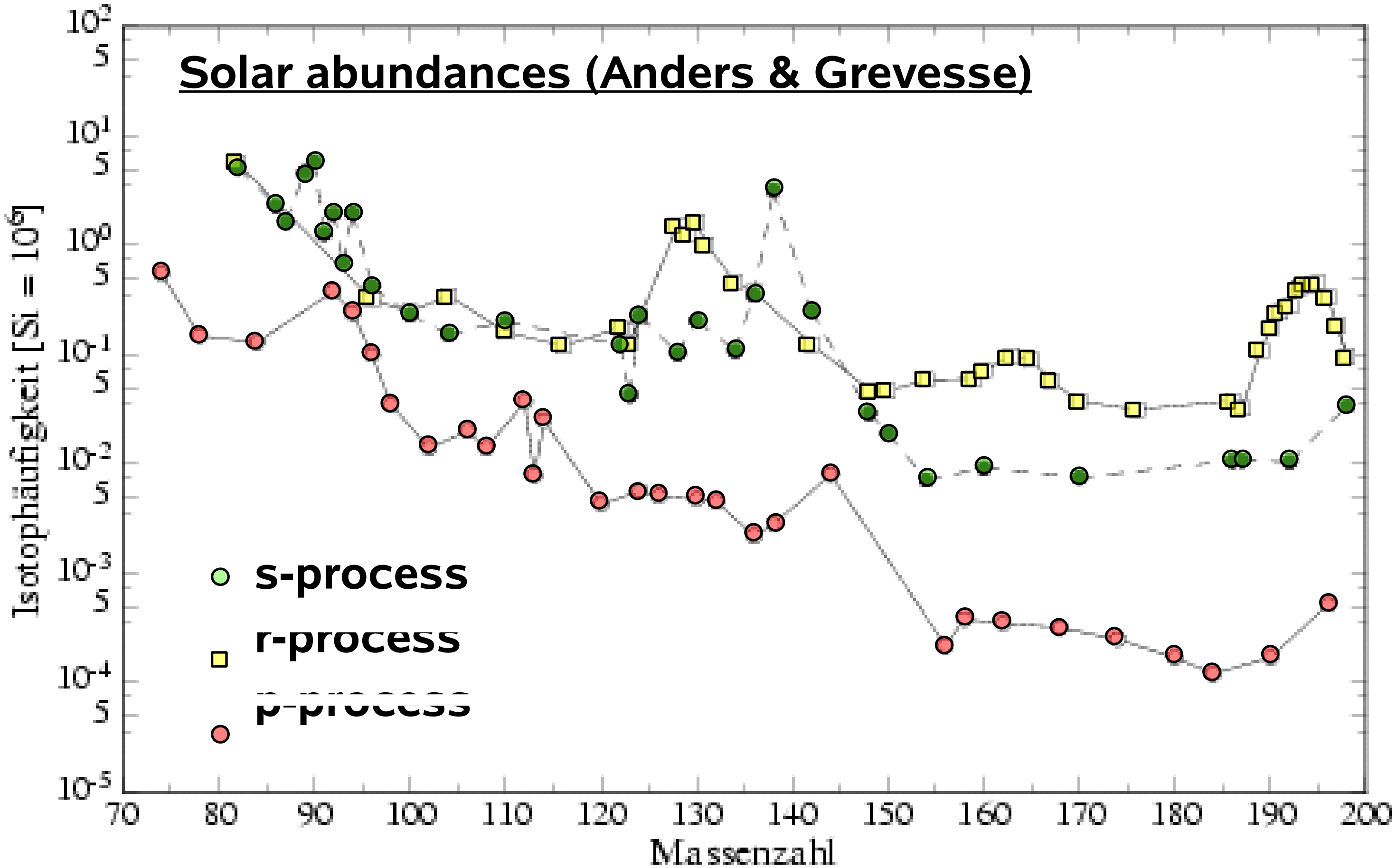
the network entry for nuclei with mass numbers A=12, 13, 14, 15 is governed in each case by a production reaction (proton reaction on A-1) and a distruction reaction (proton reaction on A). In case of a steady flow they cancel and lead to Y=0 for all A, linking all of these terms and identical to (A=14 is useful as this encounters the slowest reaction and essentially all mass assembles in ^{14}N)

$$C_{CNO} = \rho N_A \langle 14,1\rangle Y_{14}Y_1$$

$$Y_{14} \approx \frac{1.4\times 10^{-2}}{14}$$

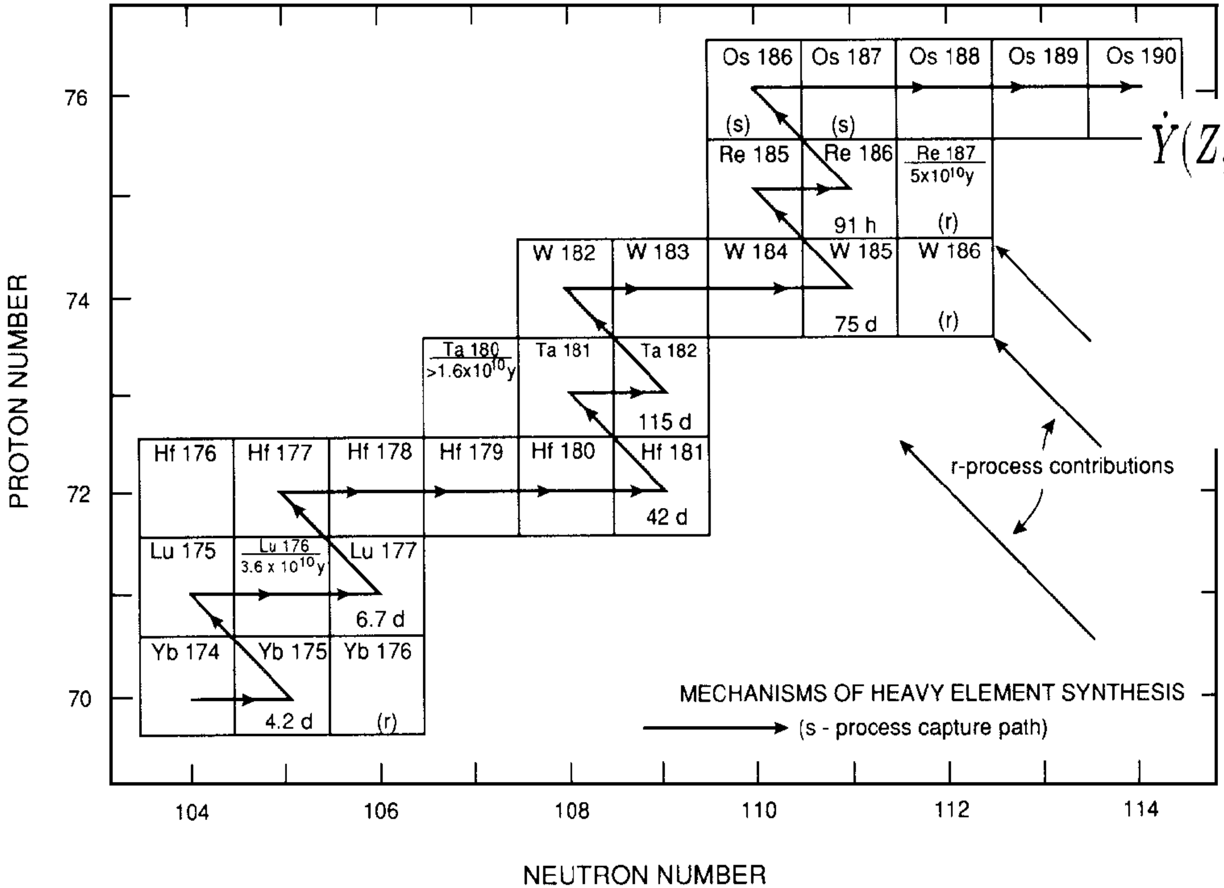
summing all mass fractions of CNO nuclei for solar metallicity

s-process and steady flow



shown are s-, r-, and p-only nuclei!

s-process and steady flow



possible destruction of nucleus (Z,A)

$$\dot{Y}(Z,A) = -\lambda_{\beta^-}(Z,A)Y(Z,A) - \rho N_A <\sigma v>_{n,\gamma} Y_n Y(Z,A)$$
$$= -\lambda_{\beta^-}(Z,A)Y(Z,A) - <\sigma v>_{n,\gamma} n_n Y(Z,A)$$
$$= -\frac{1}{\tau_\beta}Y(Z,A) - \frac{1}{\tau_{n,\gamma}}Y(Z,A).$$

$\tau_n > \tau_\beta$ beta-decay to (Z+1,A)

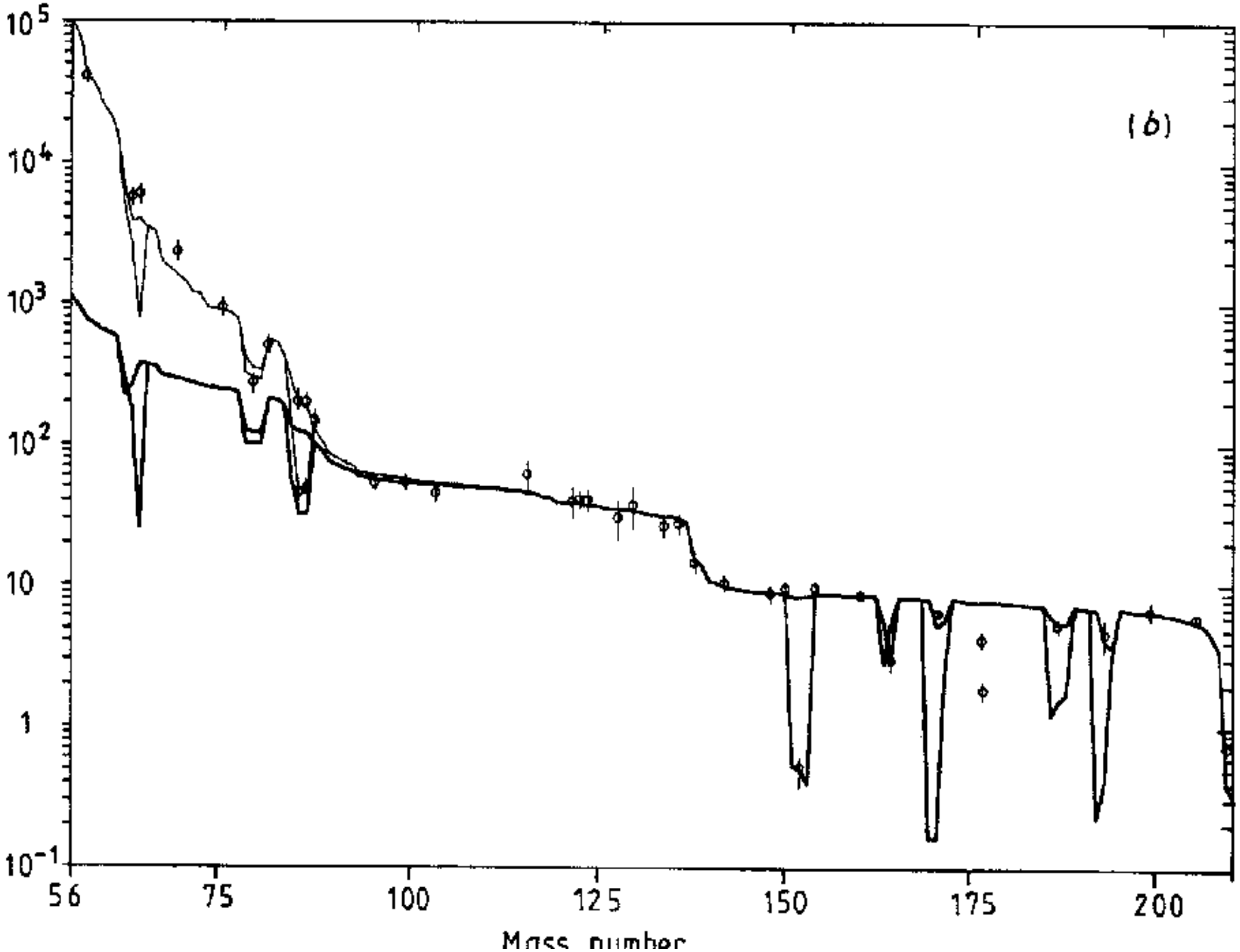
only one nucleus per A needs to be considered!

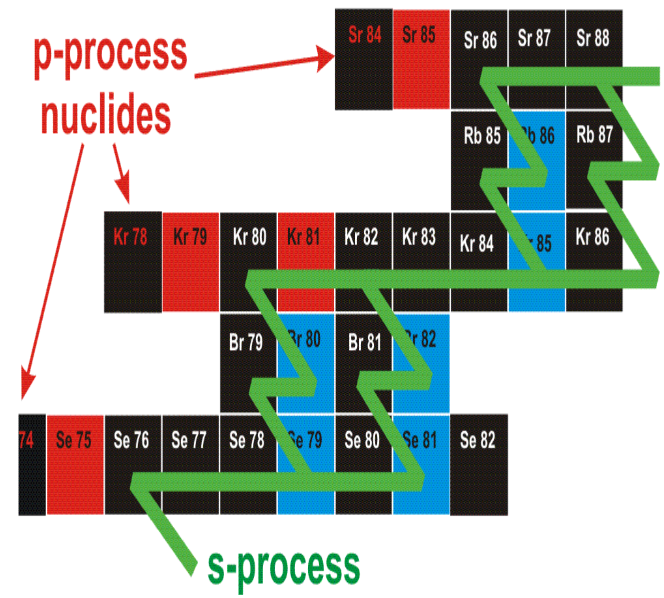
$$\dot{Y}(A) = n_n <\sigma v>_{n,\gamma} Y(A-1) - n_n <\sigma v>_{n,\gamma} Y(A)$$ in case of steady flow =0

$\sigma \approx 1/v, <\sigma v> = \sigma(v)v$ therefore

$$\sigma(A-1, 30\ \mathrm{keV})Y(A-1) = \sigma(A, 30\ \mathrm{keV})Y(A)$$

The sigma*N-curve





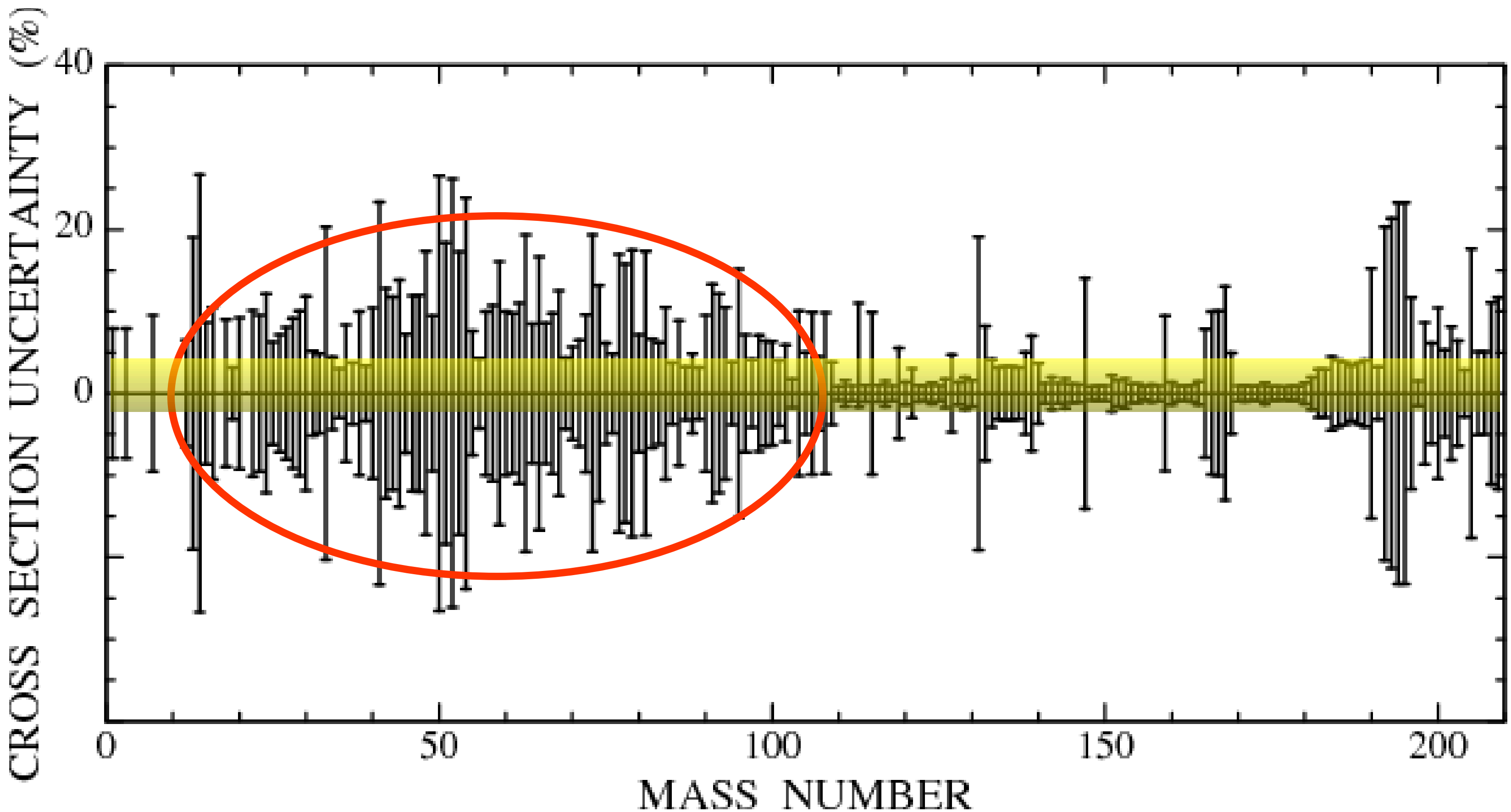
double values due to branchings

a complete steady flow is not given, but in between magic numbers (where the neutron capture cross sections are small) almost attained!

