UNIVERSITÉ DE GENÈVE

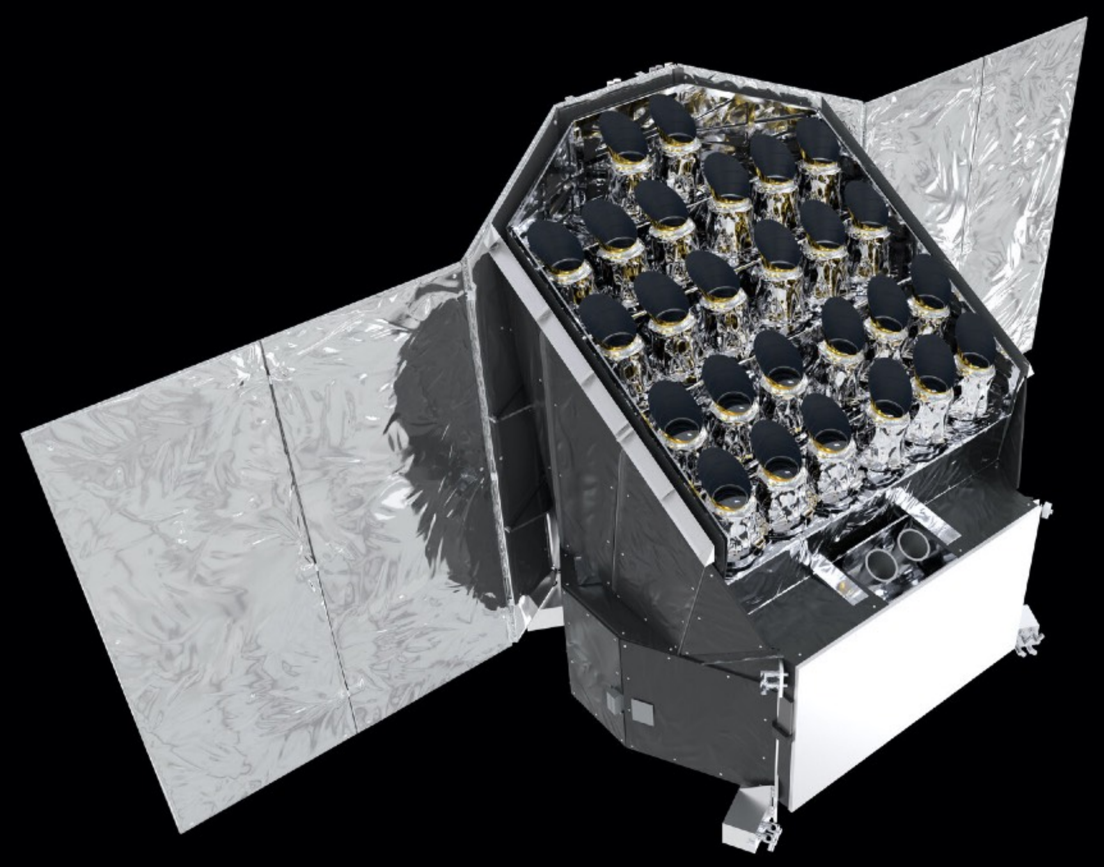
esa plato

PlanetS

Use of dynamical knowledge to improve follow-up efficiency

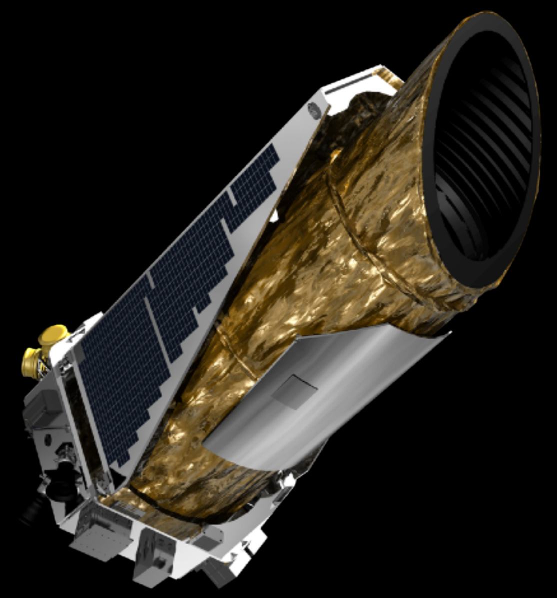
