Astronomie et astrophysique pour physiciens CUSO 2012

Instruments and observational techniques – Image formation

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The observables

Propagation of light wave from stable source at infinity:

$$\vec{E}(\vec{x},t) = \vec{A} \cdot e^{-i(\vec{k}\vec{x}-\omega \cdot t)}$$

which is a solution of wave equations if:

$$c_n = \frac{\omega}{k}, \text{ where } k = |\vec{k}|$$

c_n is the speed of light in a medium with refractive index n = n(k)

Independent observables are:

$\vec{A}$ = Amplitude of electric field

(k_x, k_y) = Direction vector projected on sky

ω = Frequency or k = Wave vector

The distance between two spatial maxima of the light wave in a given medium and at fixed *t* is called wavelength and results to be:

$$\lambda_n = \frac{2\pi}{n(k) \cdot k}$$

The observables

Astronomical spectroscopy aims at measuring:

$$\vec{A}(\nu,(k_x,k_y)) \quad \text{or} \quad \vec{A}(\lambda,(k_x,k_y))$$

At optical wavelength $\nu=\omega/2\pi$ is 10^{15} Hz, thus too fast to be resolved by detectors. The observable becomes the (surface) brightness or specific intensity I_ν or I_λ:

$$I_\nu(k_x,k_y) = \overline{\left|\vec{E}_{\nu,\lambda}(t,\vec{x}_{obs})\right|^2}^{t} = \frac{1}{2}\left|\vec{A}(\nu,(k_x,k_y))\right|^2 \quad [\mathrm{W\ m^{-2}\ sterad^{-1}\ Hz^{-1}}]$$

$$\text{or} \quad I_\lambda(k_x,k_y) = \frac{1}{2}\left|\vec{A}(\lambda,(k_x,k_y))\right|^2 \quad [\mathrm{W\ m^{-2}\ sterad^{-1}\ \mu m^{-1}}]$$

The observables

Morphology: $I(\alpha,\delta)$ and $dI/dt(\alpha,\delta)$

- ✓ Source geometry and dynamics
- ✓ Time variations
- ✓ Interactions by gravity and radiation
- ✓ Cosomology

Imaging

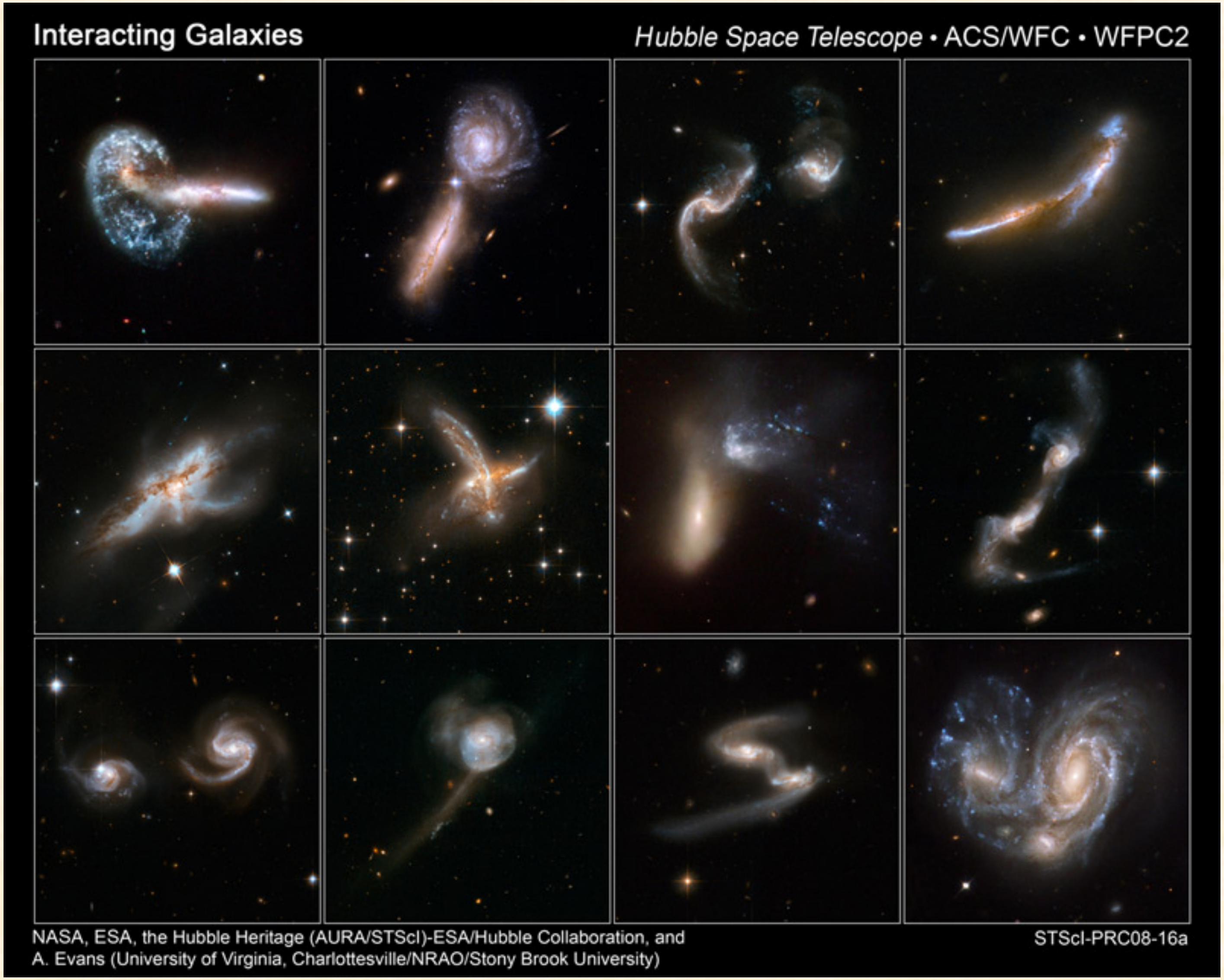
Imaging

Galileo Galilei, 1609


14 OBSERVAT. SYDEREAE

Haec eadem macula ante secundam quadraturam nigrioribus quibusdam terminis circumuallata conspicitur; qui tamquam altissima montium iuga ex parte Soli auersa obscuriores apparent, quà verò Solem respiciunt lucidiores extant; cuius oppositum in cauitatibus accidit, quarum pars Soli auersa splendens apparet, obscura verò, ac vmbrosa, quae ex parte Solis sita est. Imminuta deinde luminosa superficie, cum primum tota fermè dicta macula tenebris est obducta, clariora montium dorsa eminenter tenebras scandunt. Hanc duplicem apparentiam frequentes figurae commonstrant.

Imaging


Interacting Galaxies
Hubble Space Telescope • ACS/WFC • WFPC2
NASA, ESA, the Hubble Heritage (AURA/STScI)-ESA/Hubble Collaboration, and
A. Evans (University of Virginia, Charlottesville/NRAO/Stony Brook University)
STScI-PRC08-16a

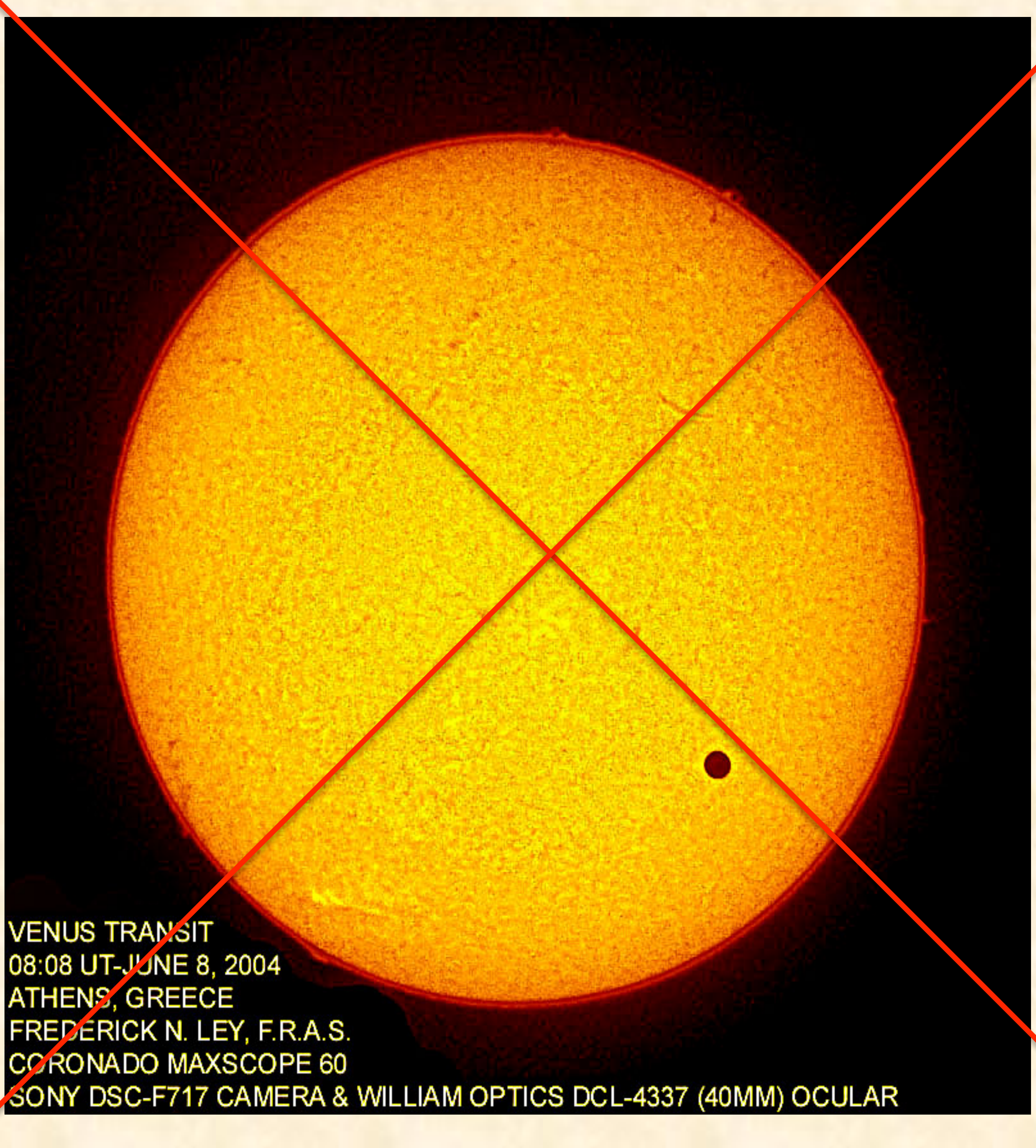
The observables

Intensity/flux: $I_{\alpha,\delta}(t)$ and $dI_{\alpha,\delta}(t)/dt$

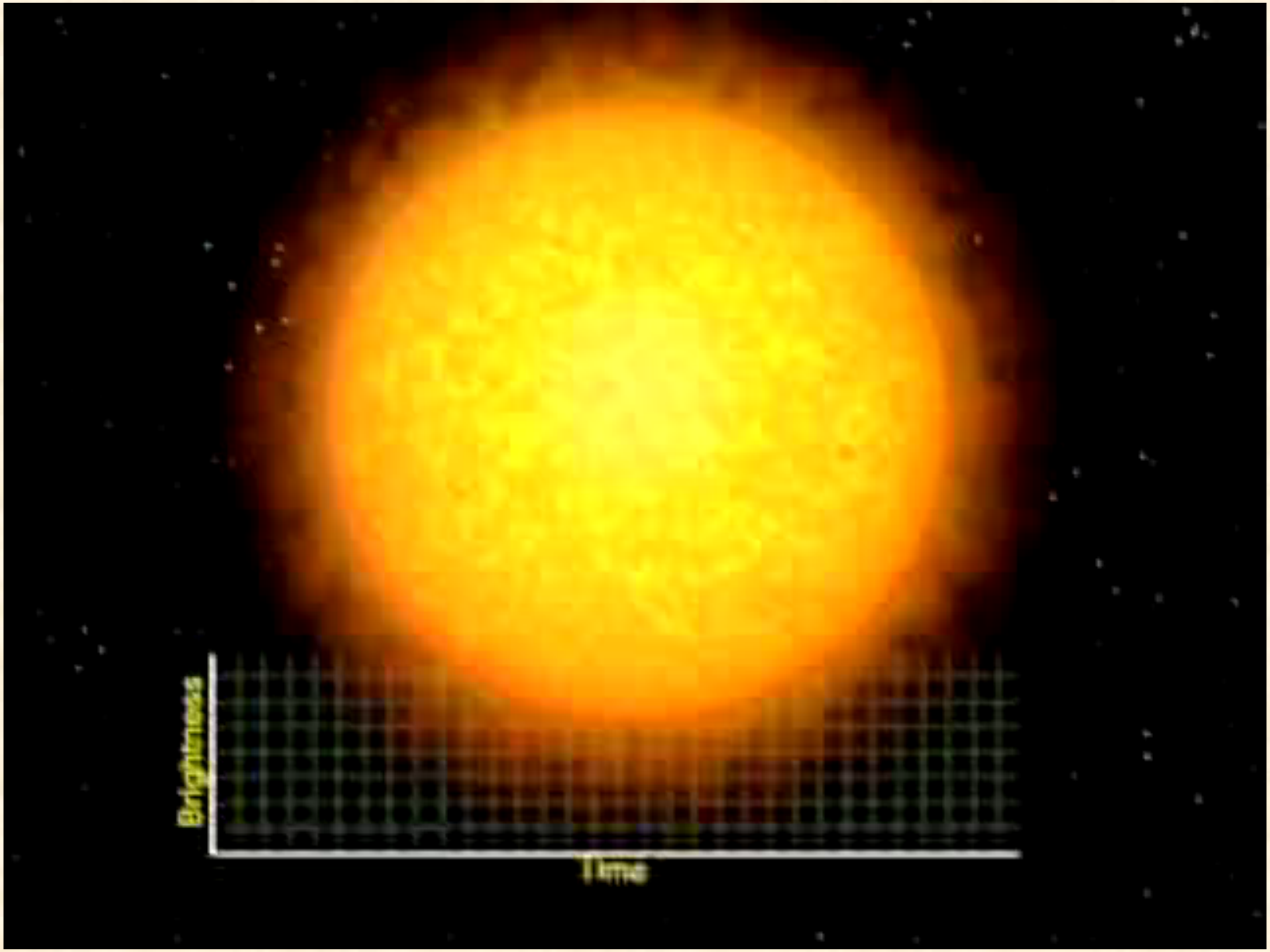
- ✓ Variability due to physics or perturbing obejct
- ✓ Periodic events
- ✓ Luminosity/distance/mass ('standard candle')