Käppeler et al. (2007): needed cross sections with uncertainties between **1** and **5%**

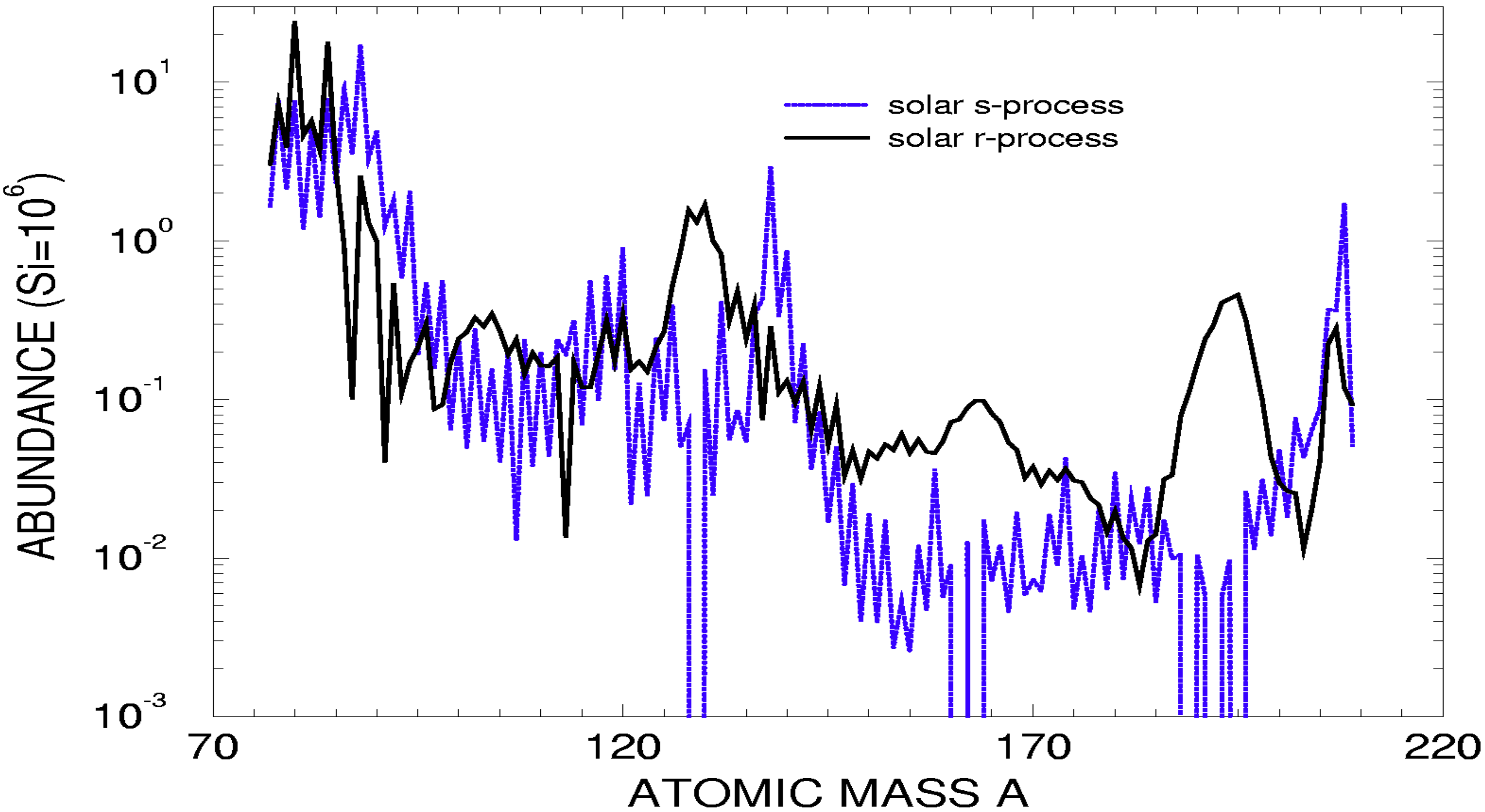
for complete set of isotopes from ^{12}C to ^{210}Po,

including unstable samples (branching points)



n_TOF (CERN), FRANZ (Frankfurt)

s- and r-decomposition



the almost constant sigma*N-curve leads to a large odd-even staggering in the abundances (due to the odd-even staggering in n-capture cross sections!

Steady flows and chem. equilibrium in stellar burning

pp-cycles and CNO-cycle lead to steady flows in H-burning

1. Hydrogen Burning $T = (1\text{-}4) \times 10^7$ K

   pp-cycles -> $^1H(p,e^+\nu)^2H$

   CNO-cycle -> slowest reaction $^{14}N(p,\gamma)^{15}O$

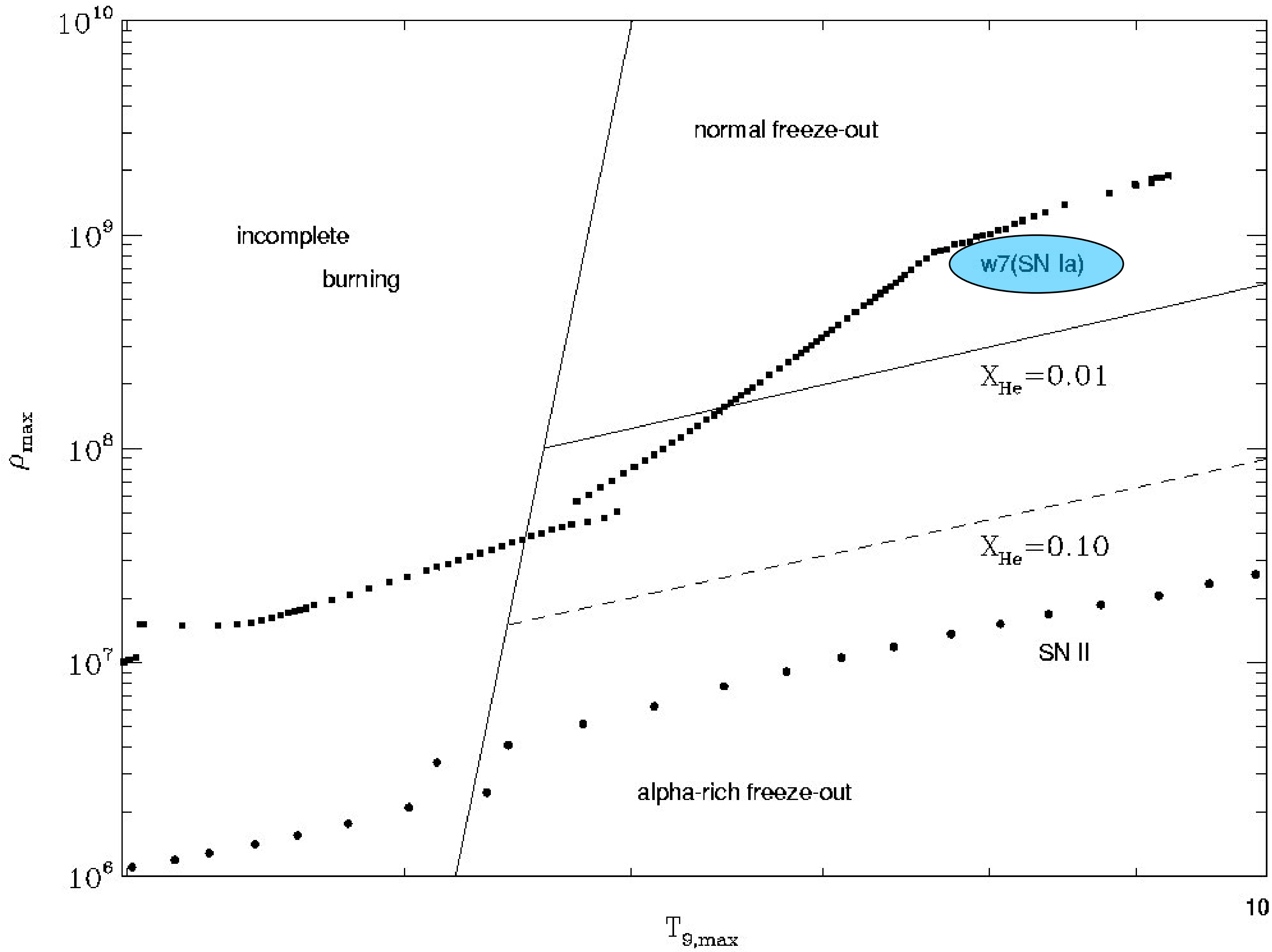
2. Helium Burning $T = (1\text{-}2) \times 10^8$ K

   $^4He + ^4He \Leftrightarrow ^8Be$ $^8Be(\alpha,\gamma)^{12}C[(\alpha,\gamma)^{16}O]$

   $^{14}N(\alpha,\gamma)^{18}F(\beta^+)^{18}O(\alpha,\gamma)^{22}Ne(\alpha,n)^{25}Mg$

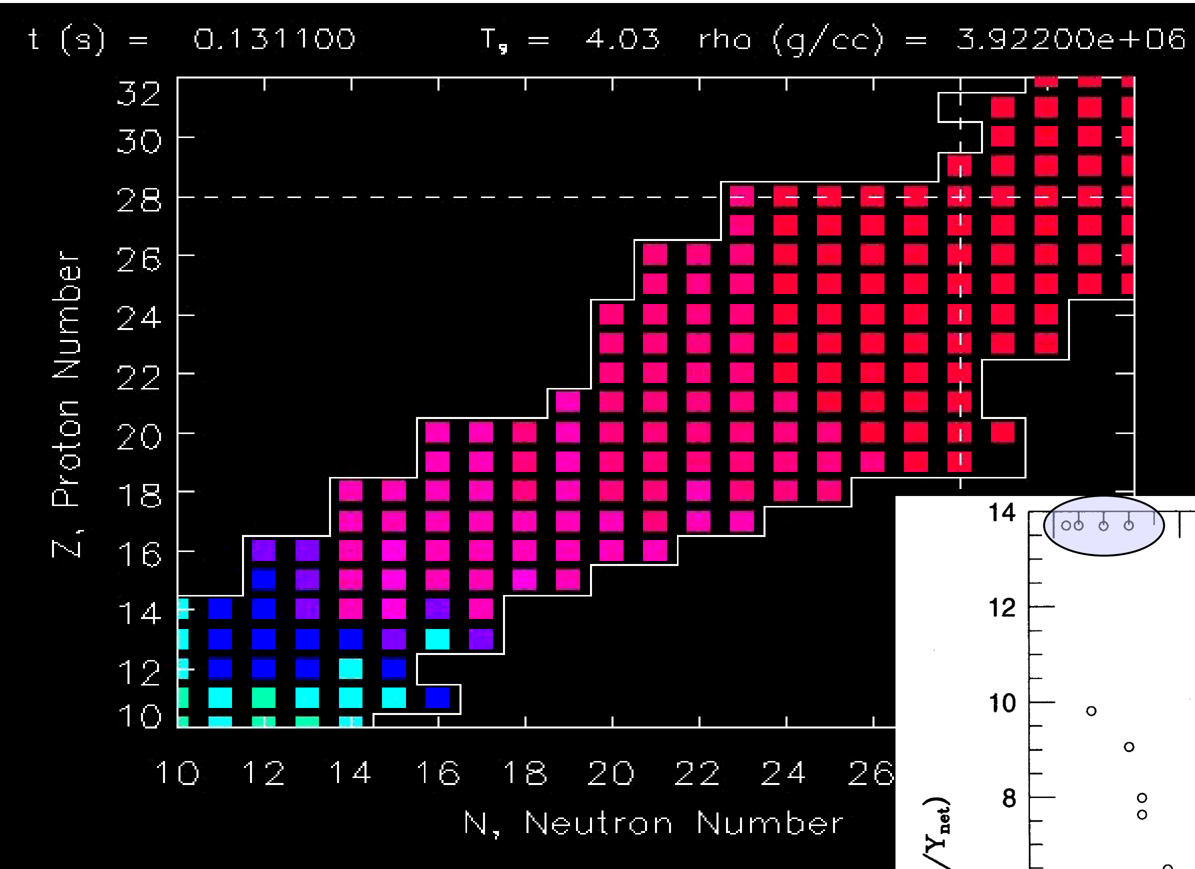
$^4He + ^4He \longleftrightarrow ^8Be$ is in chemical equilibrium
released neutrons lead to steady flow in neutron capture

Complete chem. equilibrium (NSE)

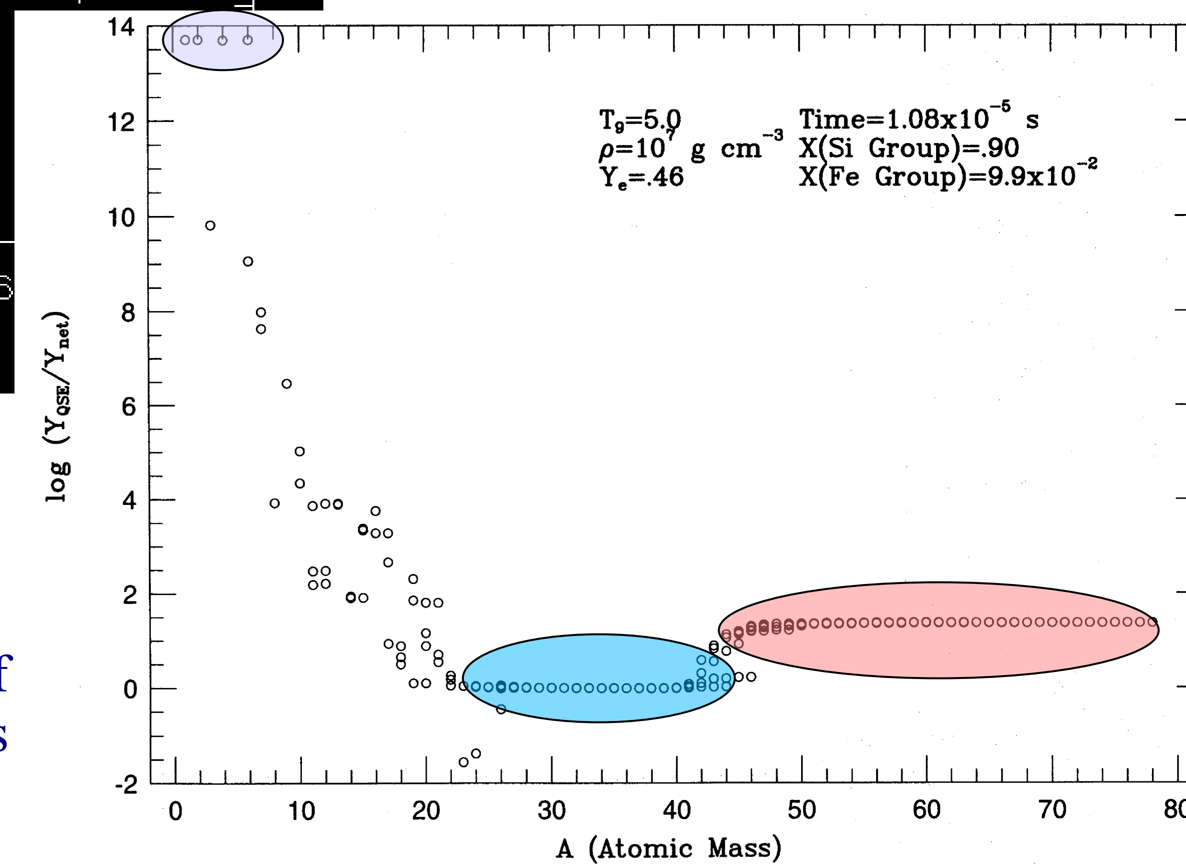


Si-burning in stellar evolution and expl. Si-burning at high densities lead to NSE!

