

Simple Estimate of a Signal-to-Noise ratio

Let S_0 and S_s be the object and sky photon counts per pixel per unit time.

For an observing time t on an aperture with n_{pix} pixels, the numbers of photons are

$$n_0 = S_0 t n_{\text{pix}} \quad \text{for object}$$

$$n_s = S_s t n_{\text{pix}} \quad \text{for the sky}$$

Assuming that the sky has poissonian noise, we have the expression for noise:

$$N_{\text{sky}} = \sqrt{n_s} = \sqrt{S_s t n_{\text{pix}}}$$

Then the signal-to-noise ratio is given by

$$\frac{S}{N} = \frac{n_0}{N_{\text{sky}}} = \frac{S_0 t n_{\text{pix}}}{\sqrt{S_s t n_{\text{pix}}}} = \frac{S_0}{\sqrt{S_s}} \sqrt{t n_{\text{pix}}}$$

$$\Rightarrow \frac{S}{N} \propto \sqrt{t}$$