Photometry

Photometry


VENUS TRANSIT
08:08 UT-JUNE 8, 2004
ATHENS, GREECE
FREDERICK N. LEY, F.R.A.S.
CORONADO MAXSCOPE 60
SONY DSC-F717 CAMERA & WILLIAM OPTICS DCL-4337 (40MM) OCULAR

Photometry



The observables

Polarisation: $I_{P,S}(\alpha,\delta,\nu)$ and $I_{P,S}(\alpha,\delta,\nu)$ /dt

- ✓ Magnetic fields
- ✓ Physics of emitting medium
- ✓ Physics of intermediate medium

Polarimetry

The observables

Position: α_S, δ_S and $d\alpha_S/dt$, $d\delta_S/dt$

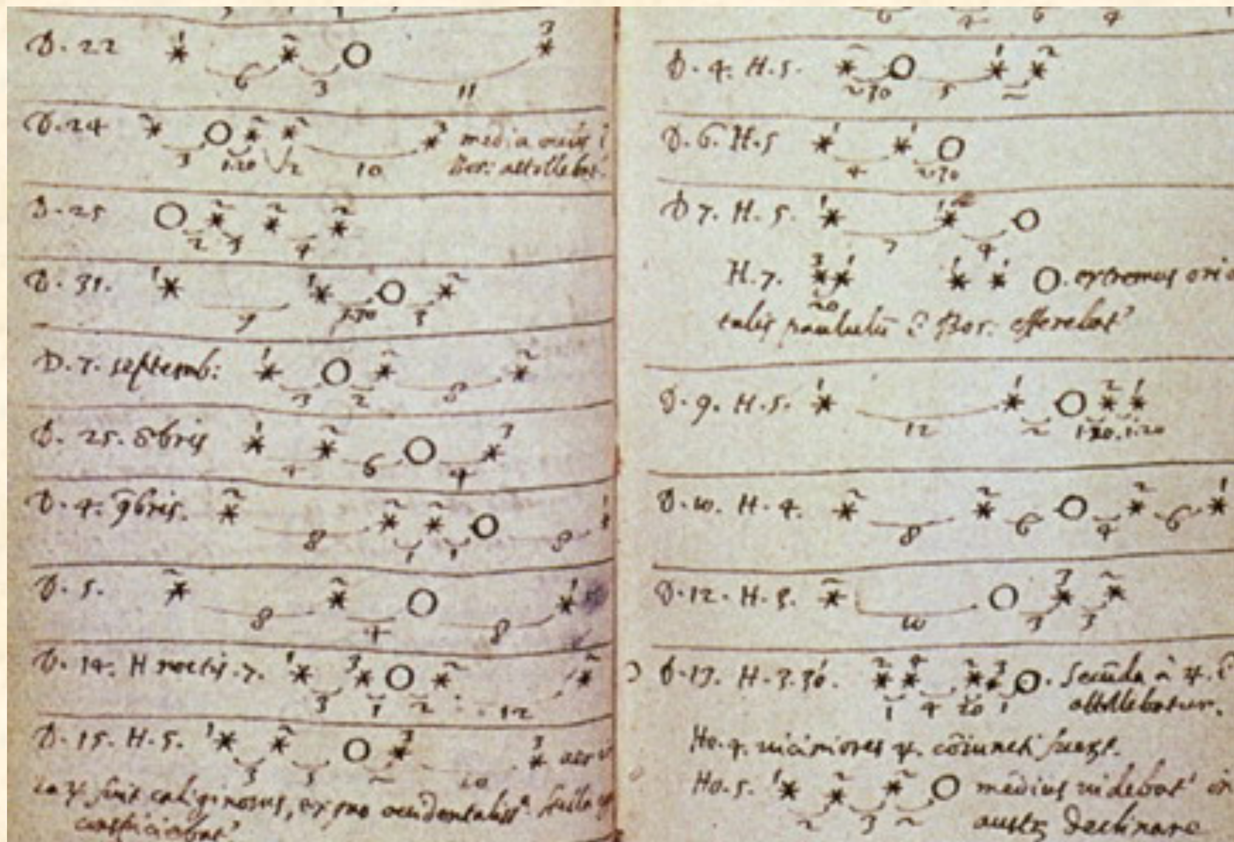
- ✓ Parallaxes -> distances
- ✓ Proper motion
- ✓ Kinematics and dynamics

Astrometry

Astrometry

Callisto
Io
Ganimede
Europa

Galileo Galilei, 1609



The observables

Spectrum: Flux density (integrated surface brightness)

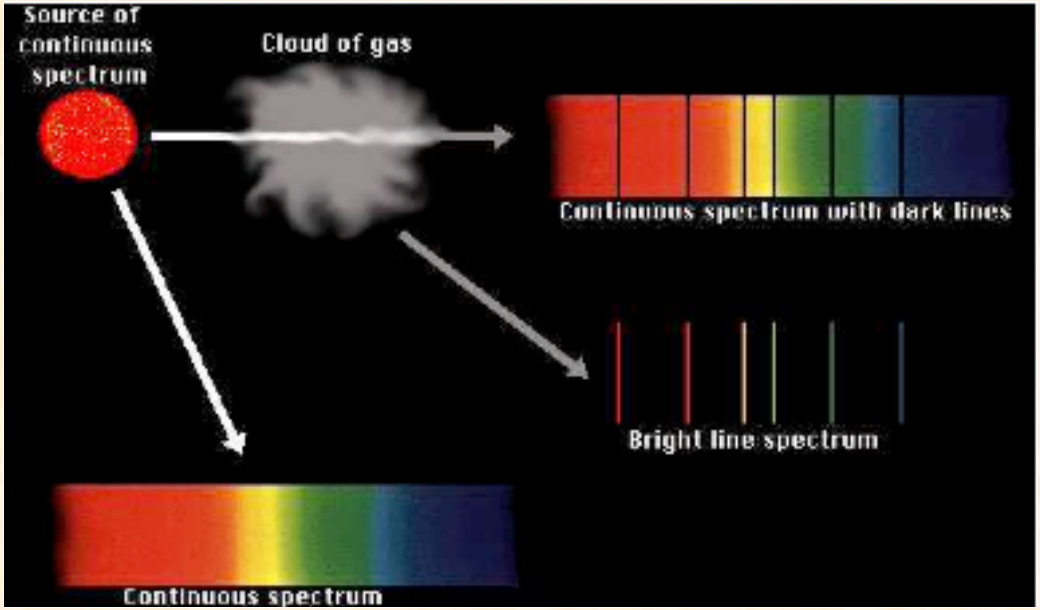
- ✓ Temperature
- ✓ Chemical composition and processes
- ✓ Source velocity and rotation (Doppler effect)

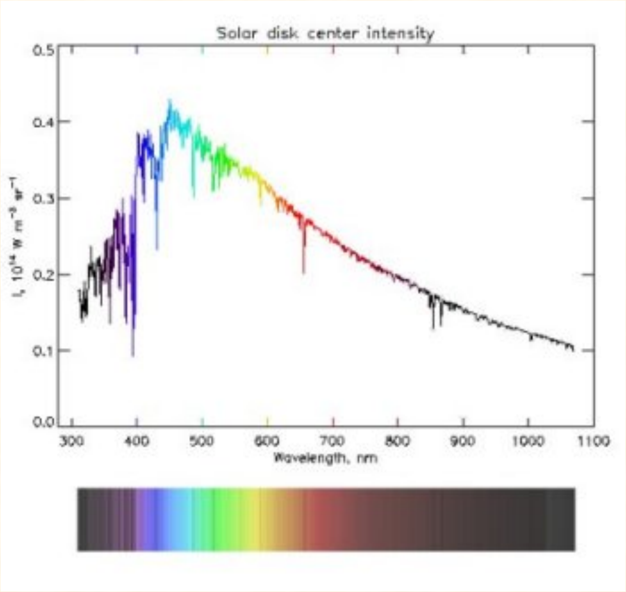
Spectroscopy

Spectroscopy

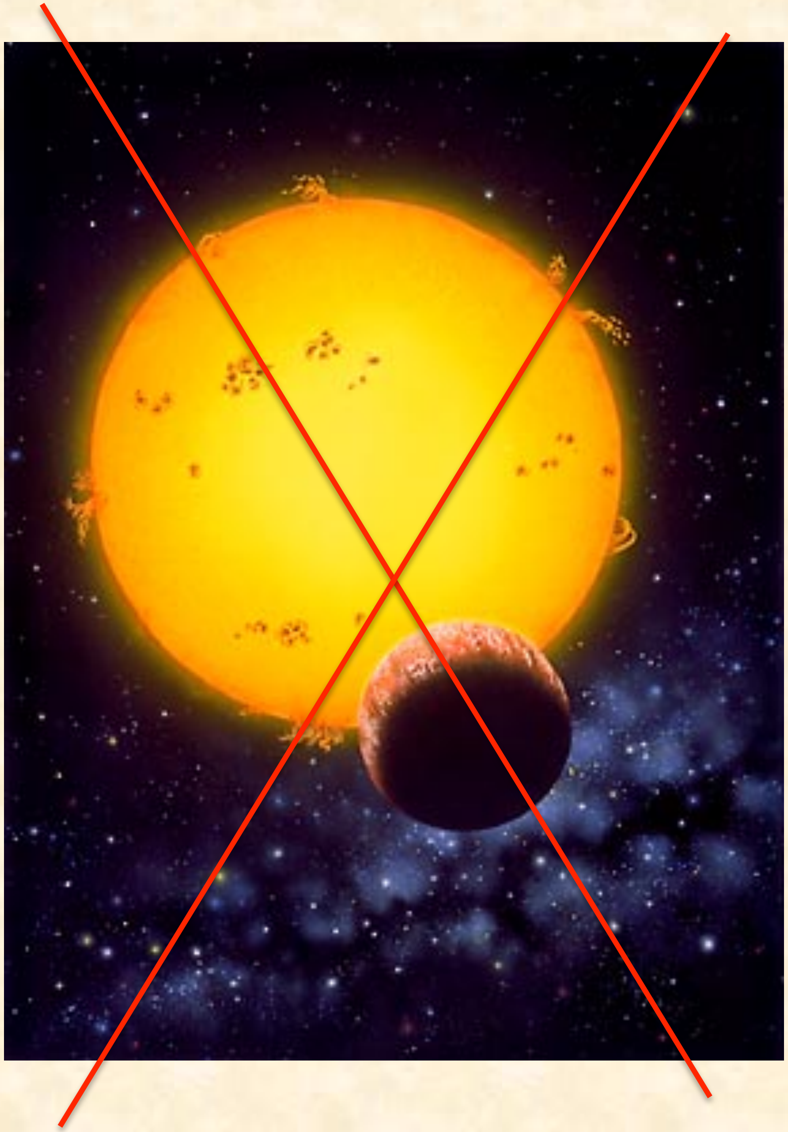
$$F_\nu = F(\nu) = \int S(\nu,(k_x,k_y)) \cdot \cos\Theta \cdot d\Omega \cong \int S(\nu,(k_x,k_y)) \cdot d\Omega$$

$$F_\lambda = F(\lambda) = \int S(\lambda,(k_x,k_y)) \cdot \cos\Theta \cdot d\Omega \cong \int S(\lambda,(k_x,k_y)) \cdot d\Omega$$