QSE in explosive Si-burning



full NSE is not attained, but there exist equilibrium groups around ^{28}Si, ^{56}Ni and n,p,^{4}He, which are separated by slow reactions



Sample Calculations from

- B.S. Meyer and
- Hix and Thielemann

small Q-values of reactions out of Z=20, N=20 cause small cross sections and hold up equilibrium

QSE Formalism

light group

$$Y_{NSE}(^AZ) = C(^AZ) Y_n{}^N Y_p{}^Z$$

Si-group

$$Y_{QSE,\mathrm{Si}}(^AZ) = \frac{C(^AZ)}{C(^{28}\mathrm{Si})} Y(^{28}\mathrm{Si}) Y_p^{Z-14} Y_n^{N-14}$$

Ni/Fe-group

$$Y_{QSE,\mathrm{Ni}}(^AZ) = \frac{C(^AZ)}{C(^{56}\mathrm{Ni})} Y(^{56}\mathrm{Ni}) Y_p^{Z-28} Y_n^{N-28}$$

$$C(^AZ) = \frac{G(^AZ)}{2^A} \left(\frac{\rho N_A}{\theta}\right)^{A-1} A^{\frac{3}{2}} \exp\left(\frac{B(^AZ)}{k_B T}\right)$$

$$\theta = \left(\frac{m_u k_B T}{2\pi\hbar^2}\right)^{3/2}$$

binding energy differences, i.e. masses enter directly

$$Y_{NG} = \sum_{i \in Lt\ group} N_i Y_i + \sum_{i \in Si\ group} (N_i - 14) Y_i + \sum_{i \in Fe\ group} (N_i - 28) Y_i$$

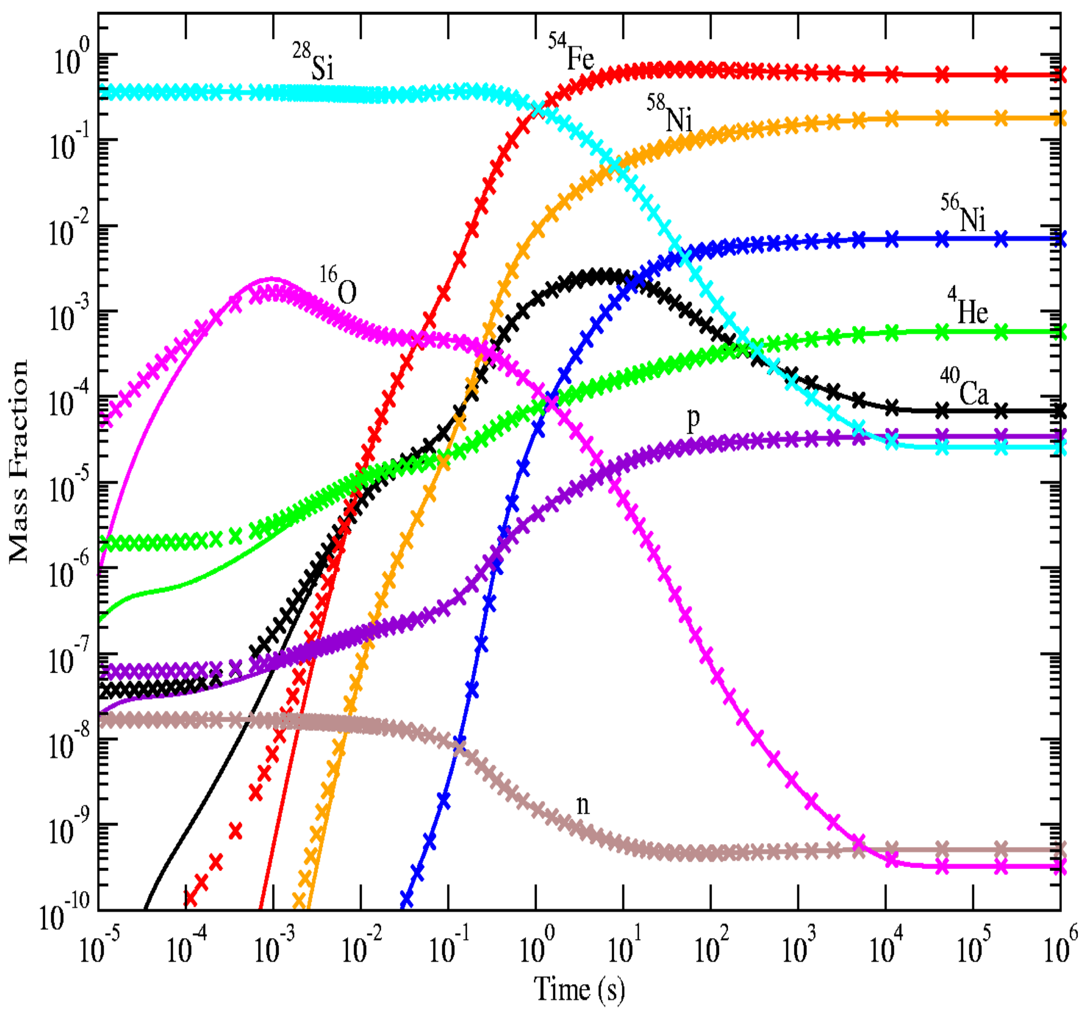
$$Y_{ZG} = \sum_{i \in Lt\ group} Z_i Y_i + \sum_{i \in Si\ group} (Z_i - 14) Y_i + \sum_{i \in Fe\ group} (Z_i - 28) Y_i$$

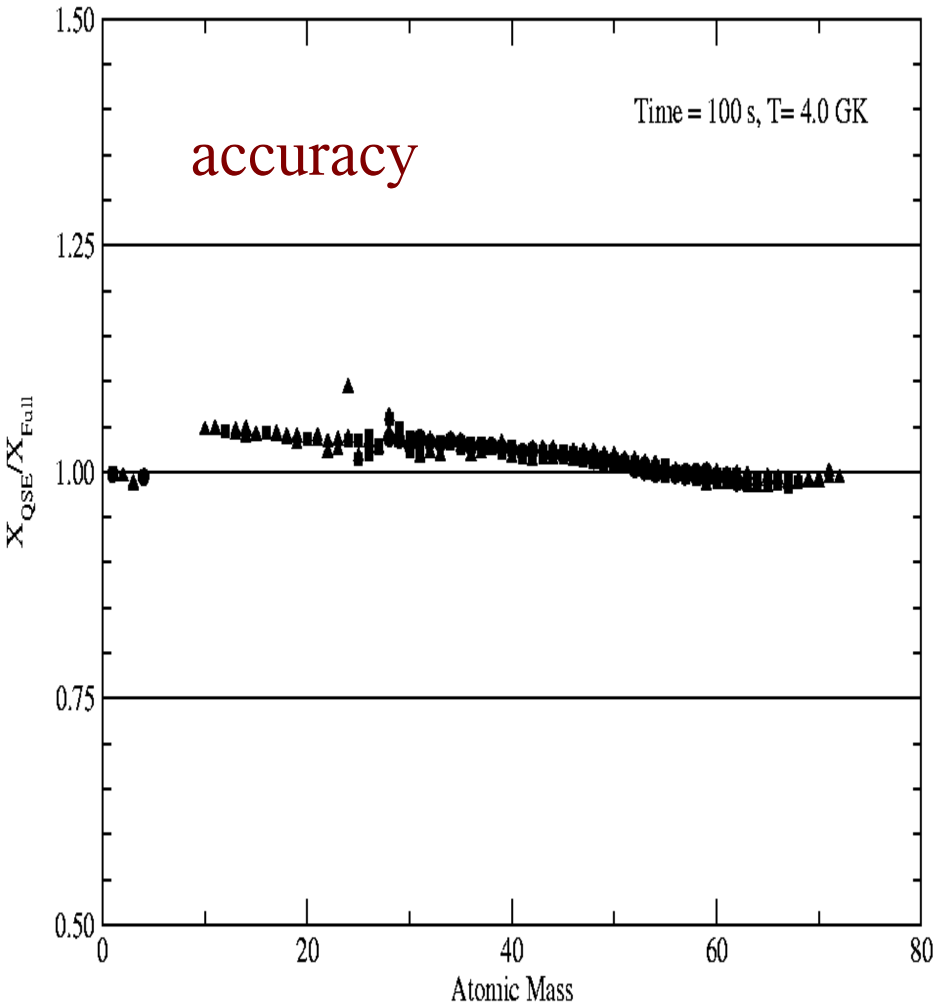
$$Y_{SiG} = \sum_{i \in Si\ group} Y_i \,,$$

$$Y_{FeG} = \sum_{i \in Fe\ group} Y_i \,.$$

time evolution for those quantities which are in equilibrium and the individual abundances of nuclei with slow reactions which link equilibrium groups (Hix, Parete-Koon, Freiburghaus, Thielemann 2007)