PLATO GOP Workshop 2022 - Adrien Leleu

Cases



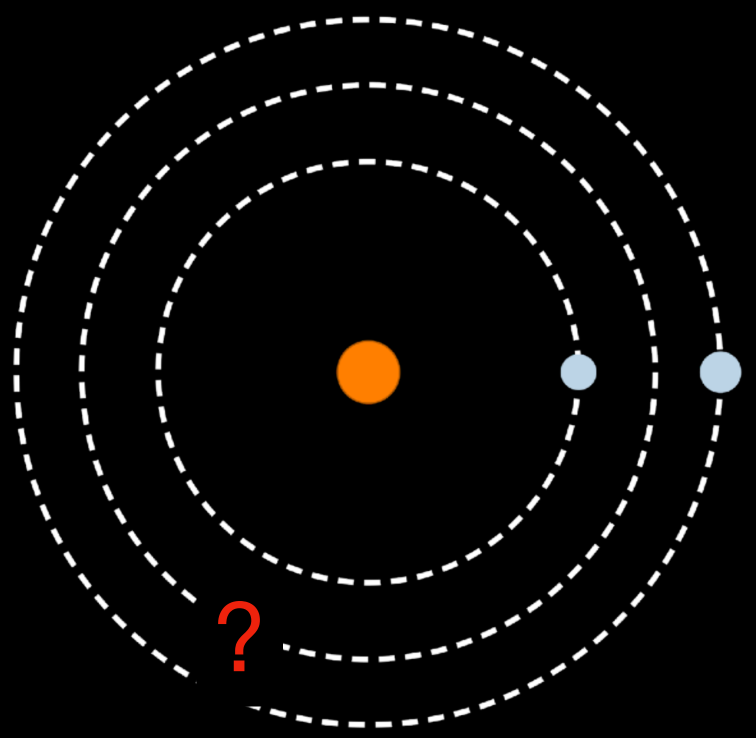
Continuous observations
example: Kepler

Step and stare
example: TESS

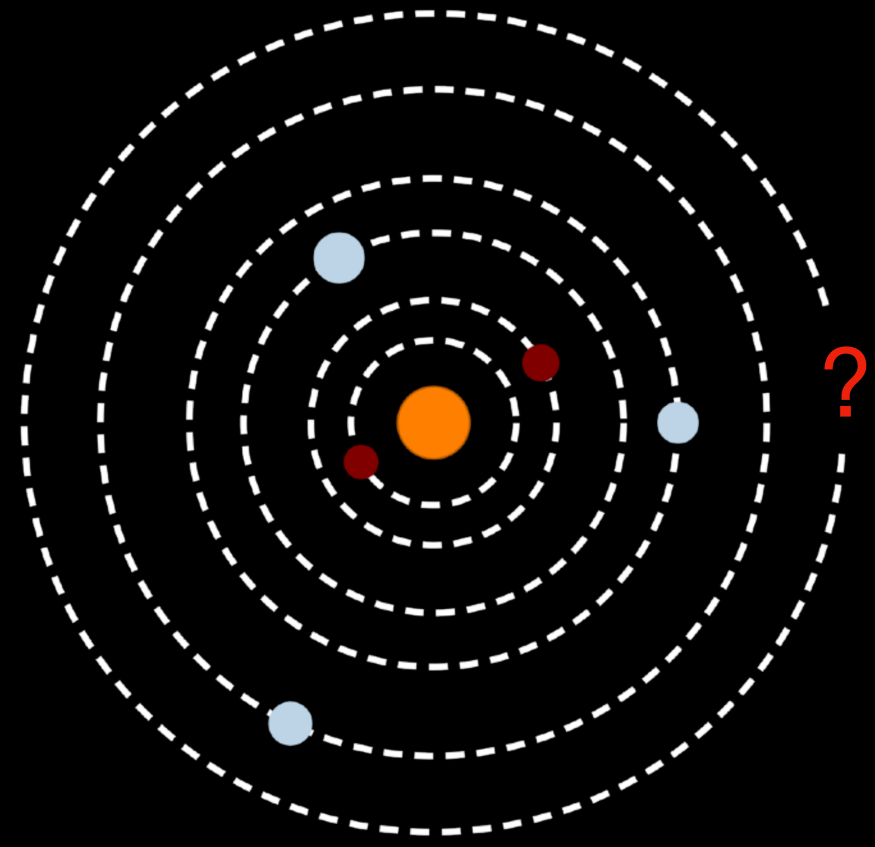