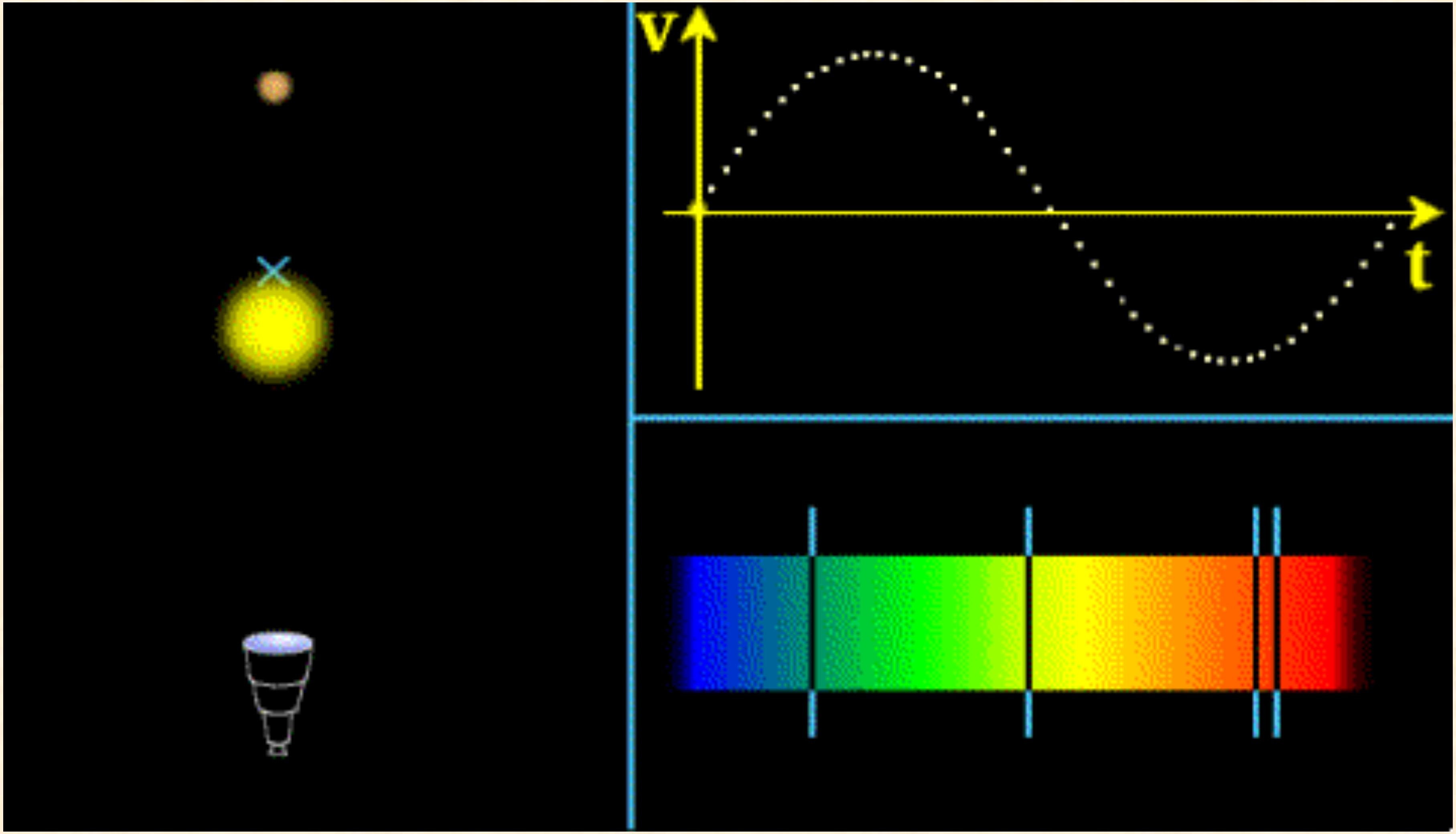


(Doppler) Spectroscopy



1995
Discovery of 51Pegb
Queloz & Mayor

(Doppler) Spectroscopy



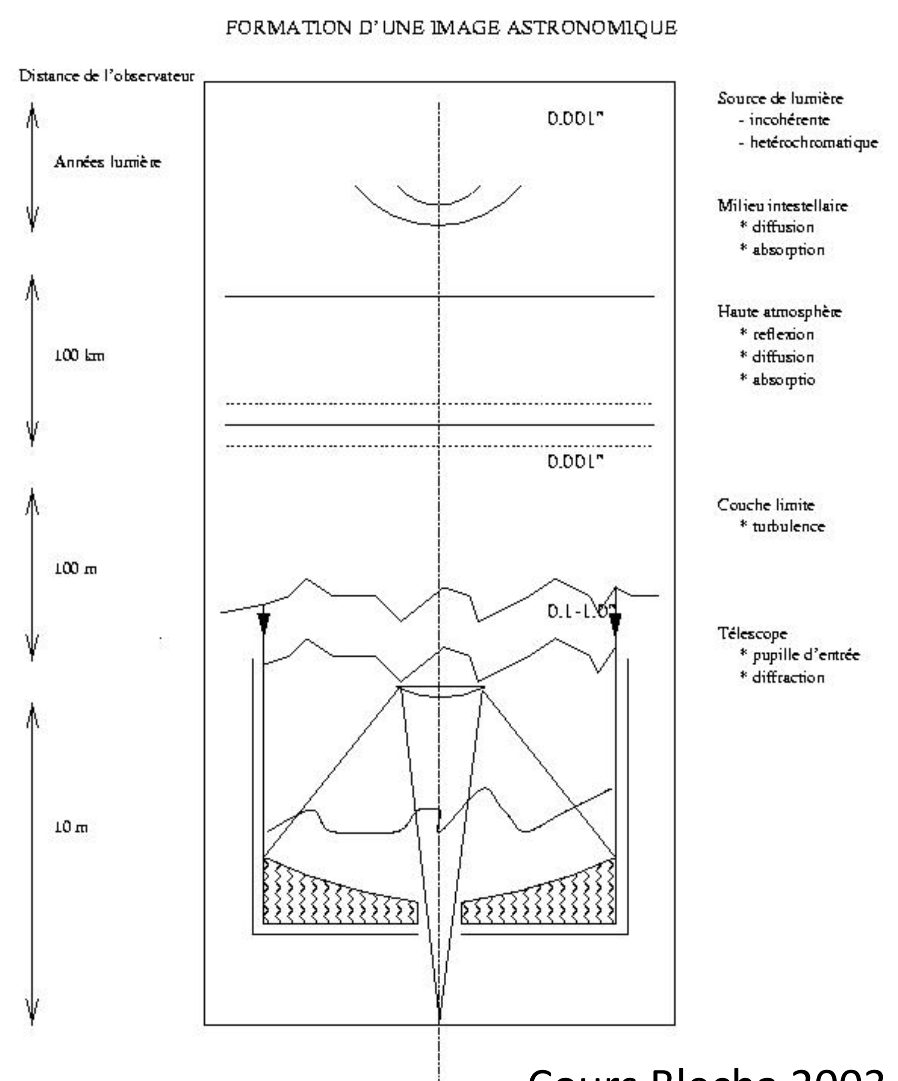
Observing with a telescope

Goals:

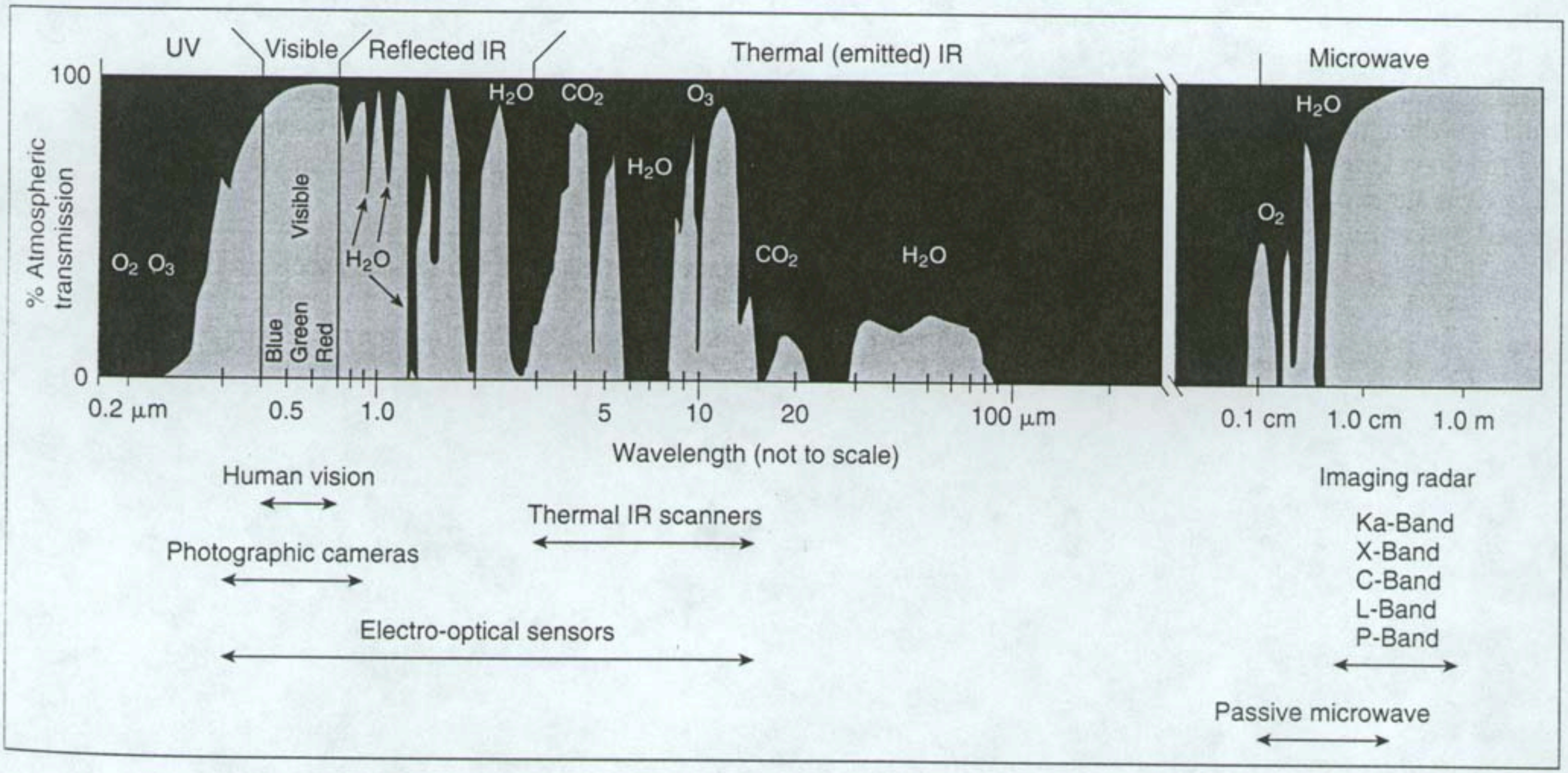
- Collect as much light as possible
- Resolve spatial 'details' (form an image)

The telescope is the 'lens' and the instrument/detector is the 'chip' of your 'camera'.