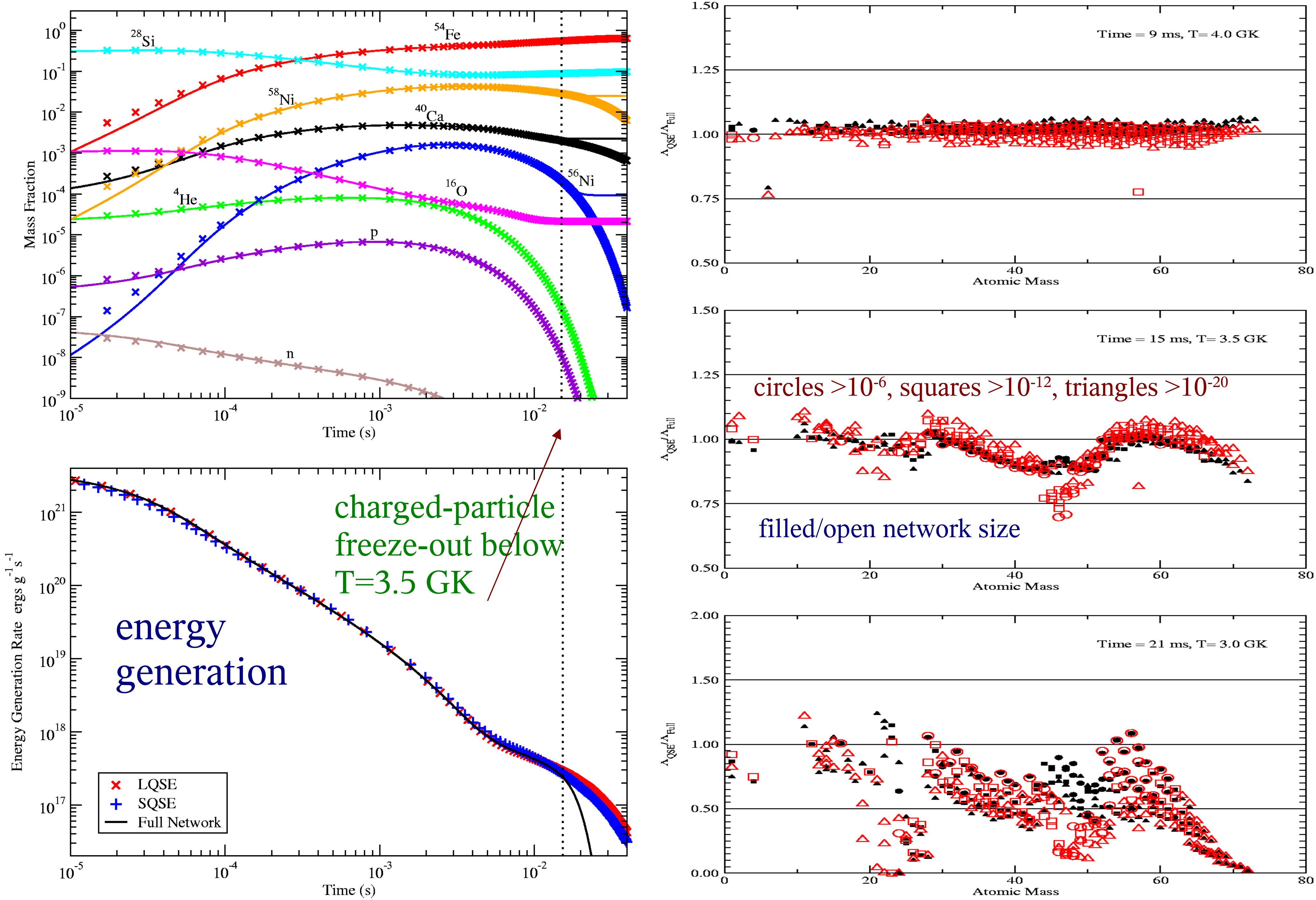
Obtaining equilibrium at high T



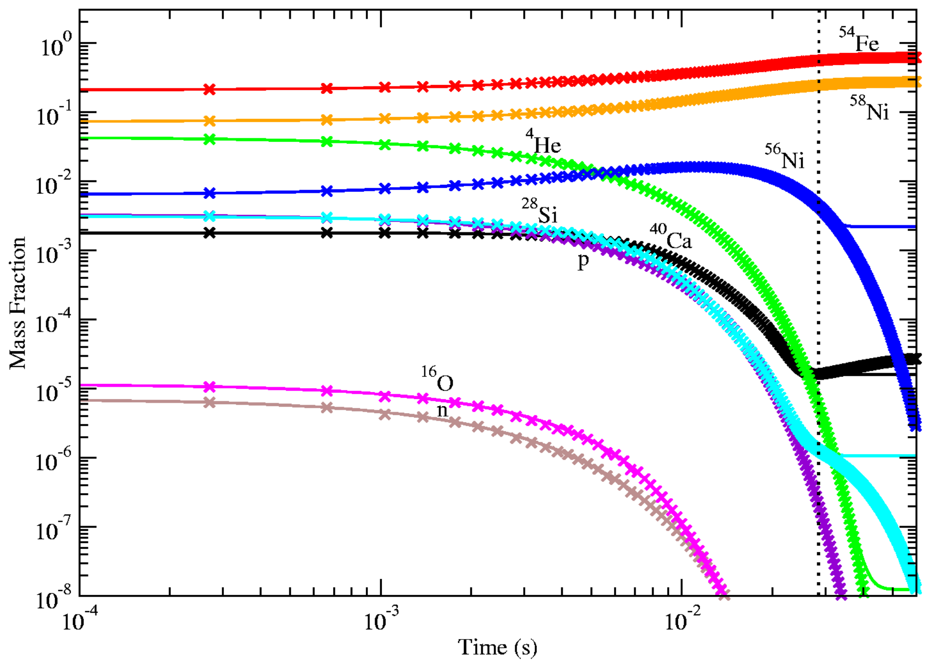


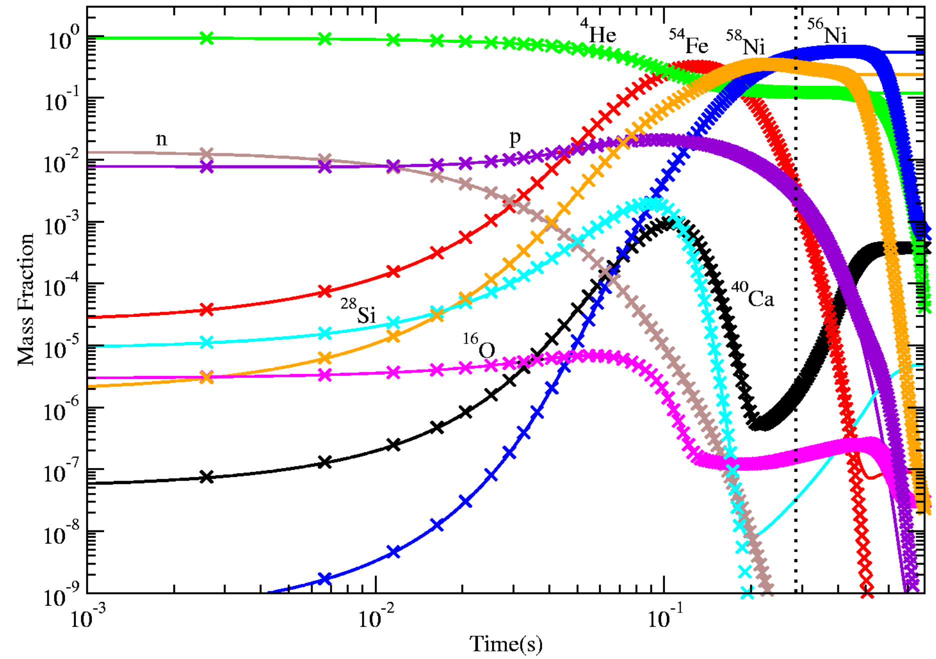
at T=4 GK the equilibrium description is correct after about 10^{-3} s!

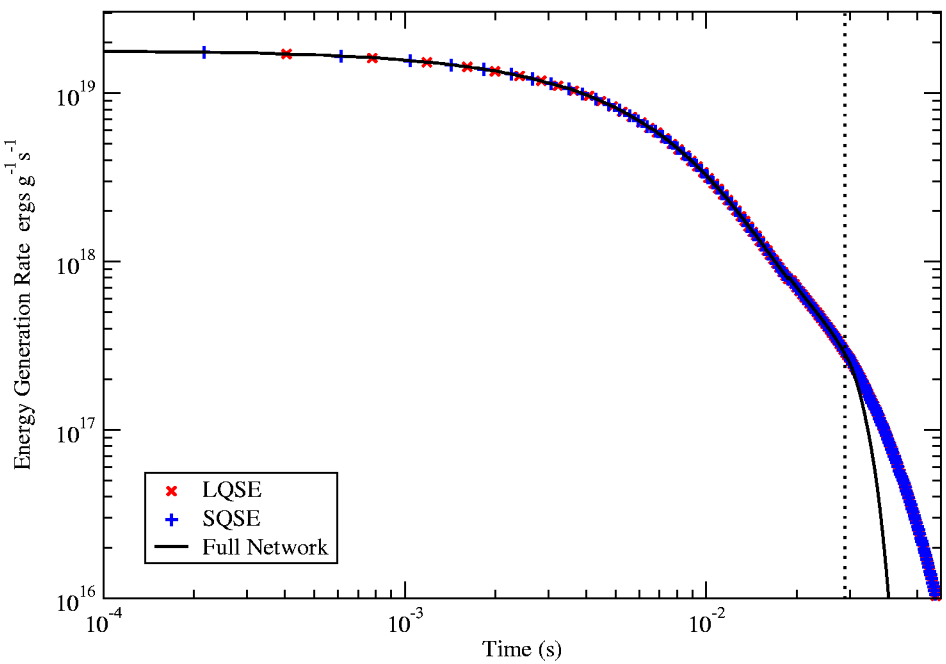
Incomplete Si-burning with freeze-out

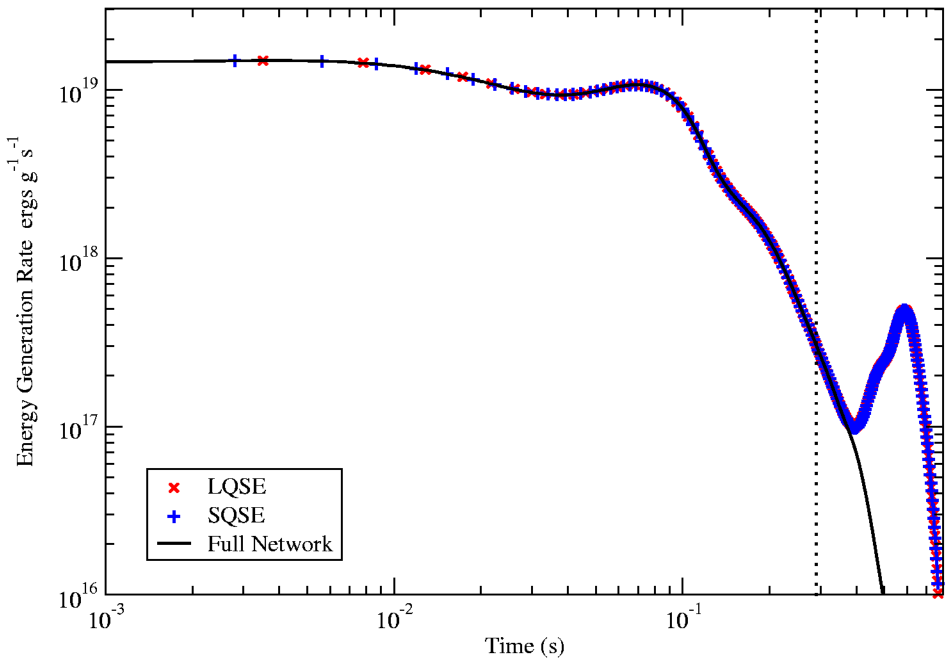


Normal and alpha-rich freeze-out









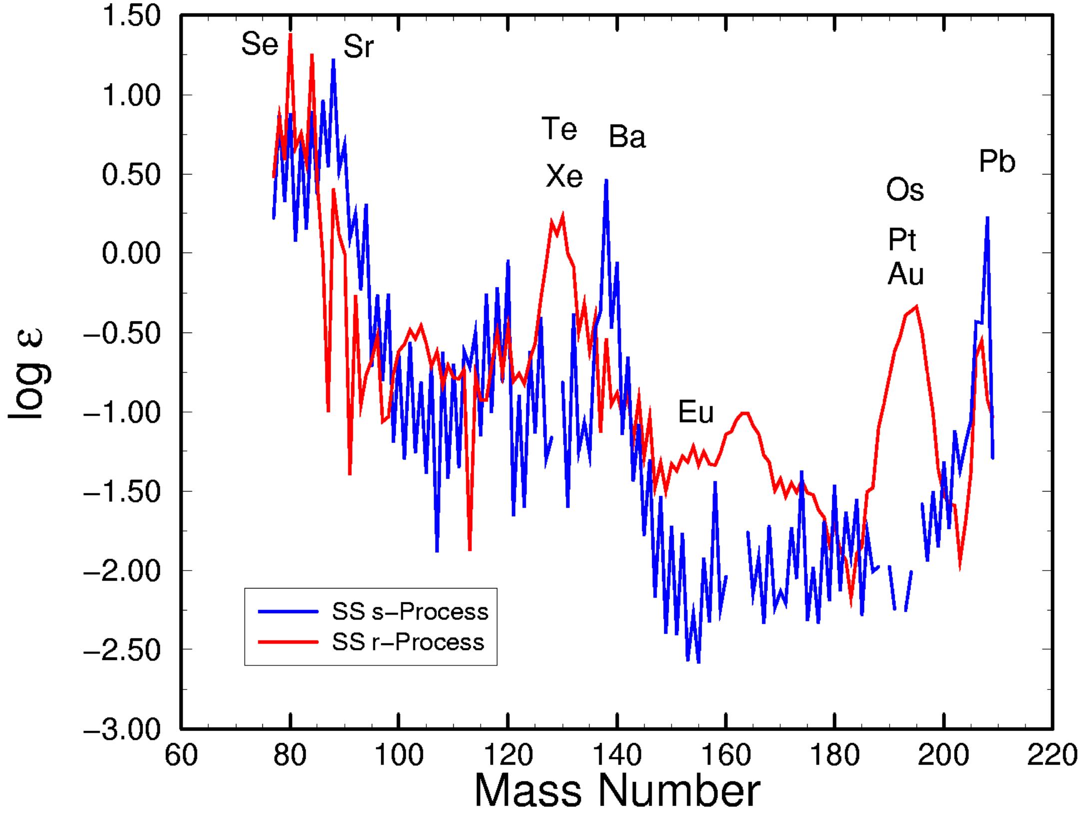
Interim conclusions

- steady flows are approached in many hydrostatic burning stages during stellar evolution, including the s-process. They are determined by rates (often the smallest ones), which are/can be related to small Q-values.
- NSE/QSE equilibria are obtained in hydrostatic Si-burning and in explosive burning. Abundance distribution depends directly on mass differences, but for these applications mostly close to stability.
- How about QSE-equilibria linked by steady flows (and far from stability)?

The classical r-process

- Assume conditions where after a charged-particle freeze-out the heavy QSE-group splits into QSE-subgroups containing each one isotopic chain Z, and a high neutron density is left over
- these QSE-groups are connected by beta-decays from Z to Z+1
- neutrons are consumed to form heavier nuclei
- is a steady flow of beta-decays conceivable?

s- and r-decoposition

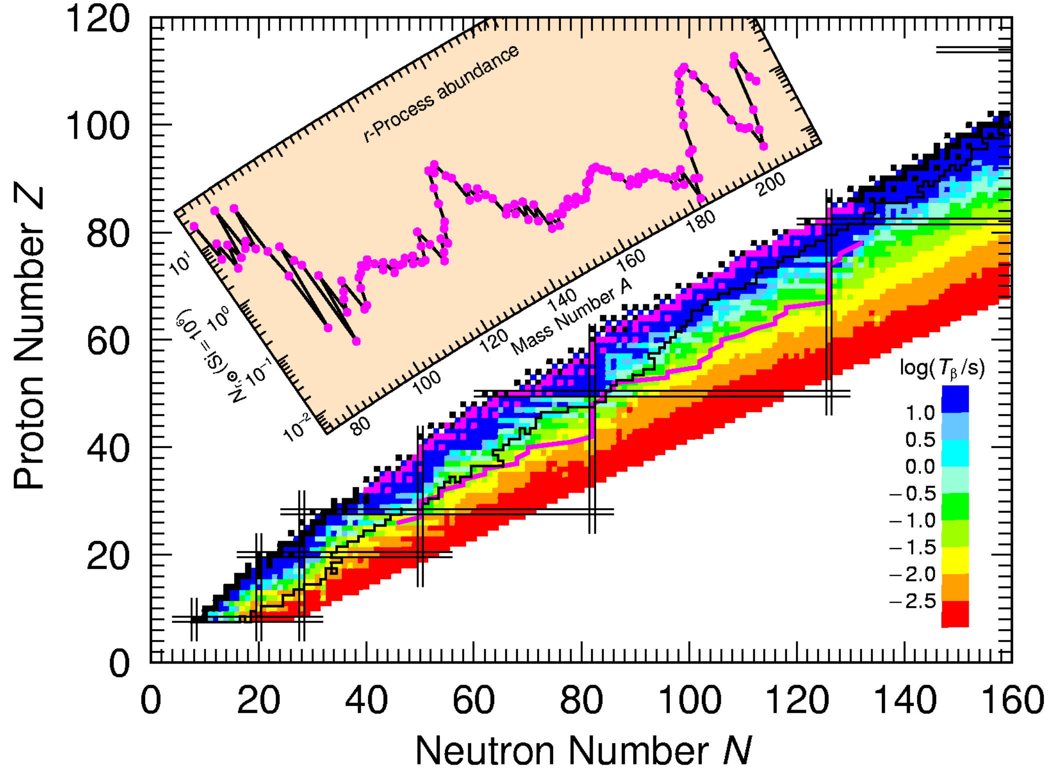


$$\dot{Y}(Z,A) = -\lambda_{\beta^-}(Z,A)Y(Z,A) - \rho N_A < \sigma v >_{n,\gamma} Y_n Y(Z,A)$$
$$= -\lambda_{\beta^-}(Z,A)Y(Z,A) - < \sigma v >_{n,\gamma} n_n Y(Z,A)$$
$$= -\frac{1}{\tau_\beta} Y(Z,A) - \frac{1}{\tau_{n,\gamma}} Y(Z,A).$$

which timescale is shorter? neutron capture inverse proportional to n_n !

Heavy Elements are made by slow and rapid neutron capture events

High neutron densities lead to nuclei far from stability



Nuclear Reactions to be considered: (n,γ), (γ,n)

(β,xn), (β,f), (n,f), inelastic ν-scattering, (ν_e,e^-) ......

The classical r-process

How to predict abundance changes?

