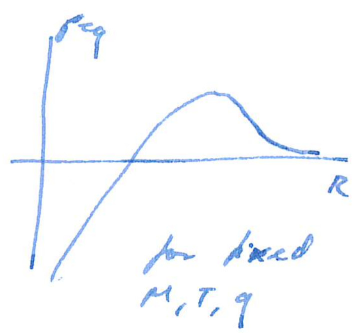
The Jeans Criterion

Consider an isothermal sphere of mass M, radius R and temperature T, surrounded by ambient pressure P_{eq} with which the interstellar cloud is in equilibrium.

Virial theorem:

$$2E_{kin} + \Omega = 3PV = 4\pi R^3 P_{eq} = 2C_V MT - q\frac{GM^2}{R}$$

$$\Rightarrow P_{eq} = \frac{C_V MT}{2\pi R^3} - q\frac{GM^2}{4\pi R^4}$$



P_{eq} has a maximum at $R_{max} = R_J$:

$$\frac{dP_{eq}}{dR} = 0 = -\frac{3C_V MT}{2\pi R_{max}^4} + \frac{q\,GM^2}{\pi R_{max}^5}$$

$$\Rightarrow R_{max} = \frac{q\,GM}{6C_V MT} \qquad \Big| \; C_V = \frac{3}{2}\frac{k}{\mu m_u}$$

$$= \frac{4}{9}\frac{\mu m_u}{k}\frac{GM}{T}$$

R_{max} is the radius of the maximum pressure that the cloud in equilibrium can sustain.

If $R < R_j$: The sustainable pressure, $P_{eq}(R)$, becomes smaller than the actual present pressure, which we assume is unchanged. The cloud starts contracting giving smaller R, and thus collapsing.

For $\rho \approx$ const, $M_{jeans} = \frac{4}{3}\pi R_j^3 \rho$

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$$= \frac{4}{3}\pi \left(\frac{4}{9} g \frac{\mu m_u}{k} \frac{M_{jeans}}{T} \right)^3 \rho$$

$$\Rightarrow M_{jeans}^2 = \frac{3}{4\pi} \frac{9^3}{4^3} \left(\frac{k}{\mu m_u} \right)^3 g^{-3} \frac{T^3}{\rho}$$

$$\Rightarrow M_{jeans} = \frac{27}{16} \left(\frac{3}{\pi g^3} \right)^{1/2} \left(\frac{k}{\mu m_a} \right)^{3/2} \frac{T^{3/2}}{\rho^{1/2}}$$