Acquiring the image

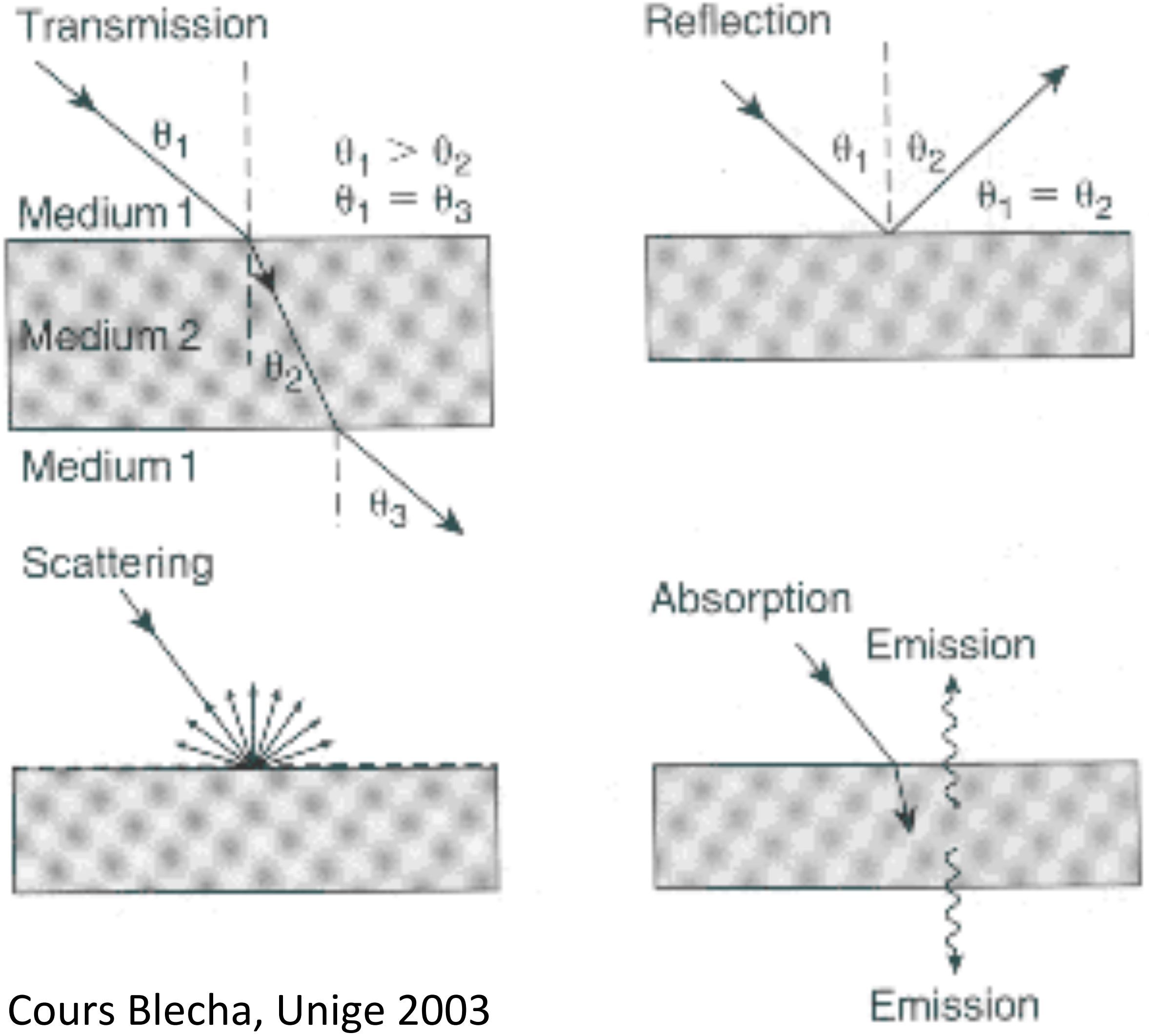


Cours Blecha 2003



Cours Blecha, Unige 2003

Effects of the atmosphere



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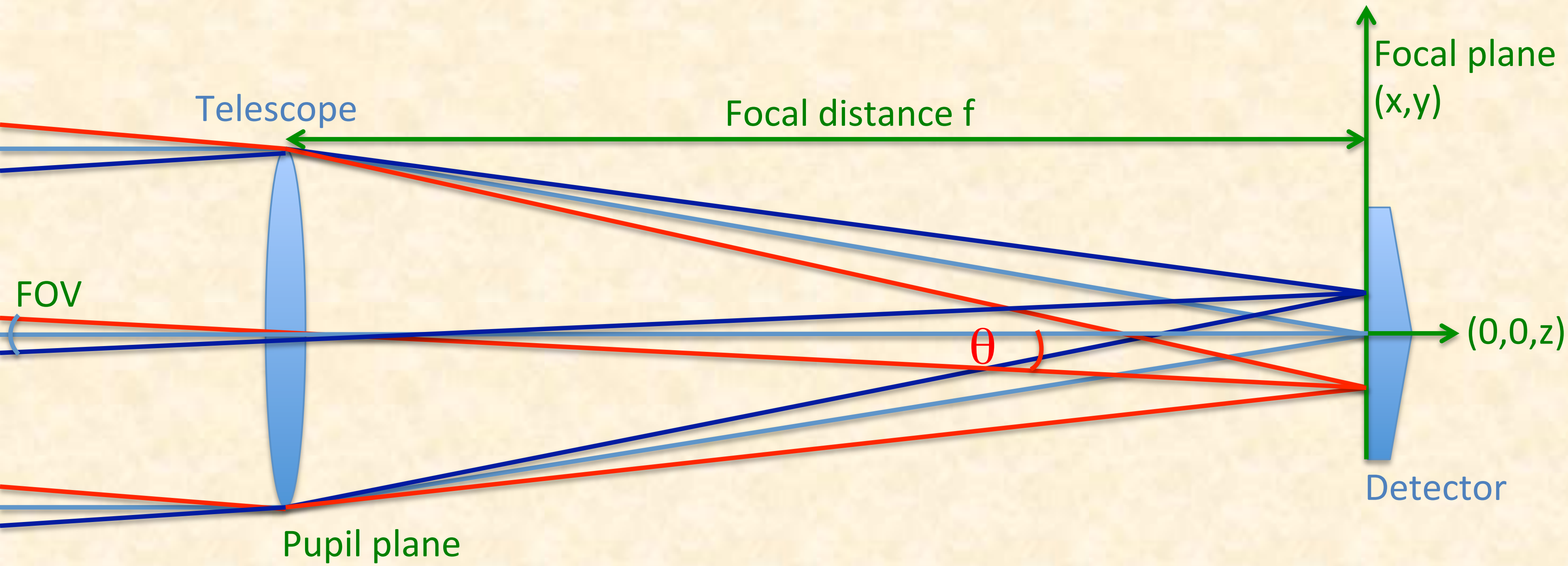
Telescope definition



Field of View: FOV [arcsec]
Telescope diameter (collecting area): D [m]
Focal length: f [m]
Source angular position: θ [arcsec]
Transmittance (efficiency): ε
Image quality: IQ

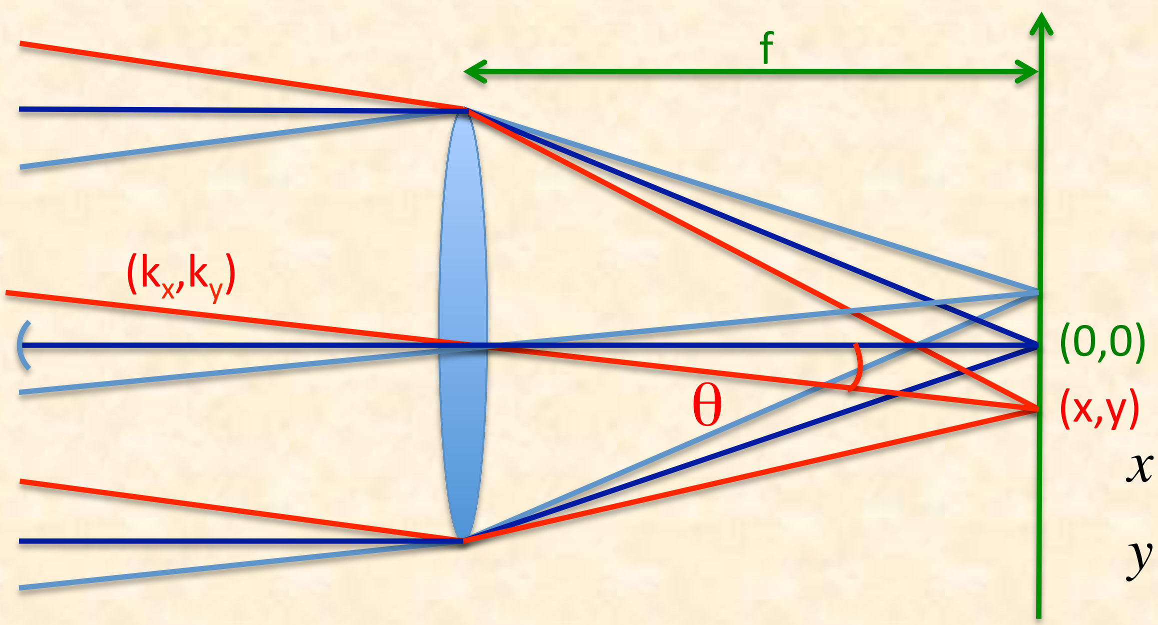
Collecting area: $S = \pi D^2/4$ [m^2]
F-number: f/D
Numerical aperture: NA = D/2f
Scale factor: $\Delta x/\Delta\theta$ [mm/arcsec]
$\Delta\theta$ = FWHM (full-width at half maximum)
= angular (spatial) resolution = $1.22\ \lambda/D$

Geometrical image

Object at infinity -> Image on detector

Angular position -> Position on detector

(k_x, k_y) -> (x, y)



$$\vec{k} = \frac{2\pi}{\lambda} \cdot \vec{j} \qquad \left|\vec{j}\right| = 1$$

$$\left|\vec{k}\right| = k = \frac{2\pi}{\lambda} = \frac{2\pi \nu n}{c}$$

$$x = f \cdot k_x / k = f \cdot \sin\theta_x \cong f \cdot \theta_x$$

$$y = f \cdot k_y / k = f \cdot \sin\theta_y \cong f \cdot \theta_y$$

Telescope classes

Refractor:

- Uses lenses
- Good AR coatings
- Chromatic
- 'Easier' to manufacture
- Lower optical power
- longer

Reflector:

- Uses mirror
- Expensive reflective coat.
- Achromatic
- More expensive
- Higher optical power
- More compact