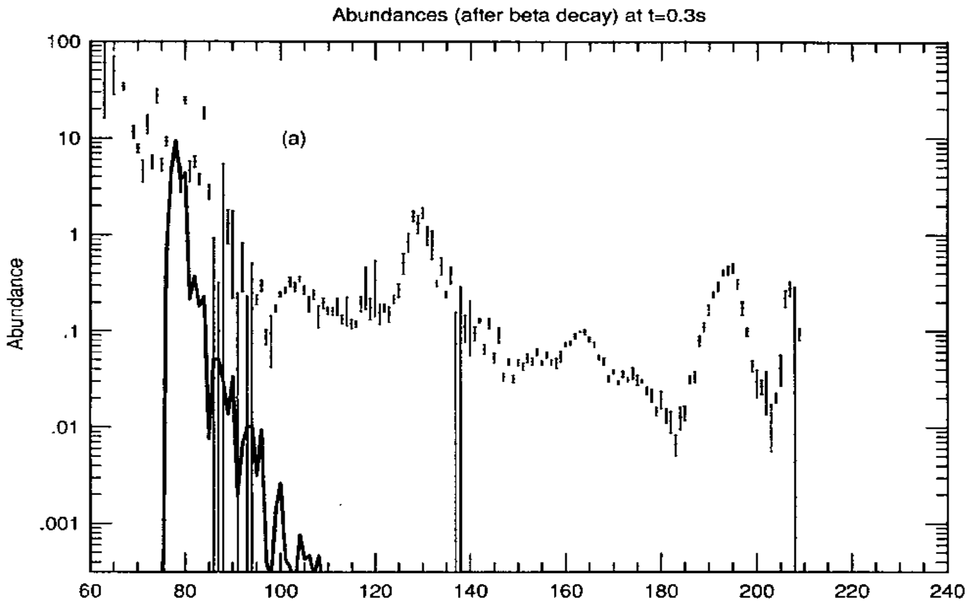
- $\dot{Y}(Z,A) = \sum \lambda_{Z',A'} Y_{Z',A'} + \sum \rho N_A < \sigma v >_{Z',A'} Y_{Z',A'} Y_n$
  with $n_n = \rho N_A Y_n$
- $\dot{Y}(Z,A) \approx \lambda_\gamma(Z,A+1)Y(Z,A+1) - < \sigma v >_{Z,A} Y_{Z,A} n_n$ in case (n,γ), (γ,n) rates dominate
- $\dot{Y}(Z,A) = 0$ in chemical equilibrium,
  $Y(Z,A+1)/Y(Z,A) = f(n_n, T, S_n)$ due to detailed balance relation between $\lambda_\gamma(Z,A+1)$ and $< \sigma v >_{Z,A}$
- abundance maxima for all Z's at same S_n
- $\dot{Y}(Z) = \lambda_\beta(Z-1)Y(Z-1) - \lambda_\beta(Z)Y(Z)$ for summed abundances in isotopic chain and averaged decay rates

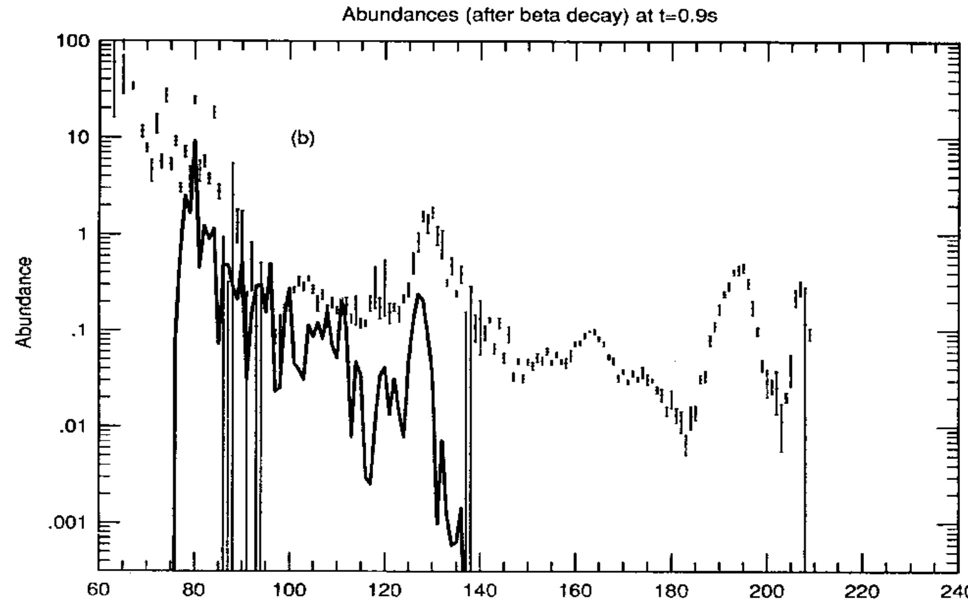
$$\frac{Y(Z,A+1)}{Y(Z,A)} = \frac{< \sigma v >_{n,\gamma}(A)}{\lambda_{\gamma,n}(A+1)} n_n$$

$$\lambda_{\gamma,n}(A+1) = \frac{2G(Z,A)}{G(Z,A+1)}\left[\frac{A}{A+1}\right]^{3/2}\left[\frac{m_u kT}{2\pi\hbar^2}\right]^{3/2} \langle\sigma v\rangle_{n,\gamma}(A)\exp(-S_n(A+1)/kT)$$

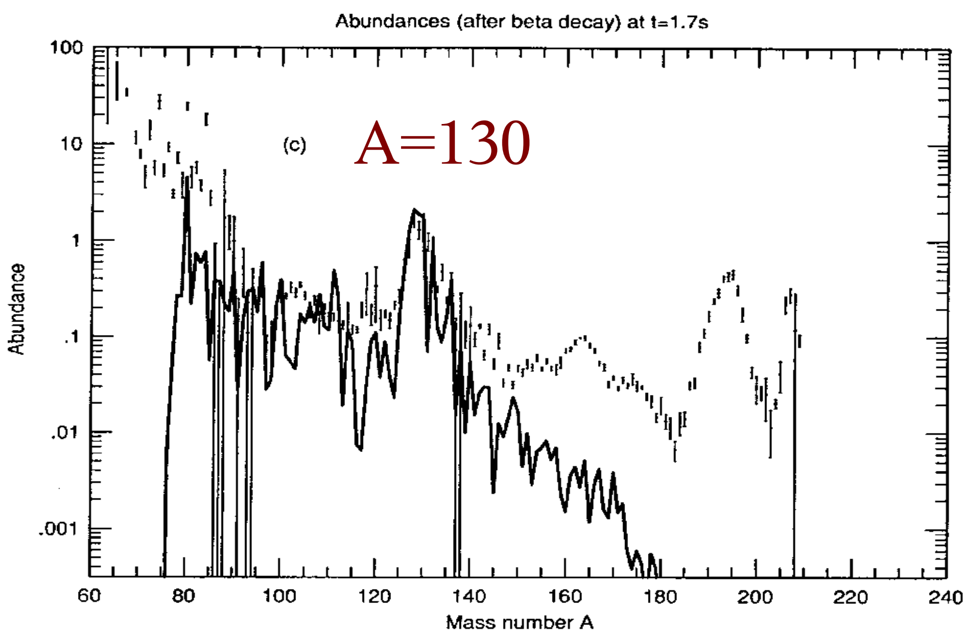
$$\frac{Y(Z,A+1)}{Y(Z,A)} = n_n \frac{G(Z,A+1)}{2G(Z,A)}\left[\frac{A+1}{A}\right]^{3/2}\left[\frac{2\pi\hbar^2}{m_u kT}\right]^{3/2}\exp(S_n(A+1)/kT)$$

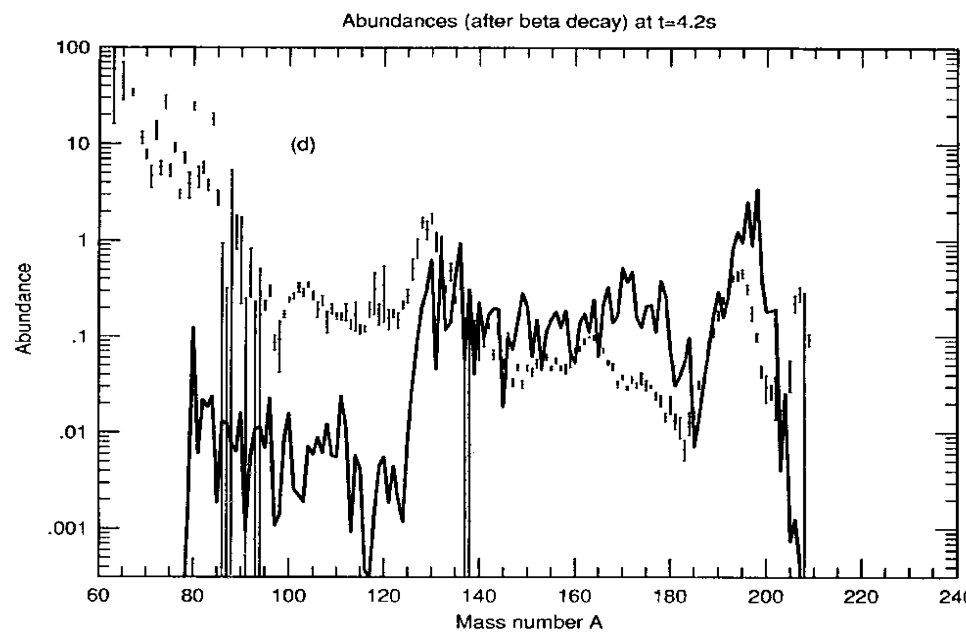
classical calculation with n_n=const and T=const



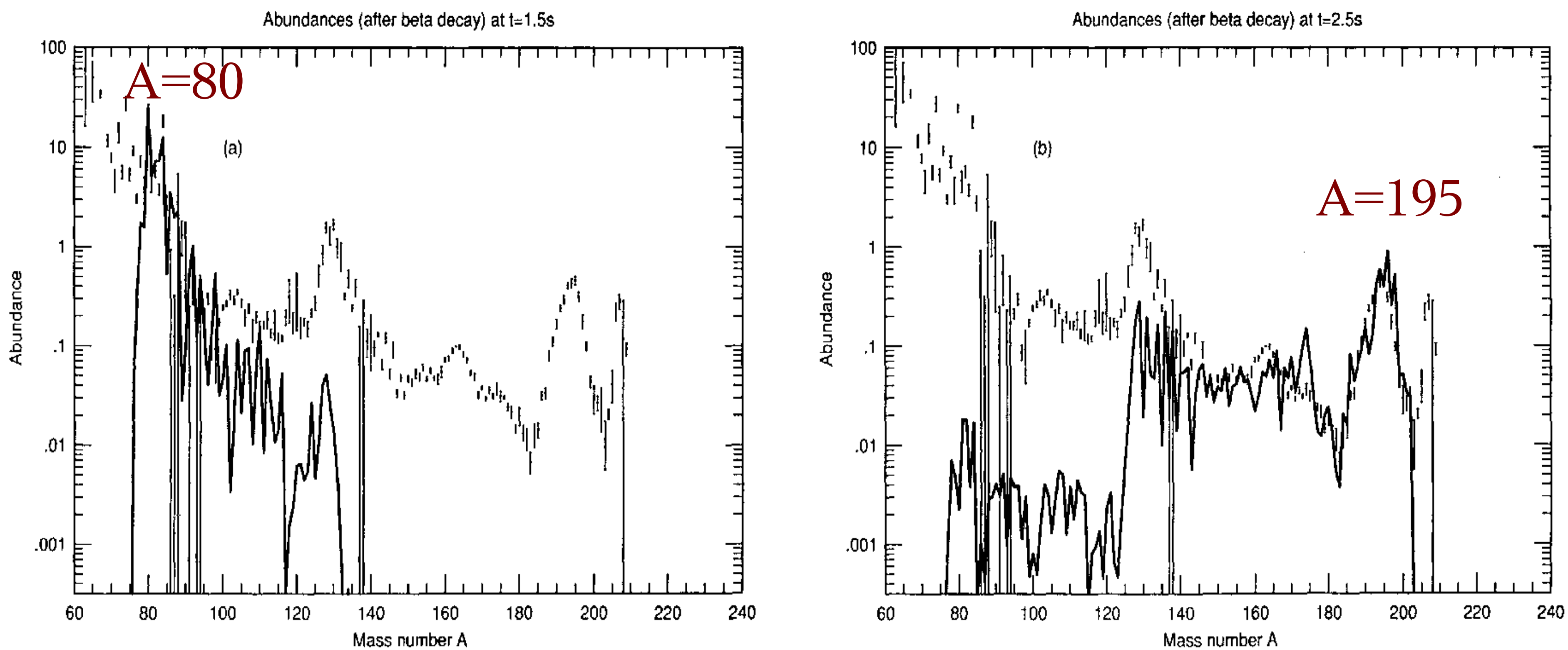


conditions with reproduce A=130 at t=1.7s





A=80 and 195 peaks

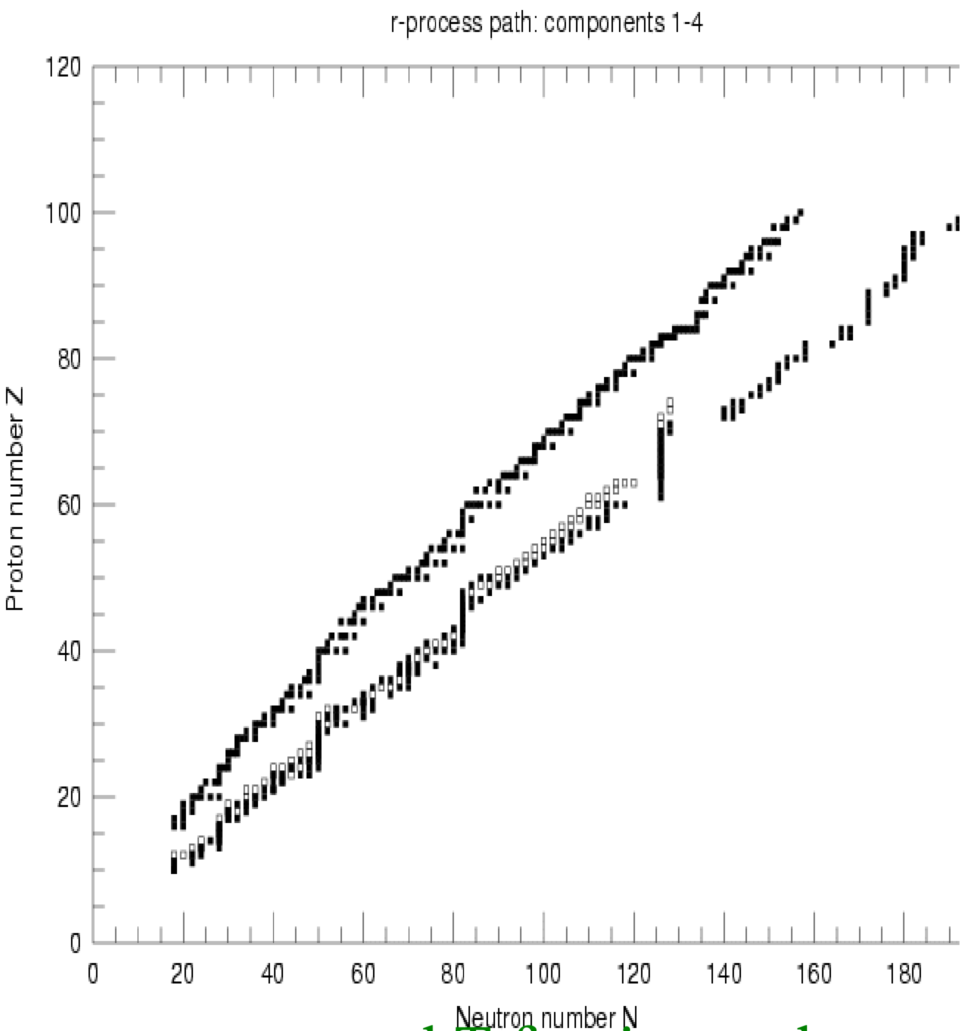


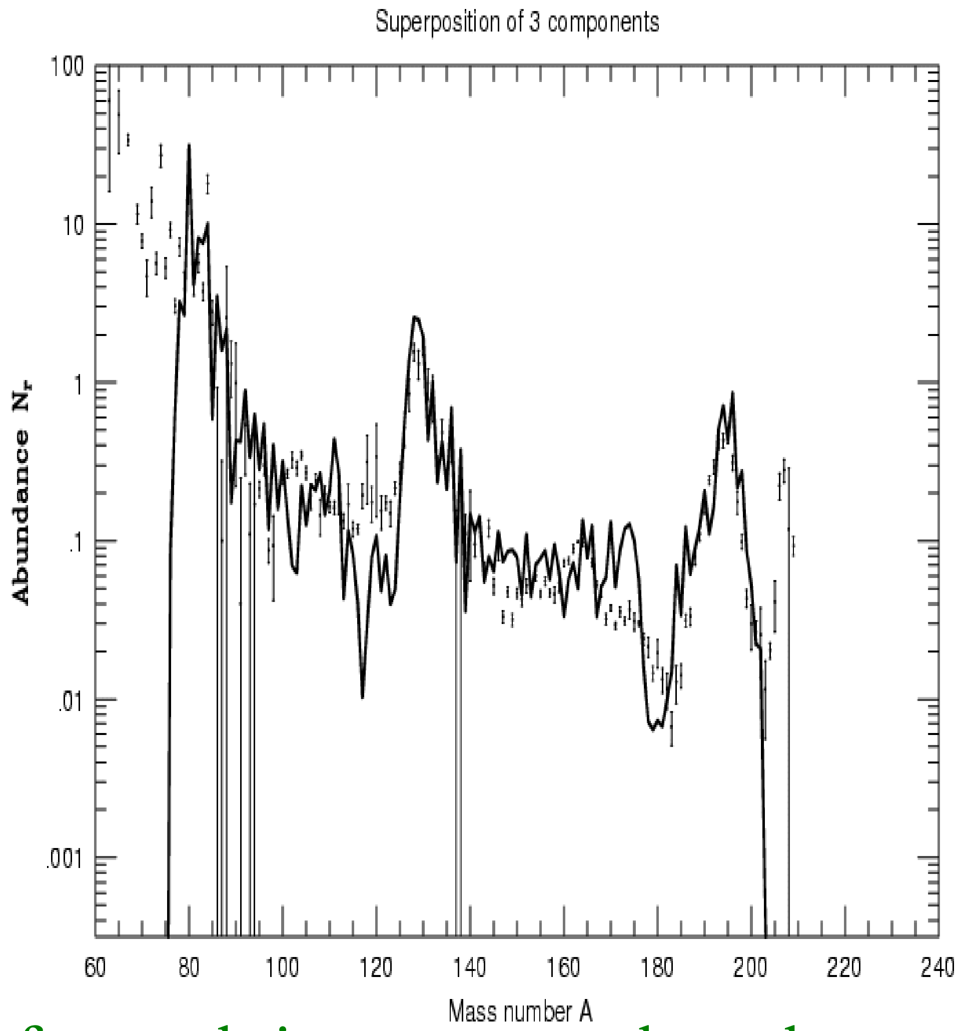
t=1.5s t=2.5s

three components produce the A=80, 130, and 195 peaks during “comparable” timescales (for the first time experimental half-lives and masses are known in the r-process path at A=80 and 130)!