Only periods and t0s necessary,
can be pushed to hundreds of days

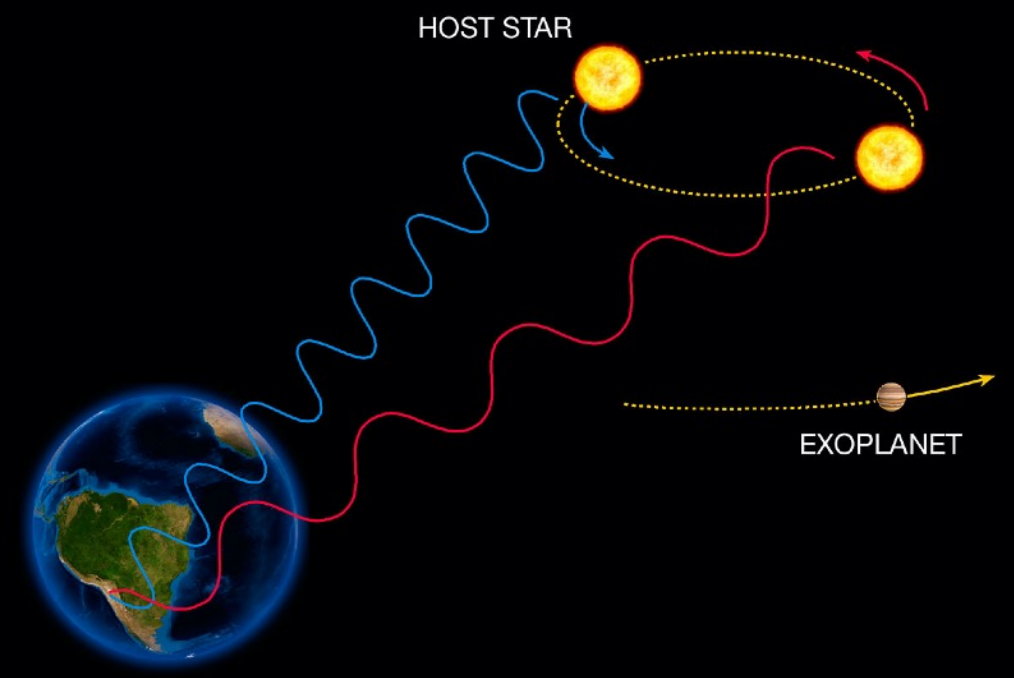
Veryfing a
potential resonant chain



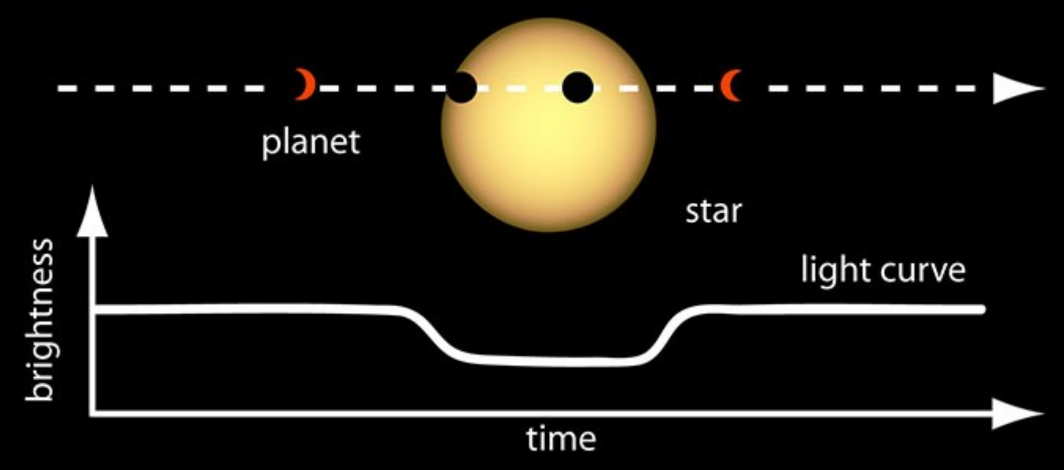