Mirror manufacturing

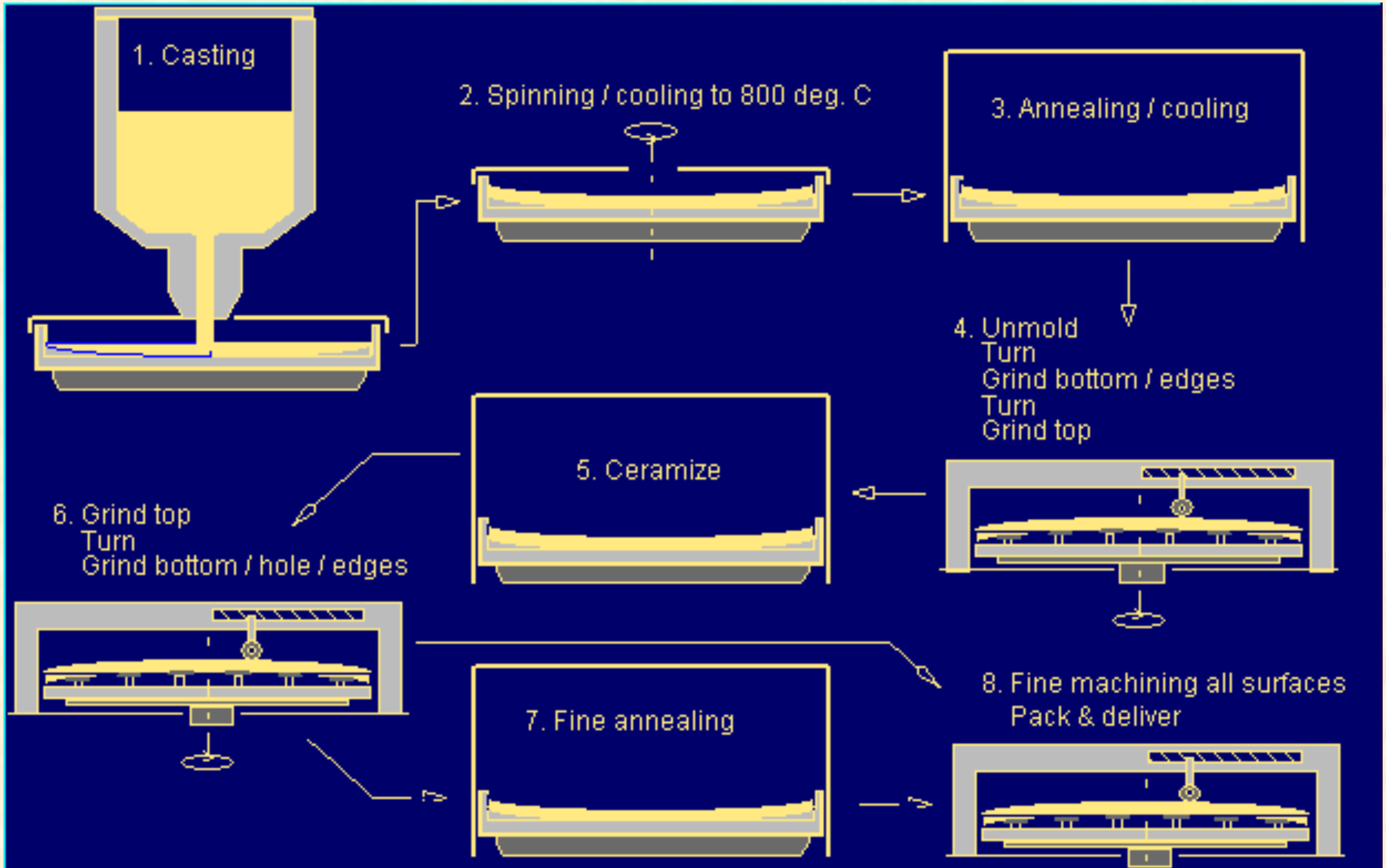


Figure 3. Mirror blanks manufacturing process

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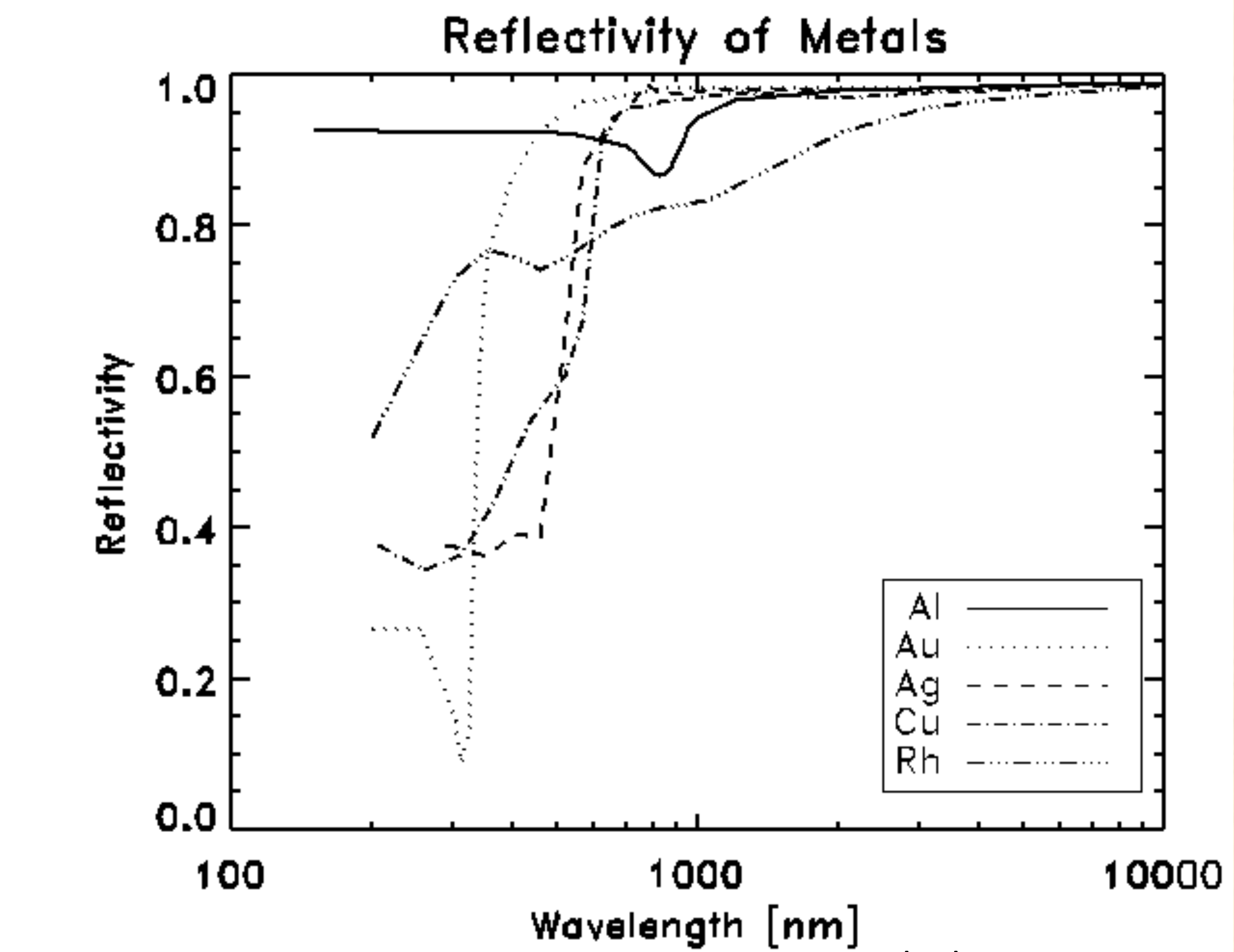
Reflective coatings



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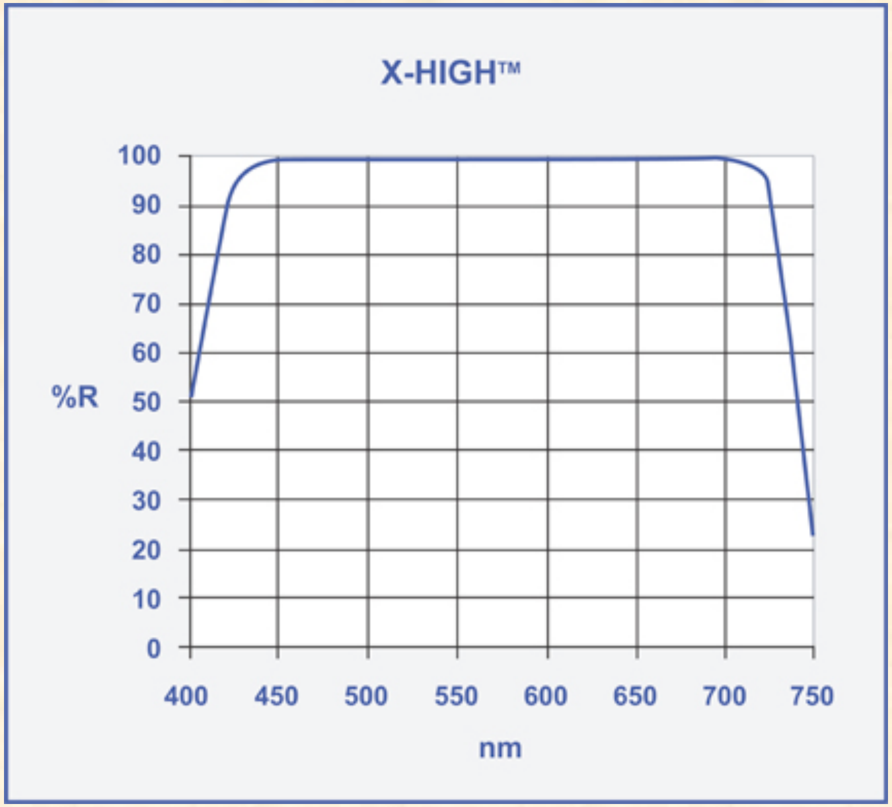
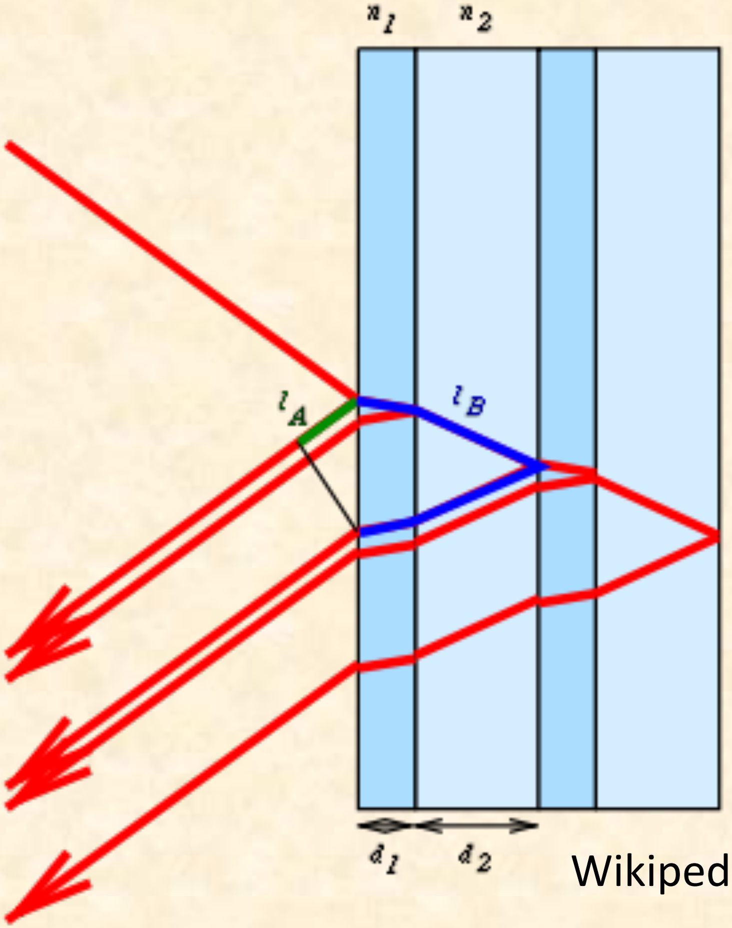
Dielectric coatings



OpcoLab

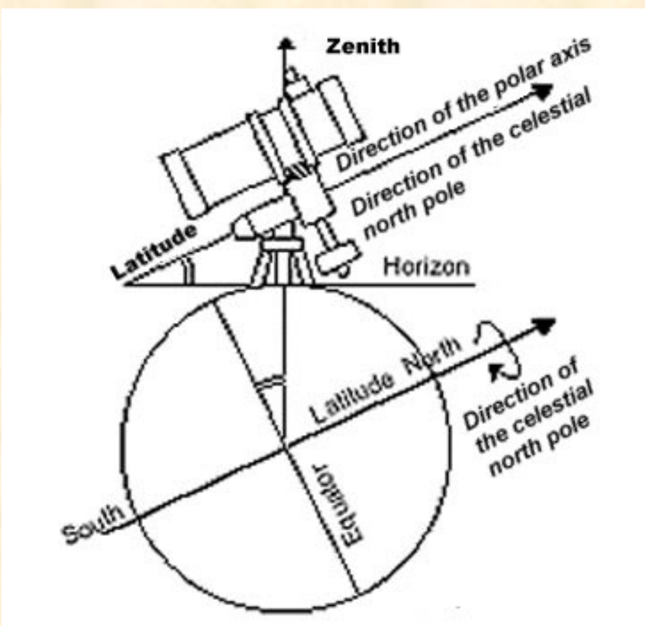


Wikipedia

Telescope mounting classes

Equatorial

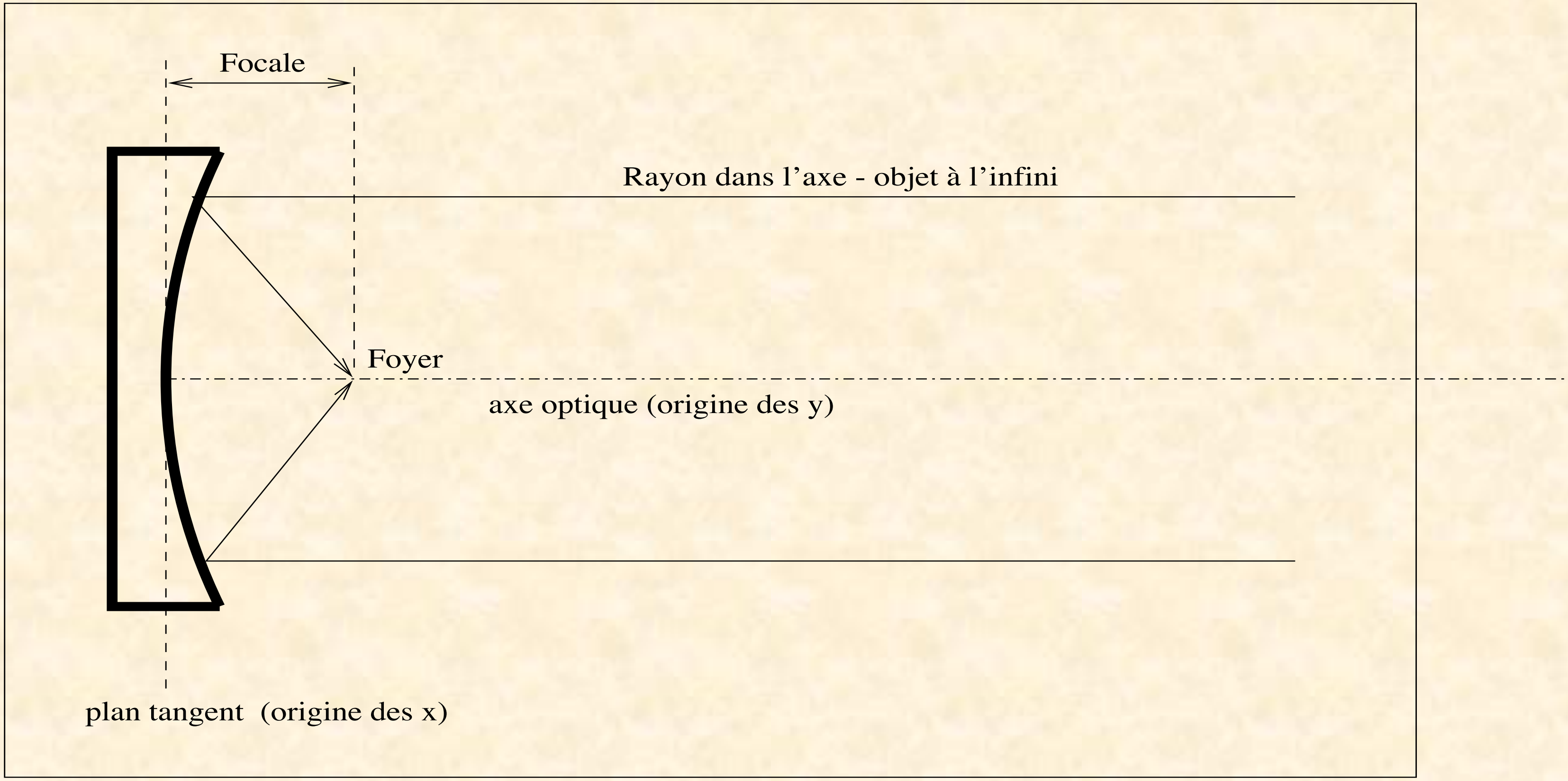
- Simple control
- Heavier and bigger mechanics



