Ideal Gases

For all ideal gasses:

$$pV = Nk_B T = nRT$$

$$C_p - C_V = nR = \frac{pV}{T}$$

Thermally perfect gas:

$$e = e(T) \,, \quad C_V = C_V(T) \,, \quad C_p = C_p(T)$$

gas is in thermodynamic equilibrium, not chemically reacting

Calorically perfect gas:

$$C_V = \text{const}, \quad C_p = \text{const}, \quad e = C_V T$$

Specific internal energy of calorically ideal gas

$$e = C_v T$$

For C_v: Use 1st law of thermodynamics

$$C_v \equiv \left.\frac{dq}{dT}\right|_\rho$$

$$du = dq + dw \qquad \text{specific values: } U = u \cdot m, \; q = \frac{Q}{m}$$

$$dq = du - dw = du + p\,dv \qquad v = \frac{V}{m} = \frac{1}{\rho}; \; dv = -\frac{1}{\rho^2} d\rho$$

$$= du - \frac{1}{\rho^2} p\,d\rho$$

$$u = \frac{N}{m}\frac{f}{2} k_B T; \qquad du = \frac{Nf}{m2} k_B\,dT$$

$$\Rightarrow dq = \left.\frac{\partial q}{\partial T}\right|_\rho dT + \left.\frac{\partial q}{\partial \rho}\right|_T d\rho = C_v\,dT + \left.\frac{\partial q}{\partial \rho}\right|_T d\rho$$

$$= \frac{Nf}{2m} k_B\,dT - \frac{1}{\rho^2} p\,d\rho$$

$$\Rightarrow C_v = \frac{Nf}{2m} k_B$$

Using $pV = N k_B T \rightarrow pv = \frac{N}{m} k_B T = \frac{p}{\rho}$

$$\rightarrow \frac{N}{m} k_B = \frac{p}{\rho T}$$

Using $\gamma = \frac{f+2}{f} = 1 + \frac{2}{f} \rightarrow \frac{2}{f} = \gamma - 1$

$$\frac{f}{2} = \frac{1}{\gamma - 1}$$

$$\Rightarrow C_v = \frac{Nf}{2m} k_B = \frac{p}{\rho T}\frac{1}{\gamma - 1} = \frac{p}{\rho(\gamma - 1)}\frac{1}{T}$$

$$\Rightarrow e = C_v T = \frac{p}{\rho(\gamma - 1)}$$