Stellar Stuff

Order of Magnitude estimates

Based on hydrostatic equilibrium:

$$\frac{1}{\rho}\frac{\partial P}{\partial r} = -\frac{GM_r}{r^2}$$

Use index c for center, s for surface

Pressure:

$$M_r \approx \frac{M_s + M_c}{2} = \frac{M_s}{2}$$

$$r \approx \frac{R_s + R_c}{2} = \frac{R_s}{2}$$

$$\Rightarrow \frac{1}{\bar{\rho}}\frac{P_s - P_c}{R_s - R_c} = -\frac{G\,M_s/2}{(R_s/2)^2} = -2\frac{GM_s}{R_s^2} =$$

$$= -\frac{P_c}{R_s}\frac{1}{\bar{\rho}} = -\frac{P_c}{R_s}\frac{1}{M_s/4/3\,\pi R_s^3}$$

$$\Rightarrow \boxed{P_c \sim \frac{3}{2\pi}\frac{GM_s^2}{R_s^4}}$$

Temperature:

$$PV = NkT \Rightarrow \quad P = \frac{N}{V} kT = nkT = \frac{\rho}{\mu m_u} kT$$

$$\Rightarrow T = \frac{\mu m_u}{k\rho} P$$

use $\bar{P} \sim \frac{1}{2} P_c$ and $\bar{\rho} = M / 4/3 \pi R^3$

$$\bar{T} \sim \frac{\mu m_u}{k\bar{\rho}} \bar{P} = \frac{\mu m_u}{k} \frac{1}{2} \frac{3}{2\pi} \frac{GM^2}{R^5} \cdot \frac{4/3 \pi R^3}{M}$$

$$= \frac{\mu m_u}{k} \frac{GM}{R}$$