Horizontal

- Simpler and more compact
- Requires computer control

Basic telescope



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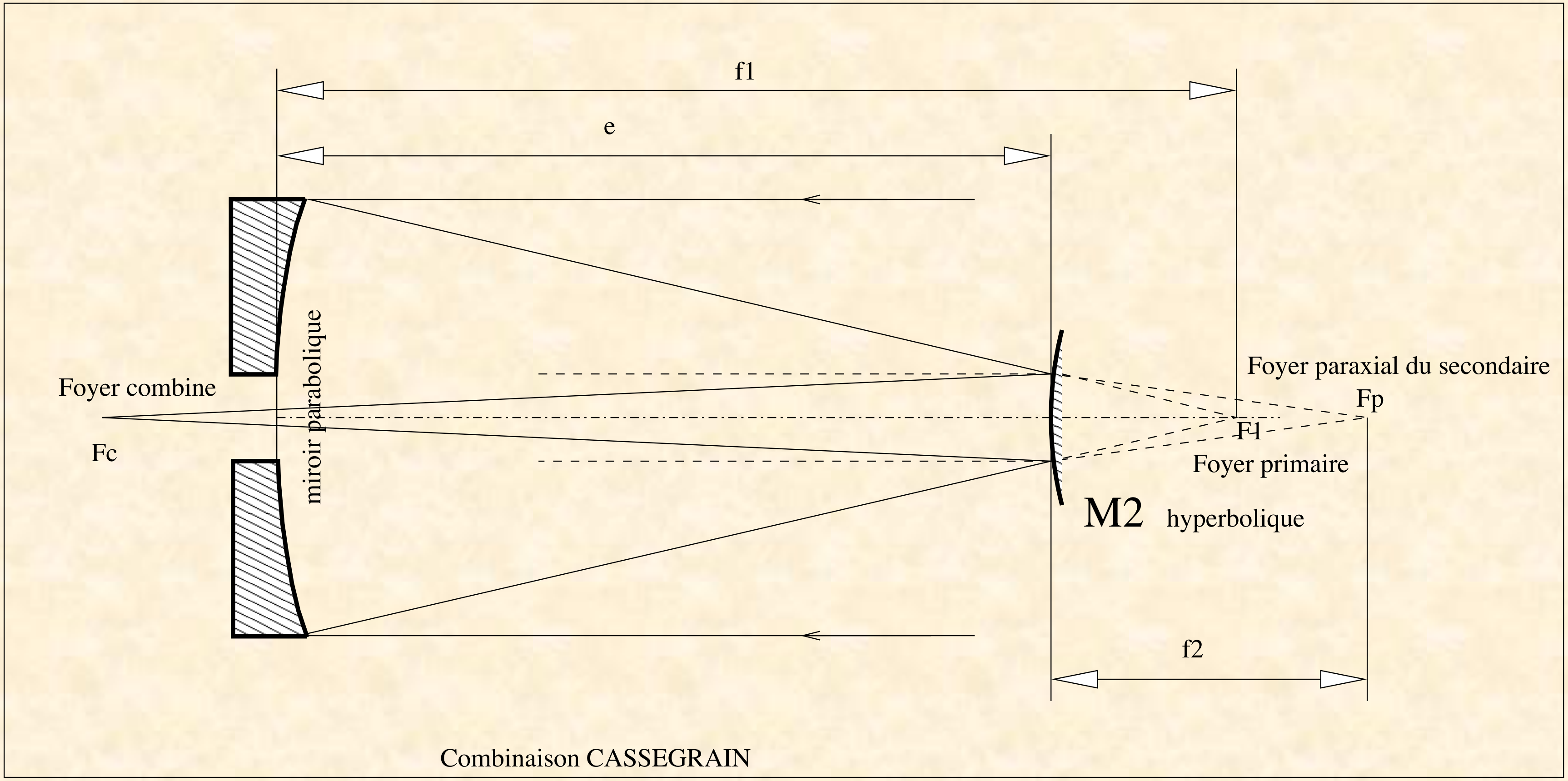
Cassegrain telescope



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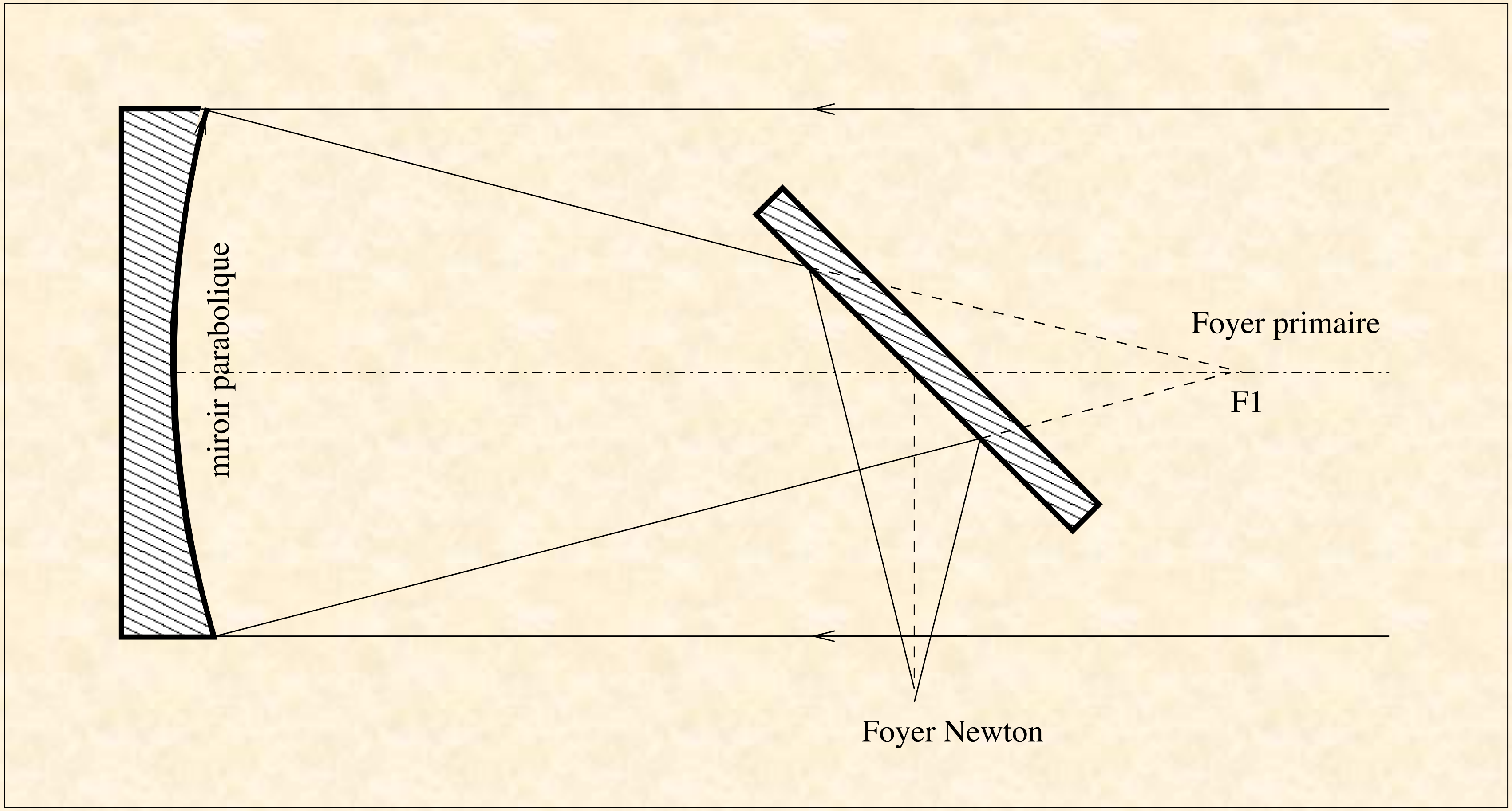
Newton telescope



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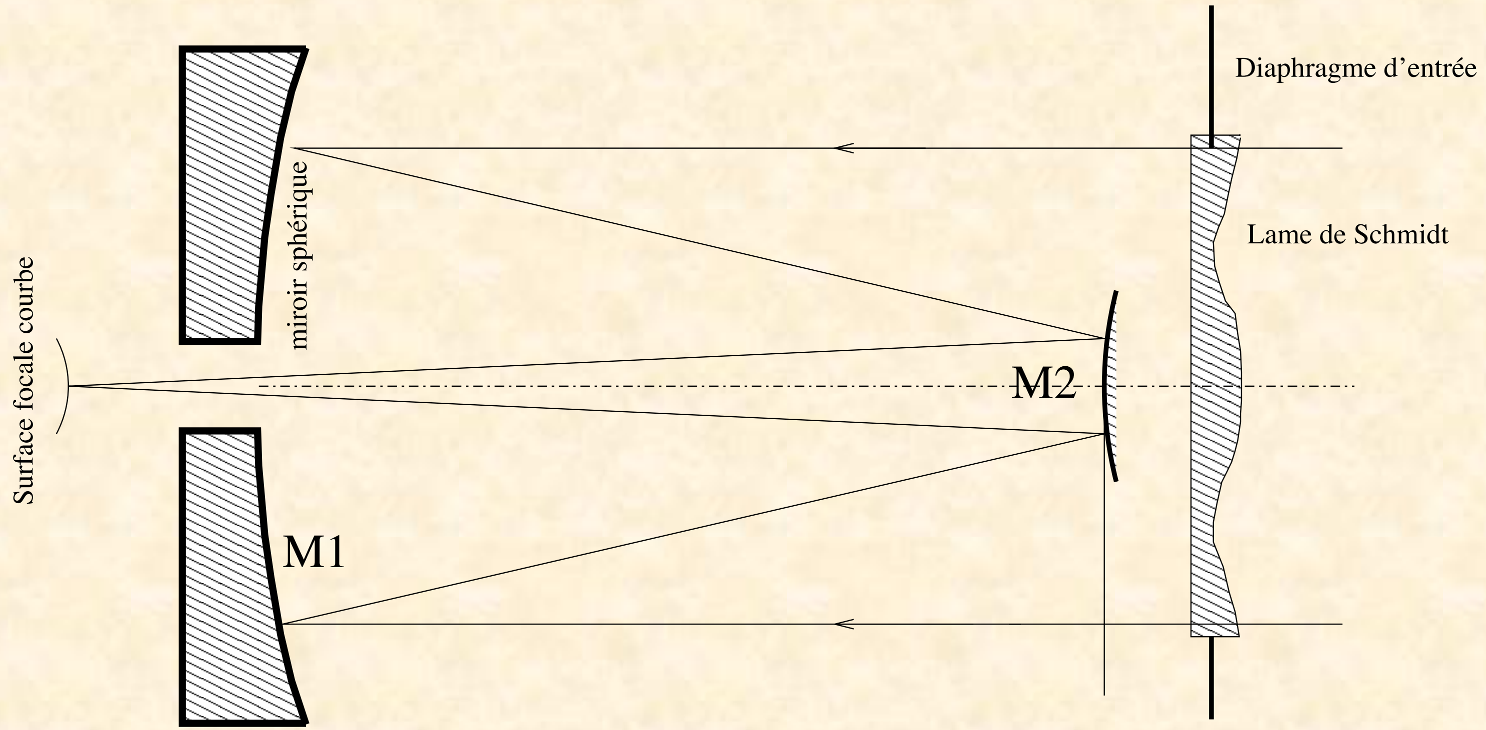
Schmidt-Cassegrain



Combinaison SCHMIDT - CASSEGRAIN

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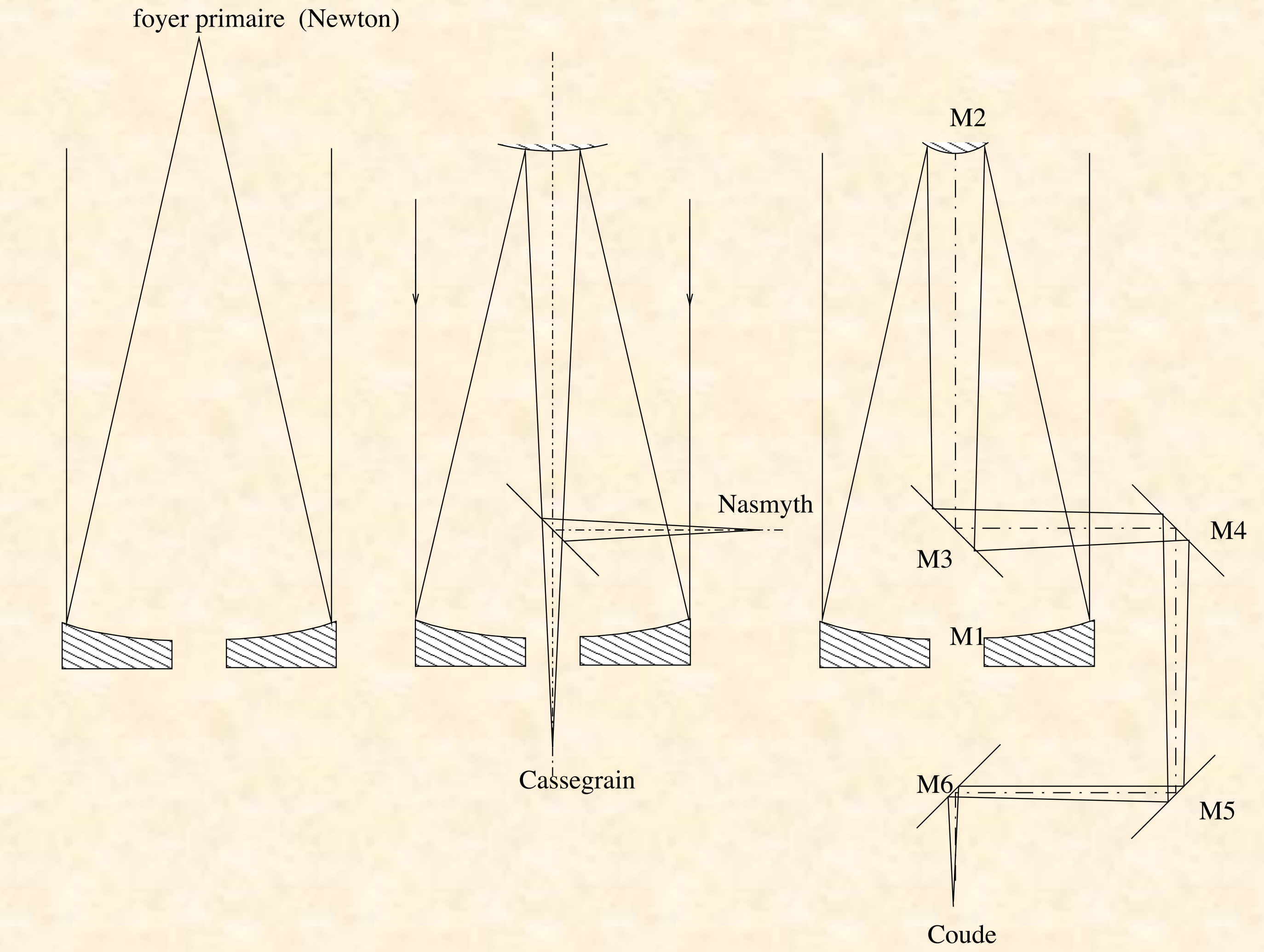
Telescope focii