Following three S_n's for timescales t_1,t_2,t_3

Kratz, Bitouzet, Thielemann, Möller, Pfeiffer and permutations 1993-1999

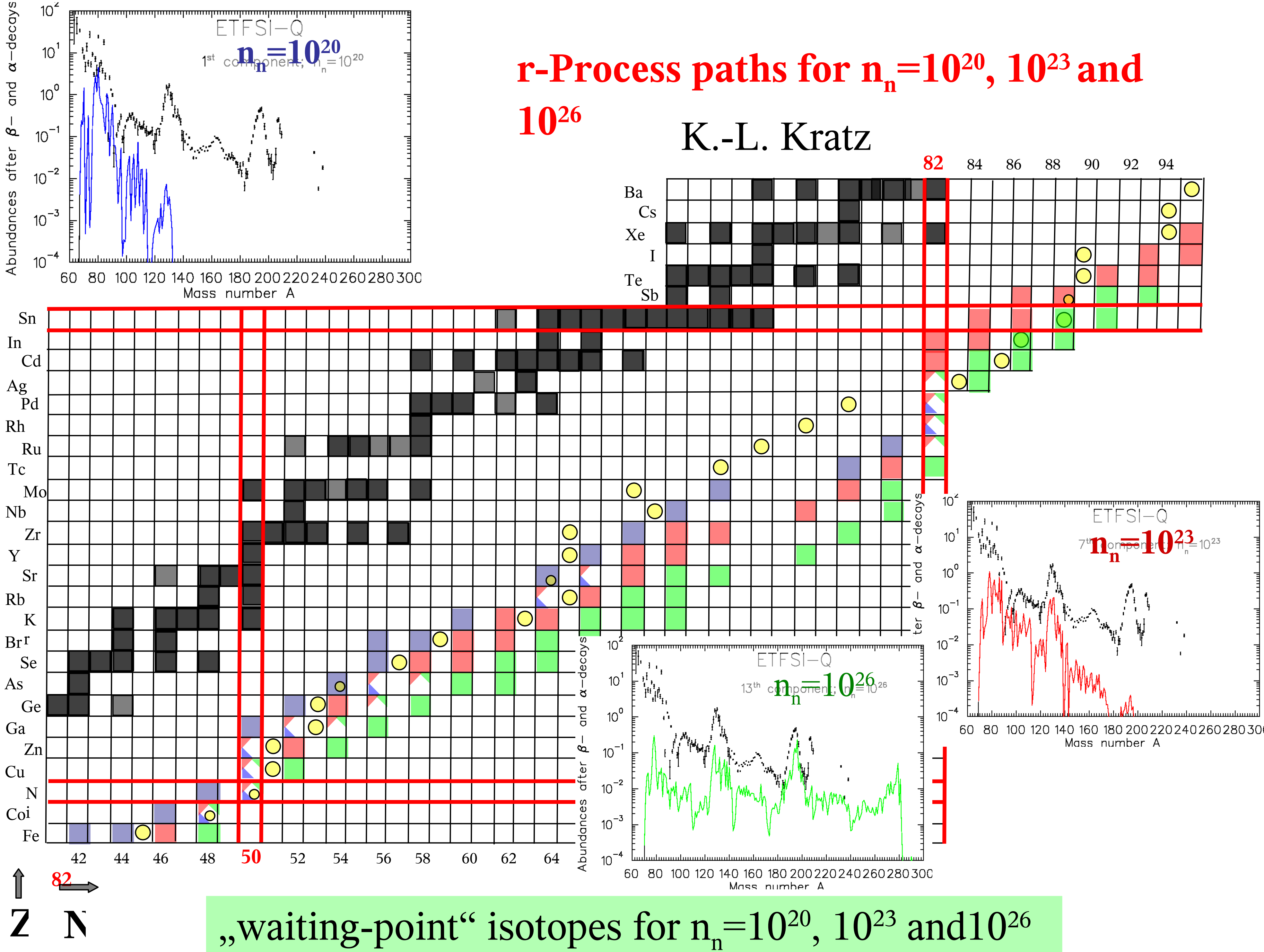




constant n_n and T for timescale t and afterwards instantaneous beta-decay

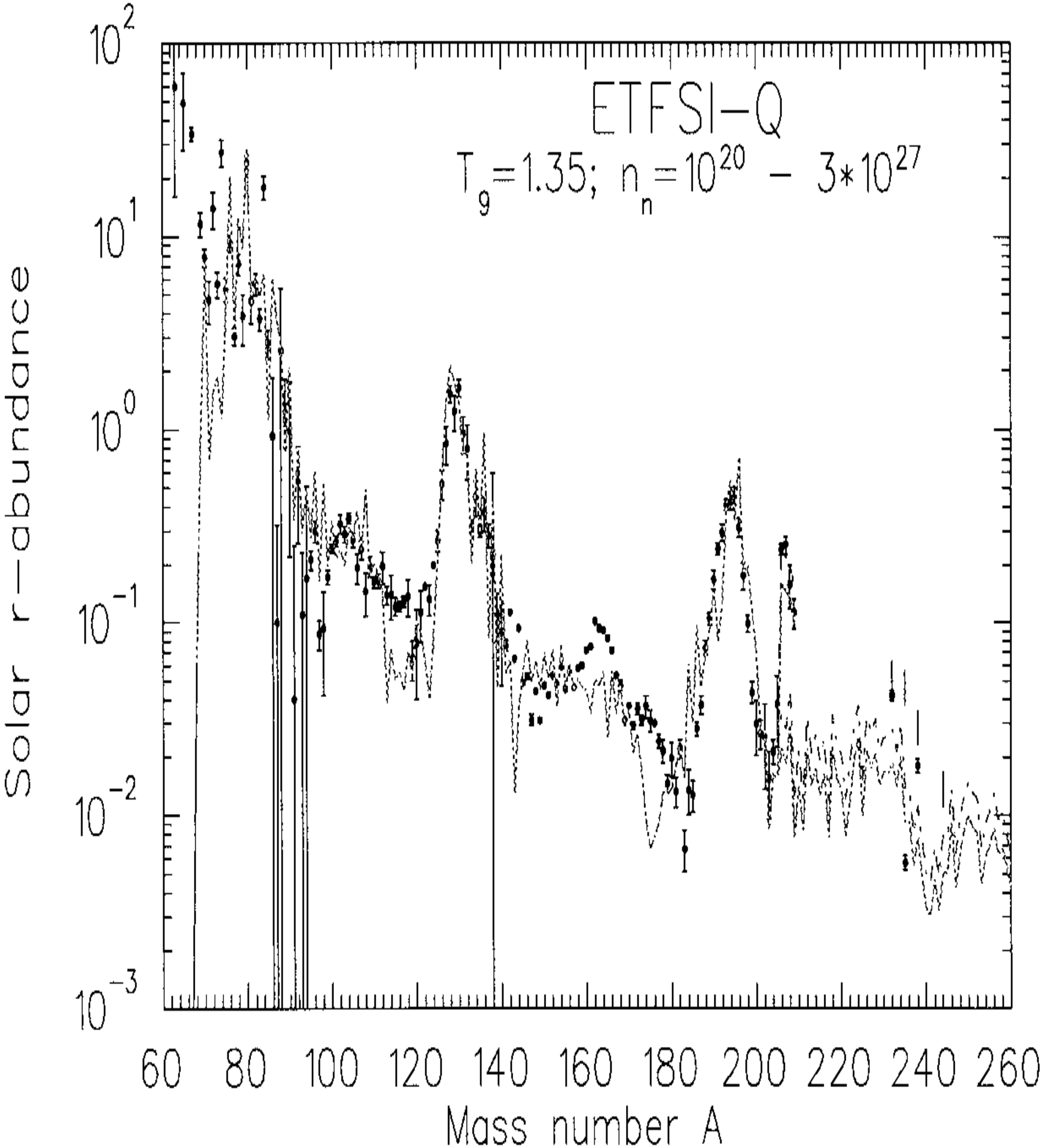
r-Process paths for $n_n=10^{20}$, 10^{23} and 10^{26}

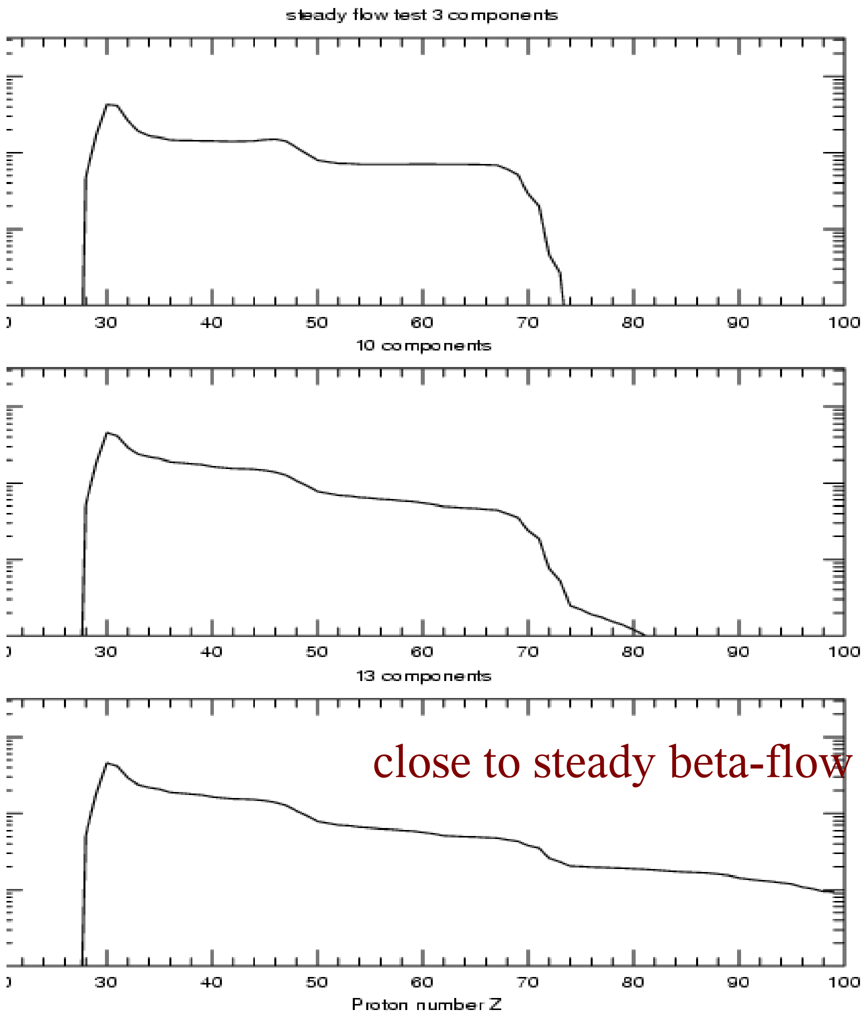
K.-L. Kratz



„waiting-point“ isotopes for $n_n=10^{20}$, 10^{23} and10^{26}

Multi-components and steady beta-flow



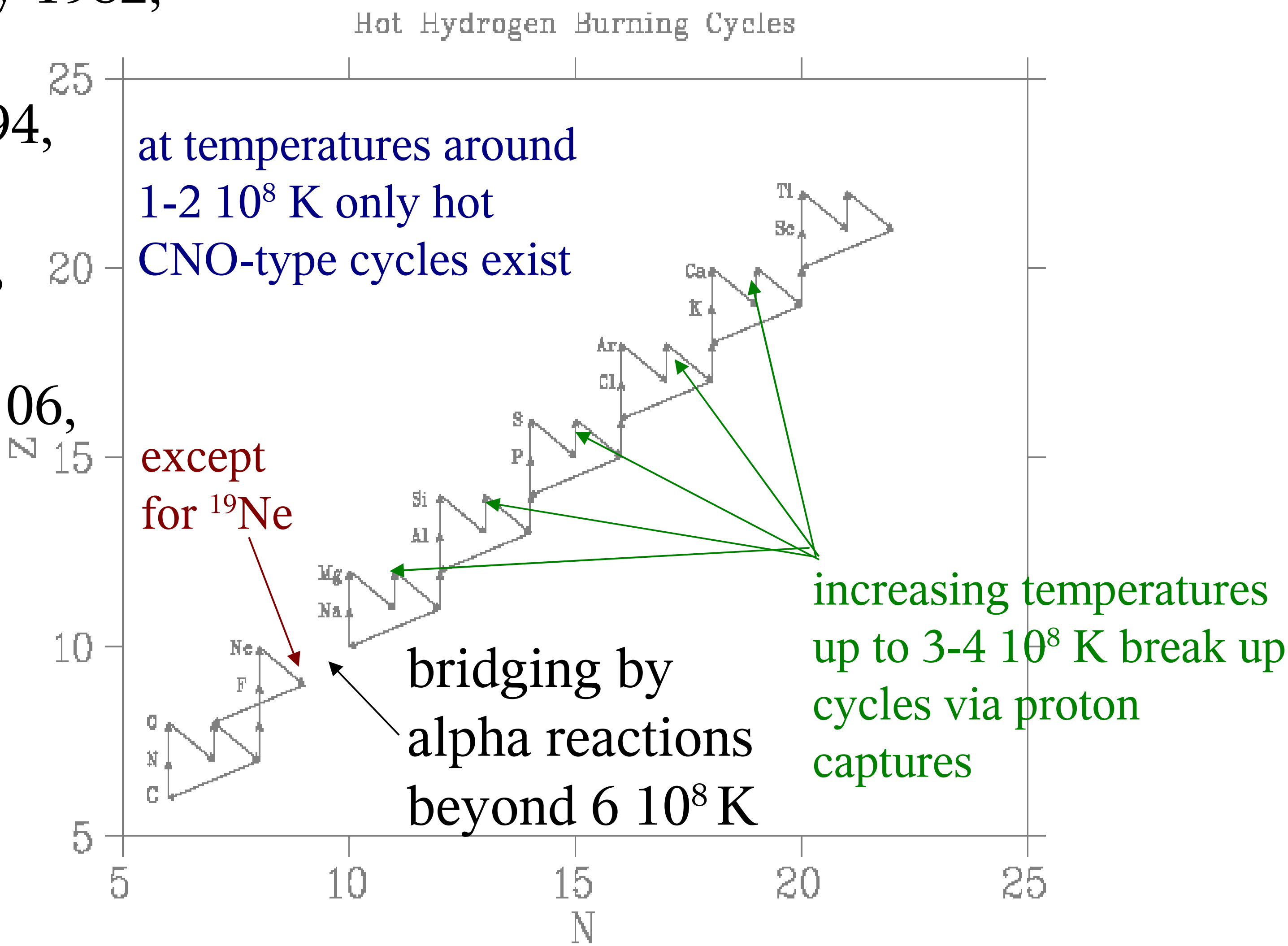


superposition with weights $w(n_n)=8.36\ 10^6 n_n^{-0.247}$ and $t(n_n)=6.97\ 10^{-2}\ n_n^{0.062}$s

decay rate of complete Z-chain multiplied with total abundance of Z-chain close to constant in between magic numbers (where long half-lives are encountered).

