Astrophysics III: Stellar and galactic dynamics

Exercises

Problem 1:

Derive the density and circular velocity corresponding to the NFW potential

$$\Phi(r) = v_s^2 \left[1 - \frac{\ln(1 + r/r_s)}{r/r_s} \right]$$

Problem 2:

Using the Tully-Fisher relationship (equation 1.11), what is the luminosity of the Galaxy given it has a rotation velocity $v_c \sim 220\,\mathrm{km\,s^{-1}}$? What about the LMC with $v_c \sim 65\,\mathrm{km\,s^{-1}}$? Andromeda (M31) with $v_c \sim 260\,\mathrm{km\,s^{-1}}$?

Problem 3:

The isochrone potential is given by

$$\Phi(r) = -\frac{GM}{b + \sqrt{b^2 + r^2}}$$

What is the density profile that gives this potential? What is the circular velocity?

Problem 4:

The velocity dispersion near the Milky Way's centre is approximately $75\,\mathrm{km\,s^{-1}}$, what is the mass of the black hole in the centre of our Galaxy? The velocity dispersion in the recently discovered ultra-compact dwarf, M60-UCD1, is $100\,\mathrm{km\,s^{-1}}$, what is its black hole mass?

The total mass of M60-UCD1 is estimated at 1.4×10^8 $\mathrm{M}_\odot$. As a fraction of their host galaxies' total mass, how much bigger is M60-UCD1's supermassive black hole compared to the Milky Way's?

Problem 5:

Using Gauss' theorem, derive the surface density for the Kuzmin disk potential at z=0

$$\Phi_K(R, z) = -\frac{GM}{\sqrt{R^2 + (a + |z|)^2}}$$