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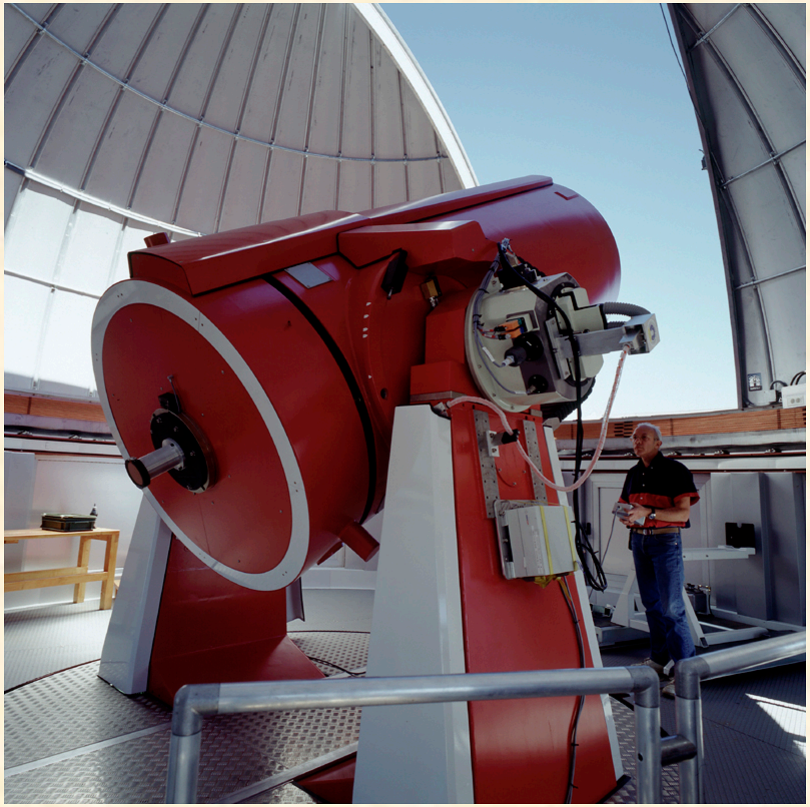
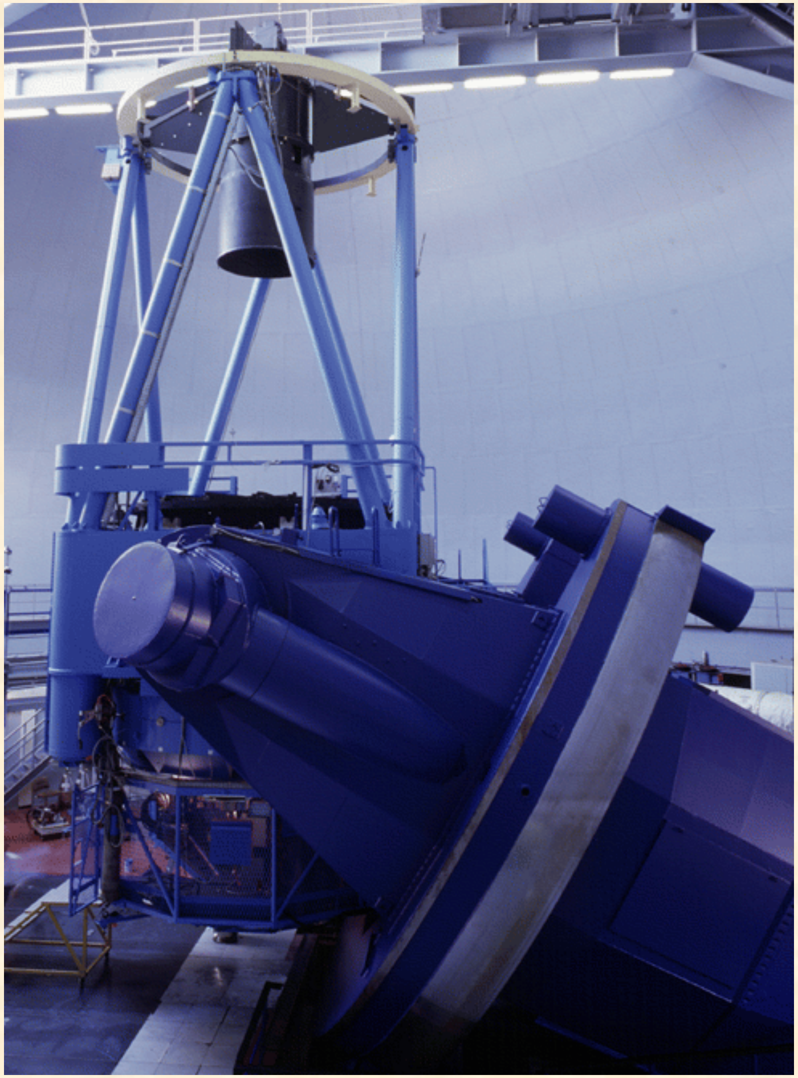
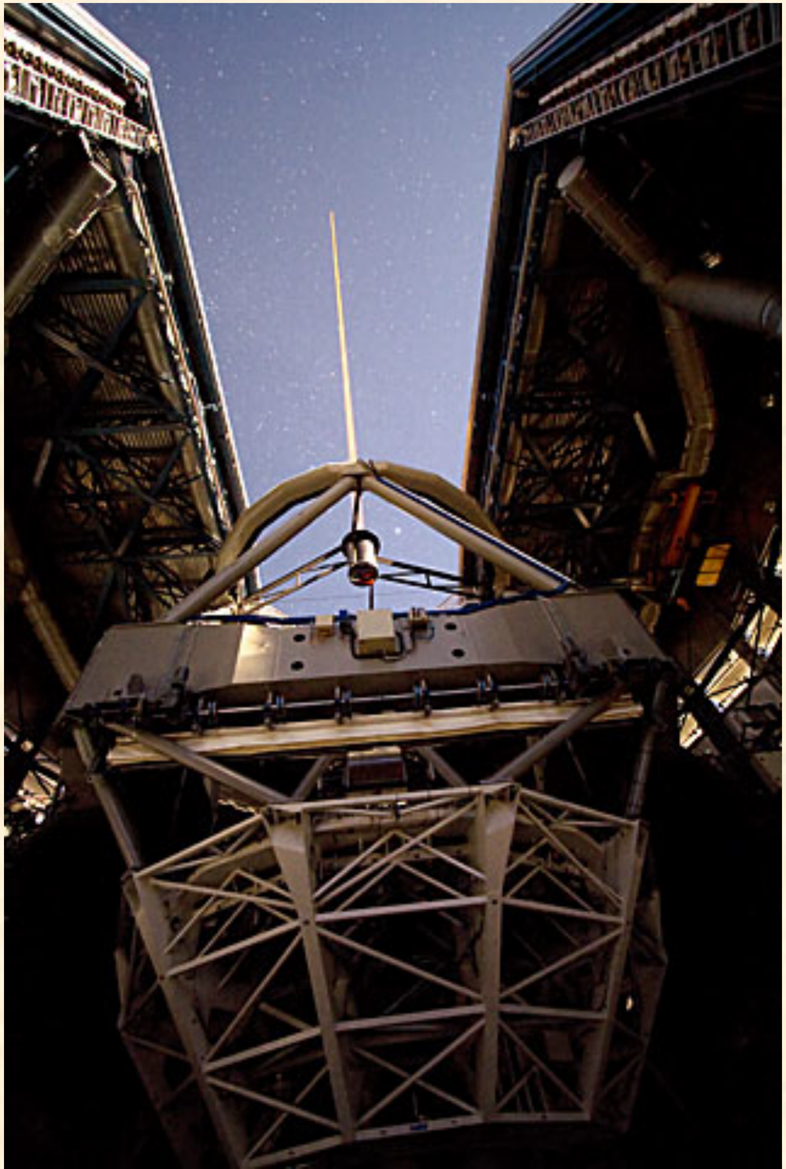
Swiss Euler@ La Silla



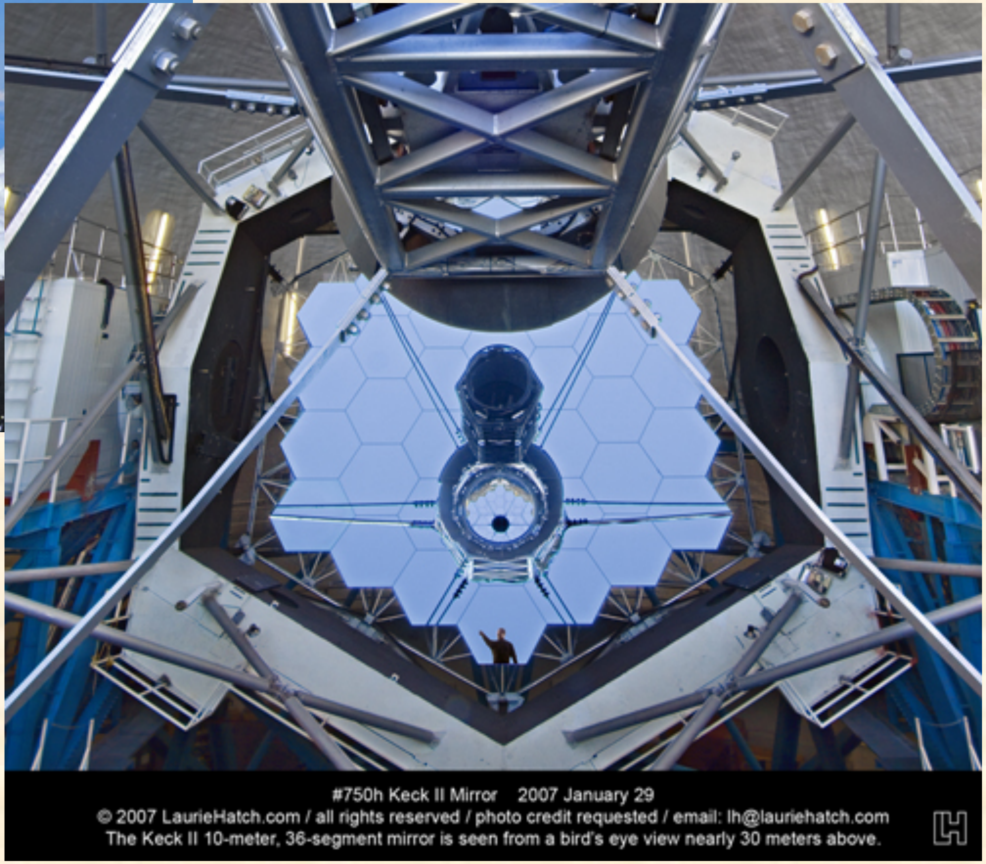
ESO 3.6-m@ La Silla



ESO VLT@Paranal



Keck@Mauna Kea



Future E-ELT@ESO

www.eso.org

Fourier optics

Prinzip 1 : L'équation de Helmholtz à comme solution les ondes planes :

$$u = u_0 \cdot e^{-i\vec{k}\vec{x}}$$

$u_0, \vec{k}$ complexes
u = scalaires

u représente 1 composante du champs électrique (vecteur $\vec{E}$) ou Magnétique ($\vec{H}$)