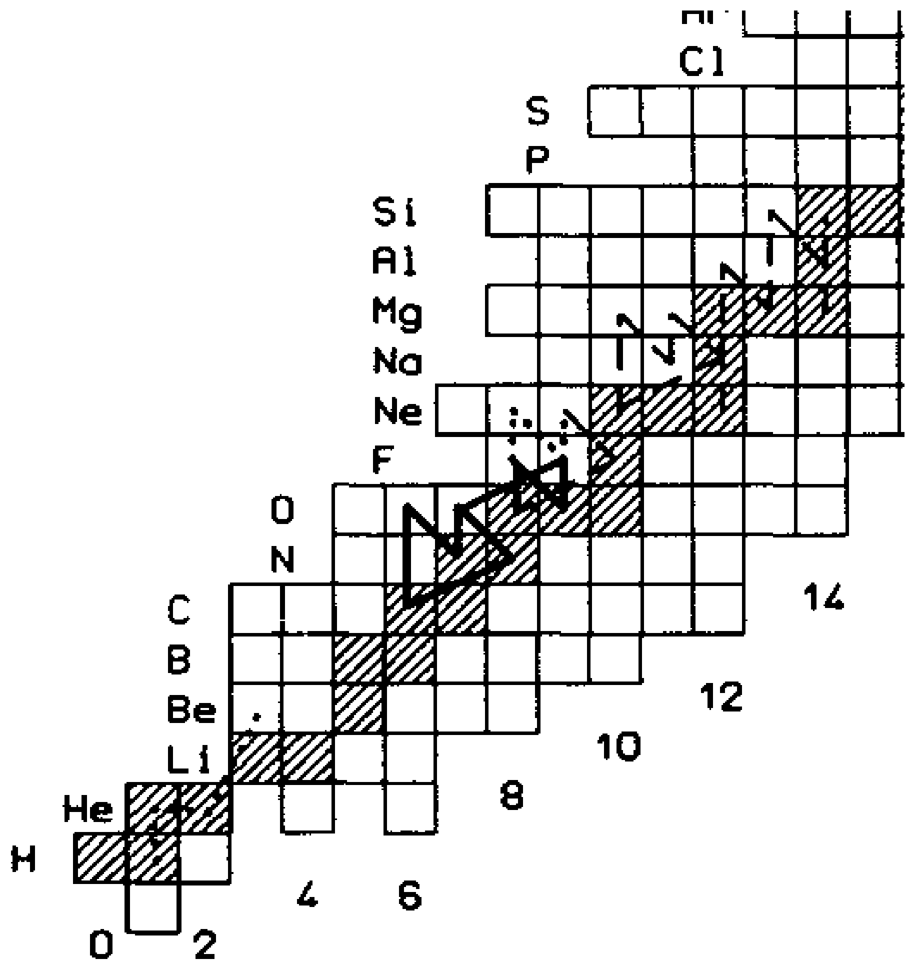
Explosive H/He-burning and the onset of the rp-process

Wallace & Woosley 1982,
Wiescher et al. 86,
van Wormer et al. 94,
Rembges et al. 97,
Schatz et al. 98, 01,
....
Cooper & Narayan 06,
...



detailed modeling is more complex and identifies reaction sensitivities and drip-line dependence (Fisker, FKT, Wiescher 2005, Fisker, Schatz, FKT 2008)

Test calculations as functions of T



T=1.5 10^8 K

hot CNO cycle

no break-out yet

for T=2-3 10^8 K more CNO-type cycles develop up to Ca

early evaluations: van Wormer et al. (1994)

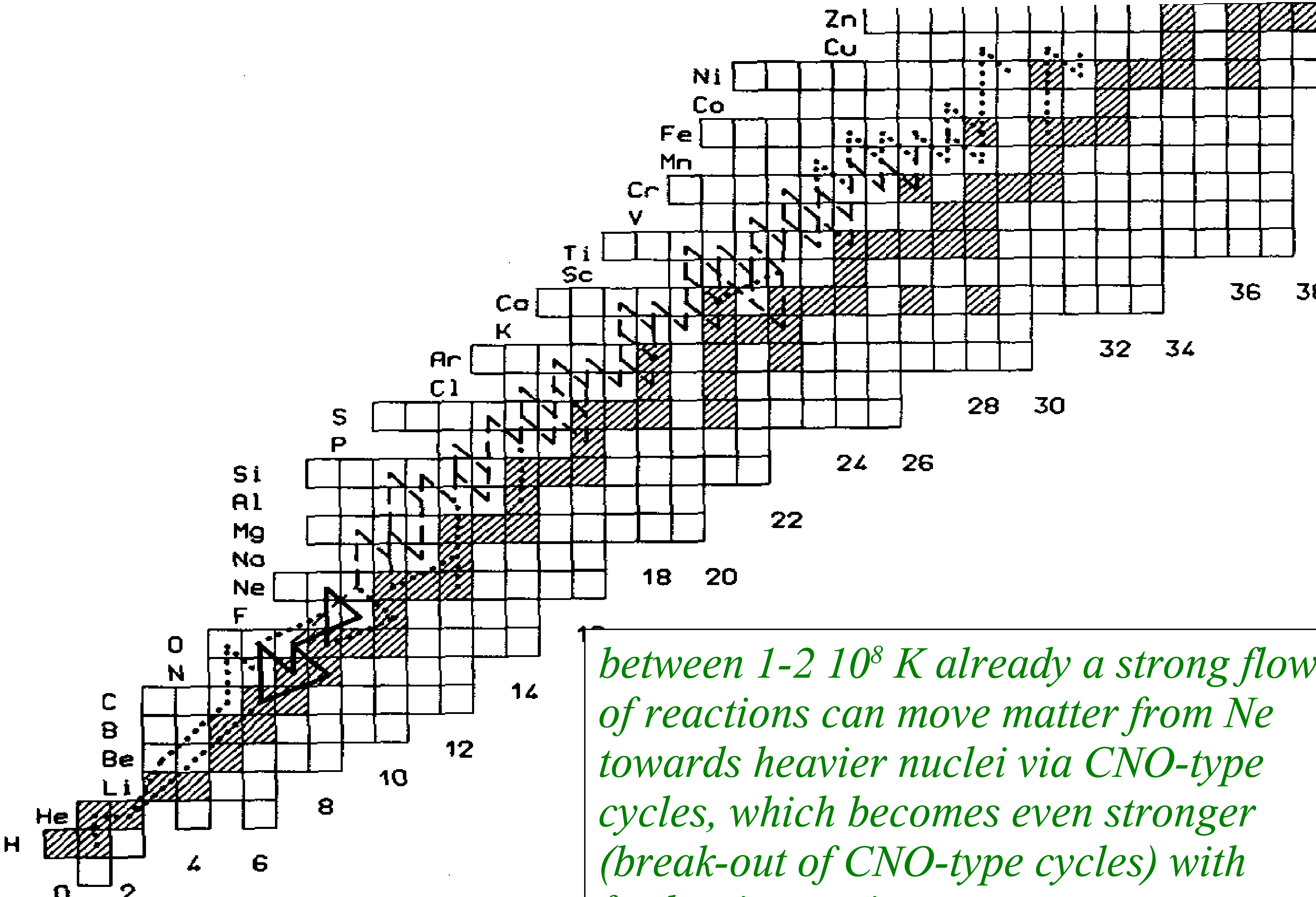
Uncertainties in:

^{14}O$(\alpha,p)^{17}$F, ^{17}F$(p,\gamma)^{18}$Ne,

^{18}Ne$(\alpha,p)^{21}$Na

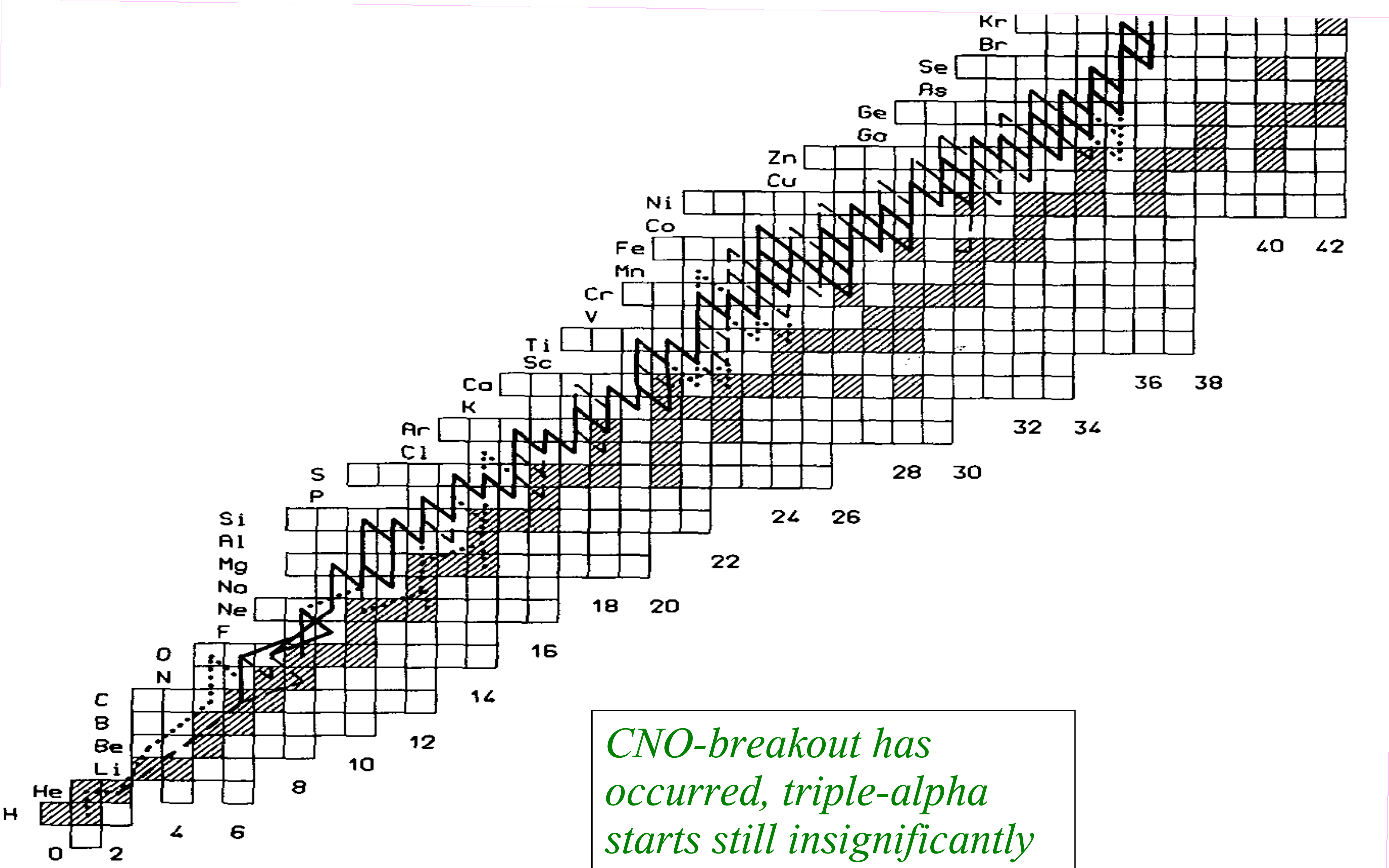
^{15}O$(\alpha,\gamma)^{19}$Ne, ^{19}Ne$(p,\gamma)^{20}$Na

T = 4 x 10^8 K



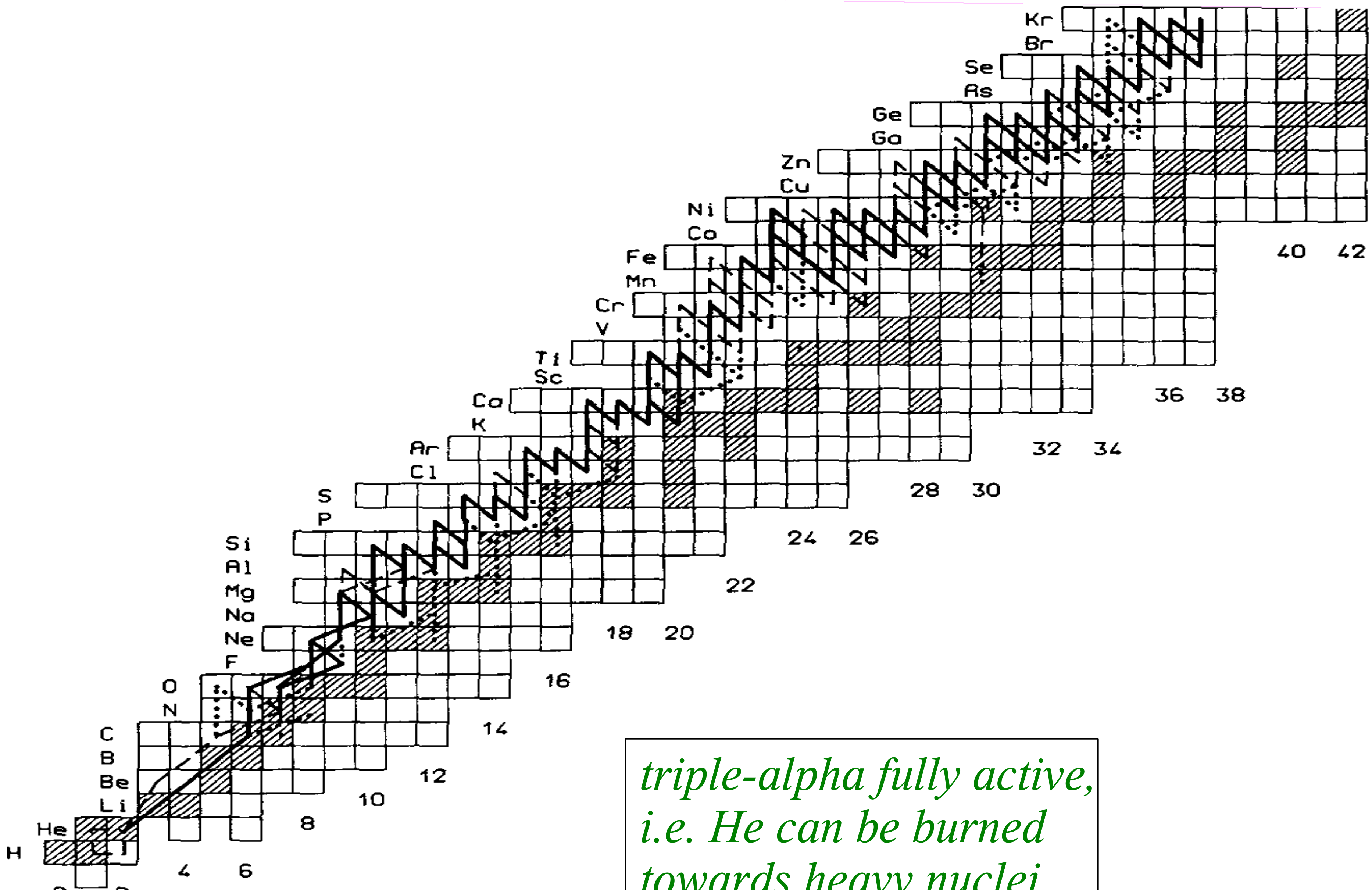
between 1-2 10^8 K already a strong flow of reactions can move matter from Ne towards heavier nuclei via CNO-type cycles, which becomes even stronger (break-out of CNO-type cycles) with further increasing temperatures

