Problem 1:
Derive the density and circular velocity corresponding to the NFW potential
\[
\Phi(r) = v_s^2 \left[ 1 - \frac{\ln(1 + r/r_s)}{r/r_s} \right]
\]

Problem 2:
Using the Tully-Fisher relationship (equation 1.11), what is the luminosity of the
Galaxy given it has a rotation velocity \( v_c \sim 220 \text{ km s}^{-1} \)? What about the LMC with
\( v_c \sim 65 \text{ km s}^{-1} \)? Andromeda (M31) with \( v_c \sim 260 \text{ km s}^{-1} \)?

Problem 3:
The isochrone potential is given by
\[
\Phi(r) = -\frac{GM}{b + \sqrt{b^2 + r^2}}
\]
What is the density profile that gives this potential? What is the circular velocity?

Problem 4:
The velocity dispersion near the Milky Way’s centre is approximately \( 75 \text{ km s}^{-1} \),
what is the mass of the black hole in the centre of our Galaxy? The velocity dispersion
in the recently discovered ultra-compact dwarf, M60-UCD1, is \( 100 \text{ km s}^{-1} \), what is its
black hole mass?
The total mass of M60-UCD1 is estimated at \( 1.4 \times 10^{8} \text{ M}_\odot \). As a fraction of their
host galaxies’ total mass, how much bigger is M60-UCD1’s supermassive black hole
compared to the Milky Way’s?

Problem 5:
Using Gauss’ theorem, derive the surface density for the Kuzmin disk potential at
\( z=0 \)
\[
\Phi_K(R, z) = -\frac{GM}{\sqrt{R^2 + (a + |z|)^2}}
\]