Prinzip 2 : Superposition.
Deux champs $u_1(\vec{x})$ et $u_2(\vec{x})$ produisent un champs qui est la somme des deux champs individuels :

$$u(\vec{x}) = u_1(\vec{x}) + u_2(\vec{x}) ,$$

Fourier optics

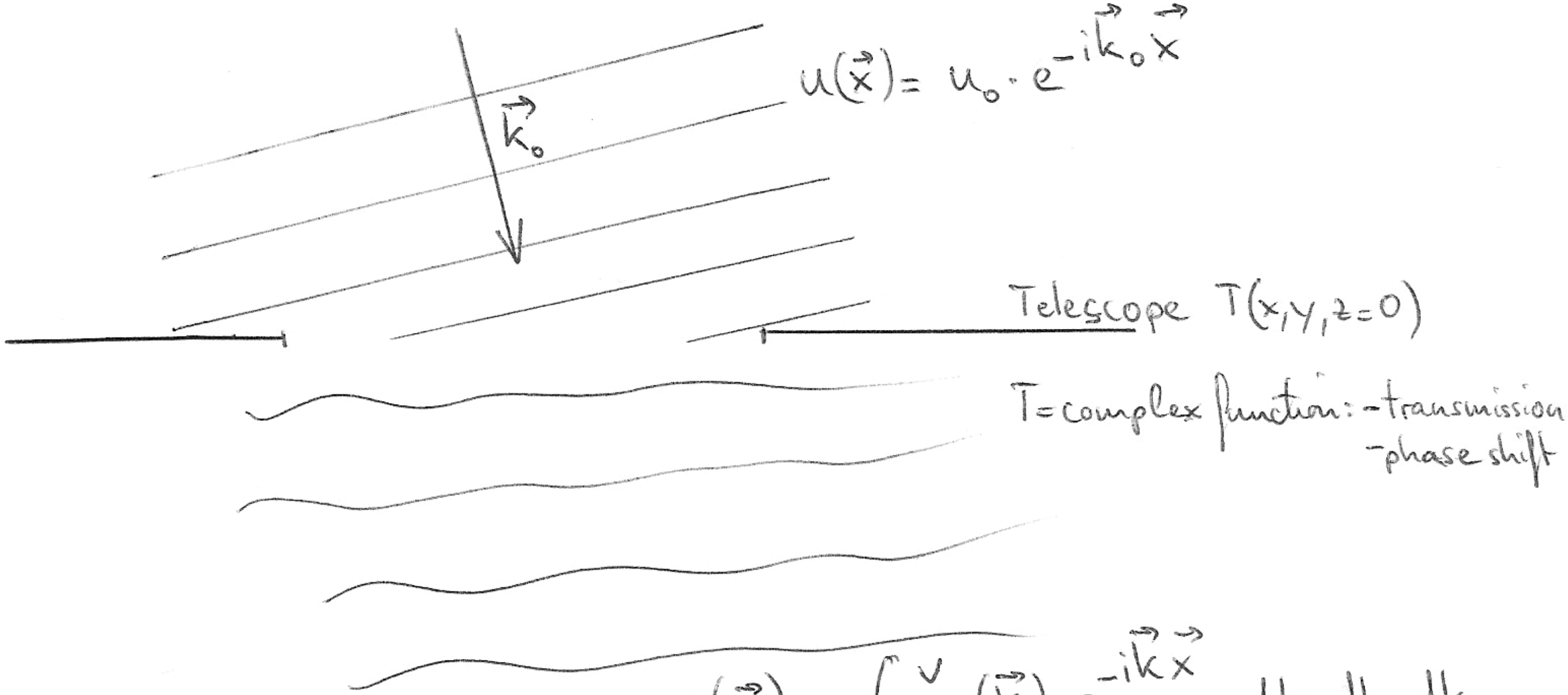
Prinzip 3: Un champs $u(\vec{x},t)$ peut être décrit en tant que superposition de ondes planes, et qui decrient automatiquement une solution de l'équation de Helmholtz.

$$u(\vec{x}) = \int \tilde{u}(\vec{k}) \cdot e^{-i\vec{k}\vec{x}} \, d\vec{k}$$

$$u(\vec{x}) = \int \tilde{u}(k_x, k_y) e^{-ik_x x} e^{-ik_y y} e^{-ik_z z} \, dk_x dk_y$$

Prinzip 4: Si un champs $u(\vec{x})|_A$ est connu partout sur une surface fermée A alors le champs $u(\vec{x})$ est connu et défini partout dans l'espace.

Image of point-like source



$$v(\vec{x}) = \int \check{u}(\vec{k}) \cdot e^{-i\vec{k}\vec{x}} \, dk_x \, dk_y \, dk_z$$

pour div v = 0 : $v(\vec{x})\big|_{z=0} \overset{!}{=} u(\vec{x})\big|_{z=0} \cdot T(x,y)$

Image of point-like source

$(z=0)$

$$\Rightarrow \int \check{u}(\vec{k}) \cdot e^{-i\vec{k}\vec{x}} \, dk_x dk_y = u_0 \cdot e^{-i\vec{k}_0 \vec{x}} \cdot T(x,y)$$

$$\Rightarrow \int \check{u}(\vec{k}) \cdot e^{-i(\vec{k}-\vec{k}_0)\vec{x}} \, dk_x dk_y = u_0 \cdot T(x,y)$$

T is FT of $\check{u}$ $\Rightarrow$ $\check{u}$ must be inverse FT of T

$$\Rightarrow \check{u}(\vec{k}) = \frac{u_0}{2\pi} \int T(x,y) \cdot e^{+i(\vec{k}-\vec{k}_0)\vec{x}} \, dx\, dy$$

$$\Rightarrow I(\vec{k}) = \frac{|\check{u}(\vec{k})|^2}{2} = \frac{u_0^2}{8\pi^2} \cdot \left|\check{T}(\vec{k}-\vec{k}_0)\right|^2$$

Def.: $PSF = |\check{T}(\vec{k})|^2$ = Point-Spread-Function

Image of an extended source

For extended source: incoherent sum!

Source $A(\vec{k}')$ = intensity distribution at emission

⇒ Measured intensity distribution $I(\vec{k})$ in focal plane:

$$I(\vec{k}) = \int A(\vec{k}') \cdot PSF(\vec{k} - \vec{k}') \, dk'_x \, dk'_y$$

(Remember: k_z defined by $|\vec{k}| = \frac{2\pi}{\lambda_n}$ $\quad \lambda_n = \frac{c}{n \cdot \nu}$)

Define optical transfer function (OTF)

- Imaging through any optical system: in intensity units

  Image = Object ⊗ Point Spread Function
  convolved with

  $I(\underline{r}) = O \otimes PSF \equiv \int d\underline{x}\, O(\underline{x} - \underline{r})\, PSF(\underline{x})$

- Take Fourier Transform: $\mathcal{F}(I) = \mathcal{F}(O)\, \mathcal{F}(PSF)$
- Optical Transfer Function is the Fourier Transform of PSF:

  $OTF = \mathcal{F}(PSF)$

PSF of '1-D' telescope

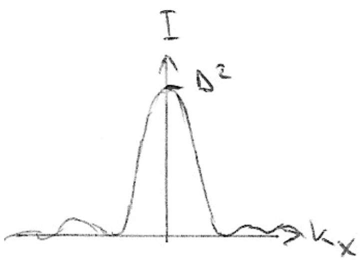
1-D Telescope

$$T(x) = \begin{cases} 0, & \text{for } |x| > D/2 \\ 1, & \text{for } |x| \leq D/2 \end{cases}$$

$$\underline{\underline{PSF(k)}} = \left| \int_x T(x)\, e^{ik_x x}\, dx \right|^2$$

$$= \left| \int_{-D/2}^{D/2} e^{ik_x x}\, dx \right|^2 = \left| \frac{1}{ik} \left(e^{i\frac{Dk_x}{2}} - e^{-i\frac{Dk_x}{2}} \right) \right|^2$$

$$= \left| D \cdot \left(\frac{e^{+i\frac{Dk_x}{2}} - e^{-i\frac{Dk_x}{2}}}{2i \cdot \frac{Dk}{2}} \right) \right|^2$$



$$= \left| D^2 \cdot \left(\frac{\sin\left(\frac{Dk_x}{2}\right)}{\frac{Dk_x}{2}} \right) \right|^2 = \underline{\underline{D^2 \cdot \text{sinc}^2\left(\frac{Dk_x}{2}\right)}}$$

PSF of '1-D' telescope

Le premier minimum est atteint pour 5.6

$$\frac{D \cdot k_x}{2} = \pi$$

Rappel : $k_x = k \cdot \sin \theta_x = \frac{2\pi}{\lambda} \cdot \sin \theta_x$

$\Rightarrow$ Minimum pour $\frac{D \cdot \pi \cdot \sin \theta_x}{\lambda} = \pi$

$\Rightarrow$ $\sin \theta_x = \frac{\lambda}{D} \simeq \delta\theta_x =$: Résolution spatiale

$\equiv$ Distance angulaire minimale à laquelle deux objets ponctuels sont distingués (résolus)

PSF of '2-D' telescope

$T(\vec{x}) = 0 \quad x^2+y^2 > R^2$

$T(\vec{x}) = 1 \quad x^2+y^2 \leq R^2$

$R = D/2$

$$\check{T}(k) = \int_A T(\vec{x}) \cdot e^{i\vec{k}\vec{x}} \, dx\, dy$$

A = Surface Télescope

$$= \int_{R<D/2} e^{ik_x x} \cdot e^{ik_y y} \, dx\, dy$$

$$= 2 \cdot \frac{J_1\left(\frac{D \cdot k}{2}\right)}{\frac{D \cdot k}{2}}$$

$$PSF(k) \propto \frac{J_1^2\left(\frac{D \cdot k}{2}\right)}{\left(\frac{D \cdot k}{2}\right)^2}$$

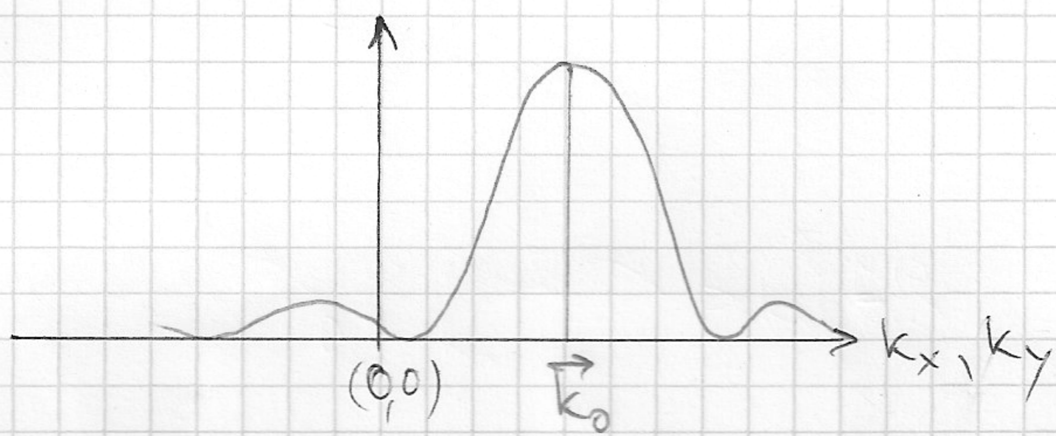
→ Premier minimum pour

$\sin\theta = 1.22 \cdot \lambda / D$ = Resolution

Image formation

Post scriptum: 5.7

Si la source ponctuelle est située à $\vec{k}_0 \neq 0$, alors l'intensité enrégistrée aura un maximum non pas à $(k_x, k_y) = (0,0)$ mais à $\vec{k} = \vec{k}_0$ $(PSF(\vec{k}-\vec{k}_0))$.



Exercises

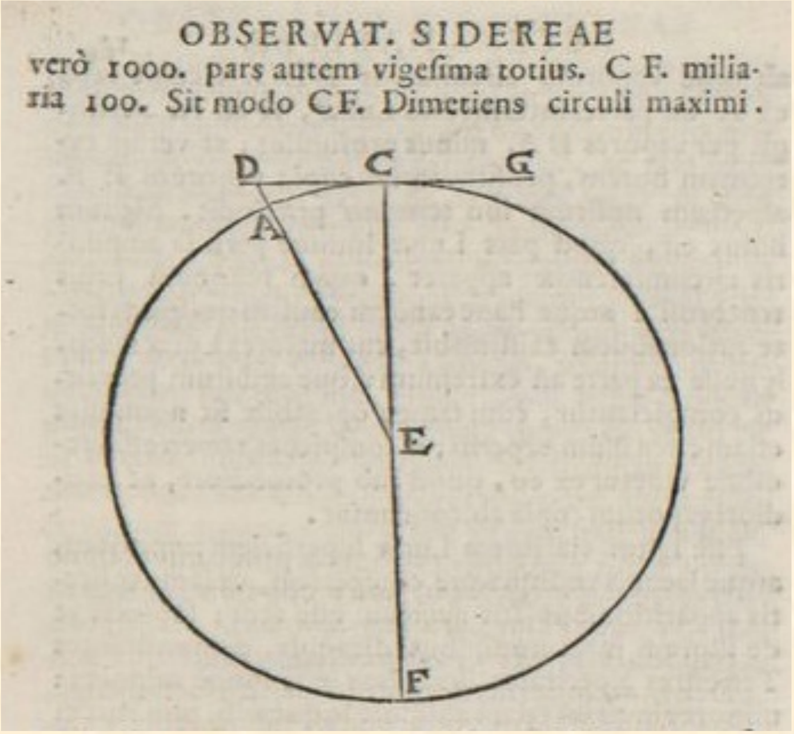
Exercise 1

Compute data rate (bytes per seconds) to observe the 'whole' universe:

- Solid angle of 4π
- Visible range (400 nm – 700 nm)
- Spectral resolution $\lambda/\Delta\lambda$ = 10'000
- Spatial resolution 1 arcsec
- 2 polaristations
- 16-bit resolution

Exercise 2

Estimate the height of the lunar 'mountains' using Galileo's approach and knowing that R_M = 3476 km:



Exercise 3

Compute the image size of a 10-arcsec galaxy in the focal plane of a telescope with effective focal length of 30m.

Exercise 4

Demonstrate that the mirror shape which reflects a parallel beam into a single focal point is a parabola. Tip: Use Fermat's principle

Exercise 5

Compute the spatial resolution of the human eye, assuming that the pupil size is of 5mm.

Give an example of an object at the limit of the eye's resolution.

Diametro della Luna= 3476km

$d/D=0.035$

$$h=((1738^2+(d/D\ 3476)^2)^{1/2} - 1738=4.25km$$