

Astrophysics III : Stellar and galactic dynamics

Solutions

Problem 1 :

The disk surface density can be expressed as a function of the total mass and radius of the disk :

$$\Sigma = \frac{f M_{\text{tot}}}{\pi R^2}. \quad (1)$$

Hence :

$$R = \sqrt{\frac{f \cdot M_{\text{tot}}}{\pi \Sigma}}. \quad (2)$$

With $f = 1/45$, $M_{\text{tot}} = 2 \cdot 10^{12} M_{\odot}$, we thus have : $R = 16.8 \text{ kpc}$.

The mean density is :

$$\langle \rho \rangle = \frac{f \cdot M_{\text{tot}}}{\pi R^2 \cdot 500} \sim 0.1 M_{\odot} \text{ pc}^{-3} \quad (3)$$

with a thickness of 500 pc, $R = 16800 \text{ pc}$ et $f = 1/45$.

The period of a circular orbit is :

$$T = \frac{2\pi R}{\sqrt{GM(R)/R}}, \quad (4)$$

So :

$$M = \frac{4\pi^2 R^3}{GT^2}. \quad (5)$$

With $G = 6.67 \cdot 10^{-11} \text{ m}^3/\text{kg}/\text{s}^2$, $R = 8 \text{ kpc} = 2.46 \cdot 10^{20} \text{ m}$ and $T = 220 \text{ Myr} = 6.9 \cdot 10^{15} \text{ s}$, we find approximately $10^{11} M_{\odot}$.

Problem 2 :

$$\begin{aligned} m_{\nu} c^2 &\leq 1 \text{ eV} = 1.6 \cdot 10^{-12} \text{ erg} \\ c &= 3 \cdot 10^{10} \text{ cm/s} \\ M_{\odot} &= 2 \cdot 10^{33} \text{ g} \\ M_{\text{halo}} &= 2 \cdot 10^{12} M_{\odot} \end{aligned}$$

$$N_{\nu} \geq \frac{M_{\text{halo}}}{m_{\nu}} = \frac{2 \cdot 10^{12} \cdot 2 \cdot 10^{33} \text{ g}}{1.6 \cdot 10^{-12} \text{ erg} / (3 \cdot 10^{10} \text{ cm/s})^2} \simeq 2 \times 10^{78}$$

Problem 3 :

For a galaxy cluster, the cluster typical radius is 1 Mpc and a typical galaxy size is 10 kpc :

$$\frac{\text{Volume (N galaxies)}}{\text{Volume (galaxy cluster)}} \simeq \frac{10^3 \cdot (10\text{kpc})^3}{(1000\text{kpc})^3} = 10^{-3}$$

For a galaxy :

$$\frac{\text{Volume (N stars)}}{\text{Volume (galaxy)}} \simeq \frac{10^{11} \cdot (10^6 \text{ km})^3}{(10^4 \text{ pc} \cdot 3.09 \cdot 10^{13} \text{ km/pc})^3} \simeq 4 \times 10^{-24}$$

We thus see that those dynamical systems are largely made of void.

Problem 4 :

In a galaxy cluster, the typical speed of galaxies is $v = 10^3 \text{ km/s} \cdot (3.09 \cdot 10^{16} \text{ km/kpc})^{-1} = 3.09 \cdot 10^{-14} \text{ kpc/s}$, hence :

$$\frac{\text{Volume (tube)}}{\text{Volume (galaxy cluster)}} \simeq \frac{\pi(10 \text{ kpc})^2 \cdot v \cdot t}{\frac{4\pi}{3}(1000\text{kpc})^3} \simeq 7.7 \cdot 10^{-4}$$

Within a galaxy, stars typically have $v = 200 \text{ km/s}$, hence :

$$\frac{\text{Volume (tube)}}{\text{Volume (galaxy)}} \simeq \frac{\pi(10^6 \text{ km})^2 \cdot v \cdot t}{\frac{4\pi}{3}(10^4 \text{ pc} \cdot 3 \cdot 10^{13} \text{ km/pc})^3} \simeq 10^{-20}$$

Given the tiny portion of the volume crossed by the components, we conclude that the probability of a collision between two components of a given dynamical system is extremely low.

Problem 5 :

We consider that the gravitational influence of an object is significant when the mutual potential energy is of the same order than the kinematic energy of the relative motion.

$$E_{\text{cin}} \simeq E_{\text{grav}} \quad \Leftrightarrow \quad \frac{1}{2}mv^2 \simeq \frac{Gm^2}{R_G} \quad \Leftrightarrow \quad R_G \simeq \frac{2Gm}{v^2}$$

We observe that this result gives the value of $b_{90} = 2Gm/v^2$ seen in the course, so that $R_G = b_{90}$.

For a galaxy moving within a galaxy cluster :

$$R_G \simeq \frac{2 \cdot 6.67 \cdot 10^{-11} \text{ N m}^2/\text{kg}^2 \cdot 10^{11} \cdot 2 \cdot 10^{30} \text{ kg}}{(10^6 \text{ m/s})^2} \simeq 10^{19} \text{ m} \simeq 1 \text{ kpc}$$

For a star moving within a galaxy :

$$R_G \simeq \frac{2 \cdot 6.67 \cdot 10^{-11} \text{ N m}^2/\text{kg}^2 \cdot 2 \cdot 10^{30} \text{ kg}}{(2 \cdot 10^5 \text{ m/s})^2} \simeq 0.7 \cdot 10^{10} \text{ m} \simeq 0.05 \text{ A.U.}$$