N-body Project

Step 0: Plot density profile

The given data was created following the profile given in Hernquist 1989;

$$\rho(r) = \frac{M_{tot}}{2\pi} \frac{a}{r} \frac{1}{(r+a)^3} \qquad M = M_{tot}$$

$$M(R) - M(R_0) = 4\pi \int_{R_0}^{R} \rho(r) r^2 dr$$

$$= -M a \left[\frac{2r+a}{(r+a)^2} \right]_{R_0}^{R}$$

$$\Rightarrow M(r) = M \frac{r^2}{(r+a)^2} \quad (\text{for } R_0 = 0,\ R = r)$$

What my program does:

- read in data
- Compute profiles:
  - Compute $r = \sqrt{x^2+y^2+z^2}$, as spherical symmetry is assumed distance from origin
  - Bin the particles by distance using logarithmic bin: $x_i = x_0 \left(\frac{x_n}{x_0}\right)^{i/n}$
    n = # of bins in total; I used 200

- Plot density profile mass profile, cumulative mass profile: both binned and from theory
  - Determine a: $M(a) = \frac{M}{4}$
  - density_profile (x_i) = $\frac{\text{mass\_profile}(x_i)}{\frac{4}{3}\pi(x_i^3 - x_{i-1}^3)}$
  - mass_profile: Find through binning of data
  - cmp: sum up mass profile starting from x_0

Add error bars to histograms:

$\varrho_{err}(x_i) = \sqrt{\varrho(x_i)}$ (Poissonian error)

tweaked for low values, when $\varrho_{err} \geq \varrho$ [for $\varrho \leq 1$], because otherwise, the error bars wouldn't plot on log scales.

Used log scales so you get something recognizible

Norm:

- Used $r_{max} \stackrel{!}{=} 1$
- Used $M_{tot} = 1000'000$ (for $M_{tot} = 1$, the error bars escalated)

Scaling a quantity: $a = A_0 \, a'$

$$a = \frac{Gm}{r^2} \; ; \qquad a' = \frac{G'm'}{r'^2} \; ;$$

$$\Rightarrow \; a = \frac{R_0}{T_0^2} a' = \frac{R_0}{T_0^2} \frac{G'm'}{r'^2} = \frac{R_0}{T_0^2} \frac{G'm}{\cancel{M_0} \, r^2} \cdot \frac{R_0^2}{M_0} = \frac{Gm}{r^2}$$

$$\Rightarrow \; G' = \frac{G \, T_0^2 M_0}{R_0^3} \overset{!}{=} 1$$

Choose M_0, R_0 according to your system:

$$R_0 = R_{max}$$

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$$\Rightarrow \; T_0 = \sqrt{\frac{R_0^3}{G M_0}}$$

$$\frac{1}{r^2}\partial_r\left(r^2\partial_r\varphi\right) = 4\pi G\rho(r)$$

$$r^2\partial_r\varphi = 4\pi G\int\rho(r)r^2dr = M(r)$$

$$\partial_r\varphi = \frac{GM(r)}{r^2}$$

$$\varphi(r) = \int\frac{GM(r)}{r^2}dr = \int\frac{GM}{r^2}\frac{r^2}{(r+a)^2}dr$$

$$= GM\int\frac{1}{(r+a)^2}dr =$$

$$\Big| \int x^n = \frac{1}{n+1}x^{n+1}$$

$$= -MG\left[\frac{1}{(r+a)}\right] + \varphi_0$$

Chose $\varphi(r\to\infty) = 0 \Rightarrow \varphi_0 = \left(\frac{MG}{r+a}\right) \to 0$

$$\Rightarrow \boxed{\varphi(r) = \left(\frac{-MG}{r+a}\right)}$$

$$F = -\nabla\varphi = -\frac{\partial\varphi}{\partial r} = \frac{MG}{(r+a)^2}$$

$$\overline{F}[a,b] = \frac{1}{b-a}\int_a^b F(x)\,dx$$

$$F(r) = \frac{MGm}{(r+a)^2}$$

$$\Rightarrow \overline{F}(r_1, r_2) = \frac{1}{r_2 - r_1}\int_{r_1}^{r_2} \frac{MGm}{(r+a)^2}\,dr$$

$$= -\frac{MGm}{r_2 - r_2}\,\frac{1}{(r+a)}\bigg|_{r_1}^{r_2}$$

$$= -\frac{GMm}{r_2 - r_1}\left[\frac{1}{(r_2+a)} - \frac{1}{(r_1+a)}\right]$$