$T = 6 \times 10^8$ K



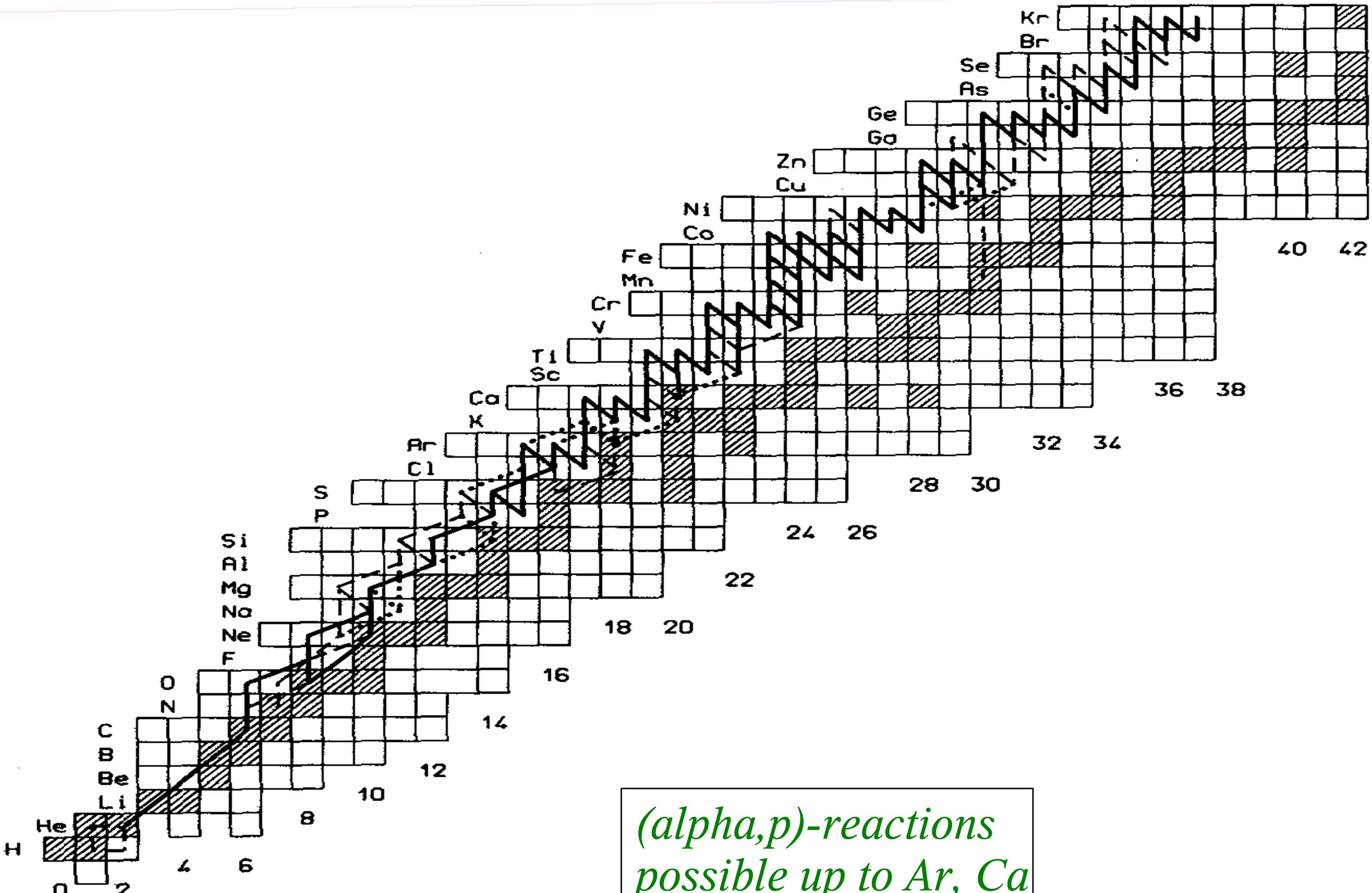
CNO-breakout has occurred, triple-alpha starts still insignificantly

T = 8 x 10^8 K



triple-alpha fully active, i.e. He can be burned towards heavy nuclei

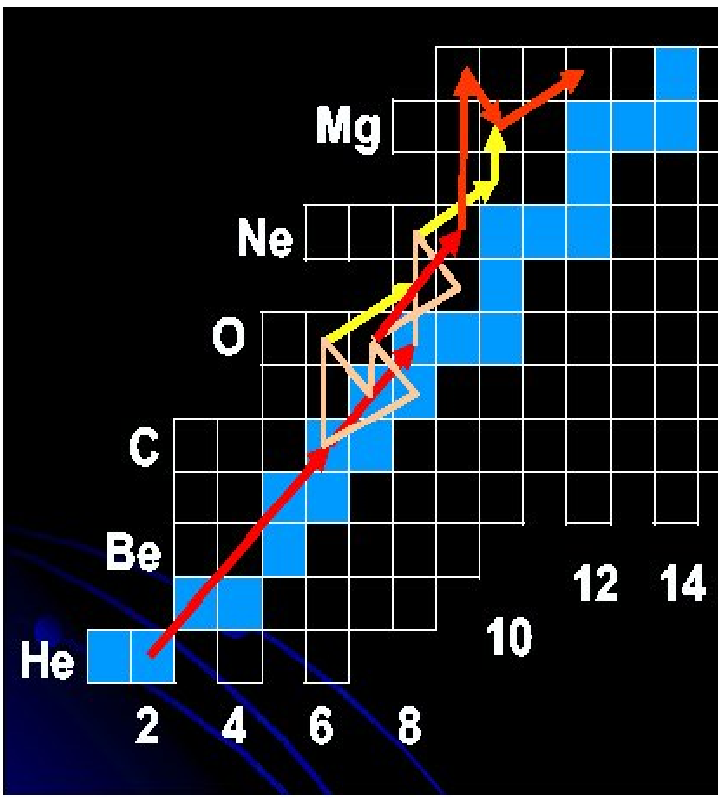
T = 1.5 10^9 K

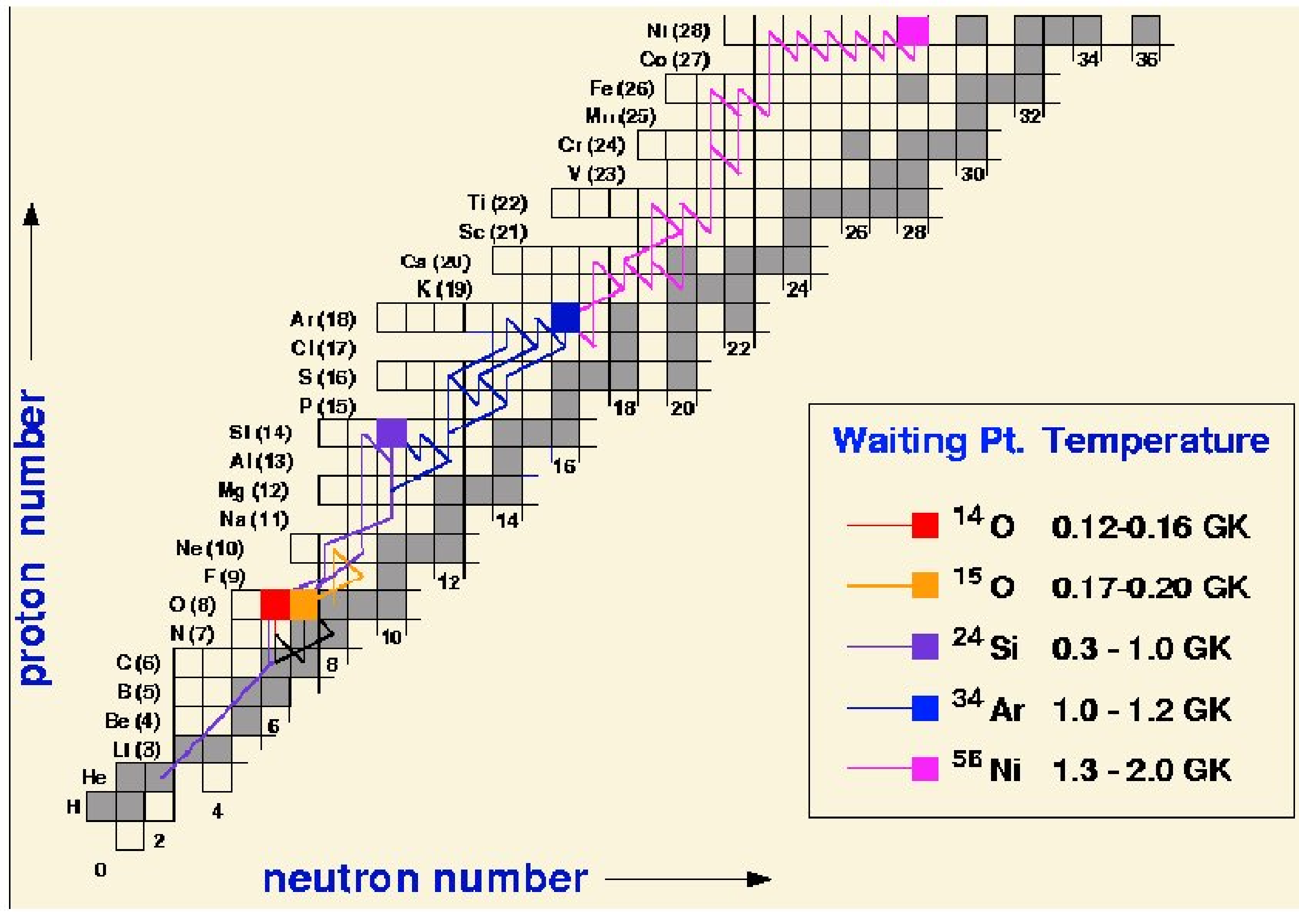


(alpha,p)-reactions possible up to Ar, Ca

Break-out from hot CNO at 4-5 10^8 K

Wiescher, Görres, Thielemann, van Wormer, Schatz, Rembges ..

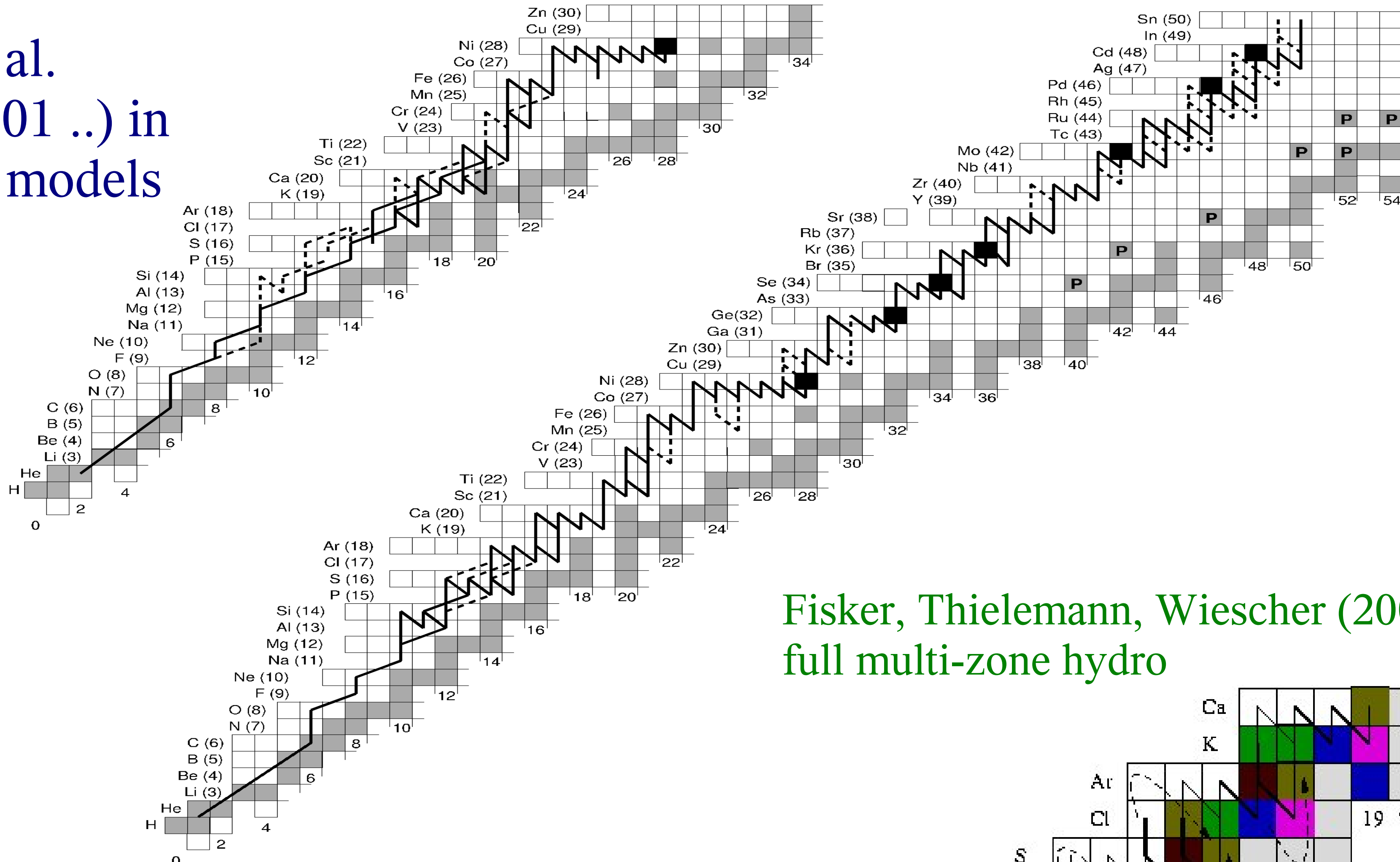




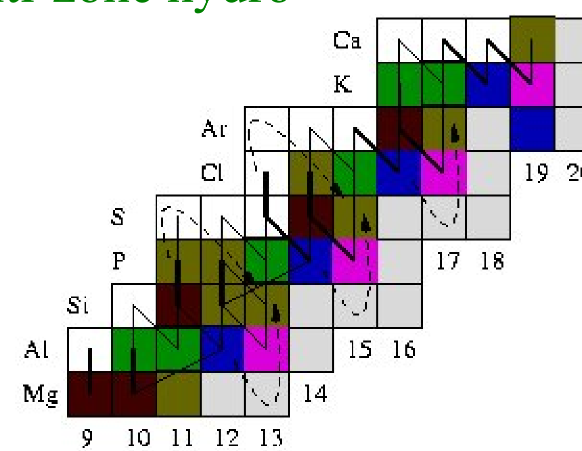
at Z above 20 a sequence of rapid proton captures and beta-decays (rp-process), for smaller target charges (Coulomb barriers) (a,p) reactions possible

rp-process (encounters p-drip line, endpoint Sb-Te cycle)

Schatz et al. (1998, 2001 ..) in one-zone models



Fisker, Thielemann, Wiescher (2004), full multi-zone hydro



develops features of **QSE-groups along isotonic lines** with links via beta-decays (waiting points) and alpha-induced reactions (tested for energy generation by Rembges et al. 1997)