Completing a
resonant system



Radial velocity follow-up



Photometry follow-up



Can take place at any period: HR 8799

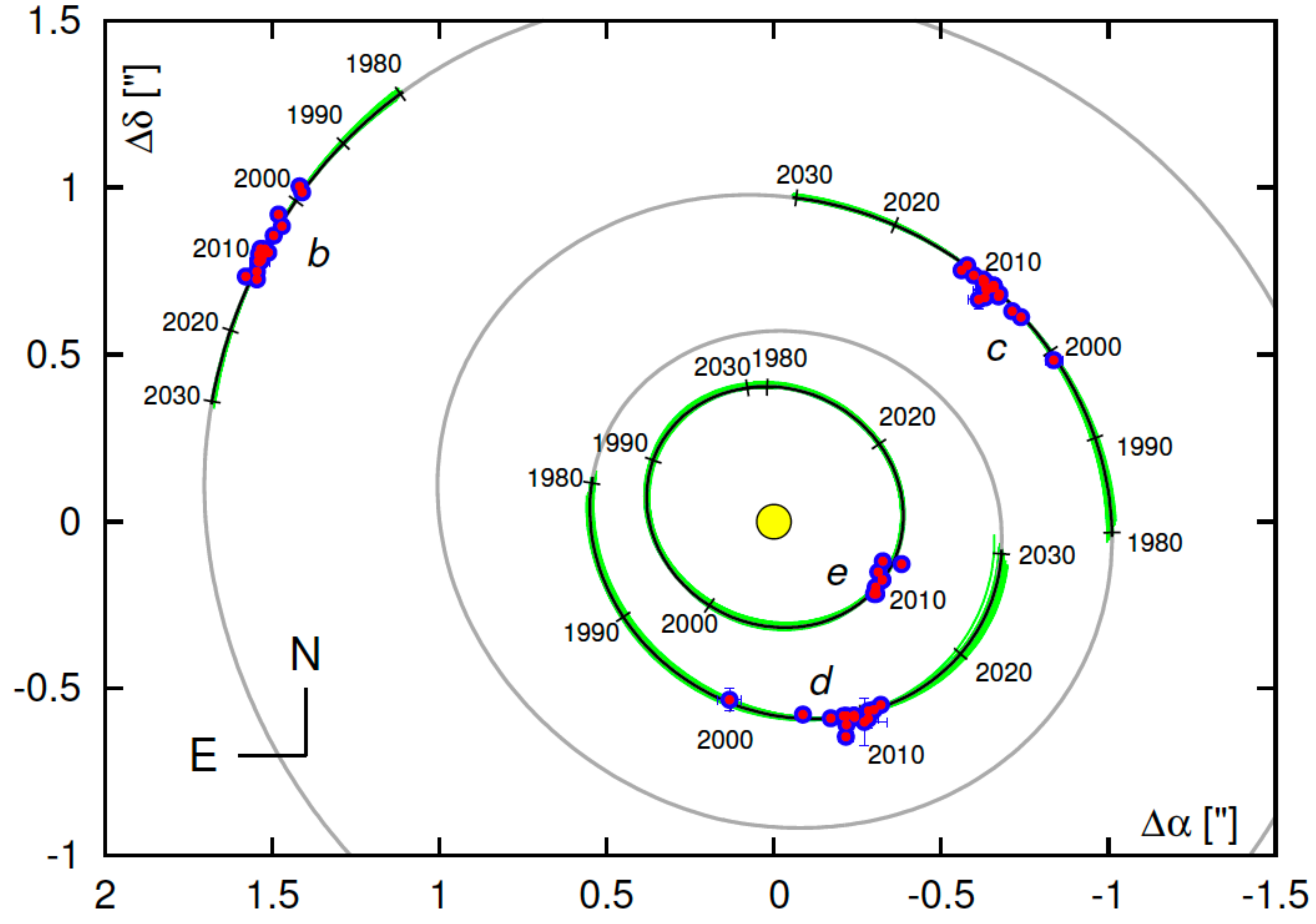
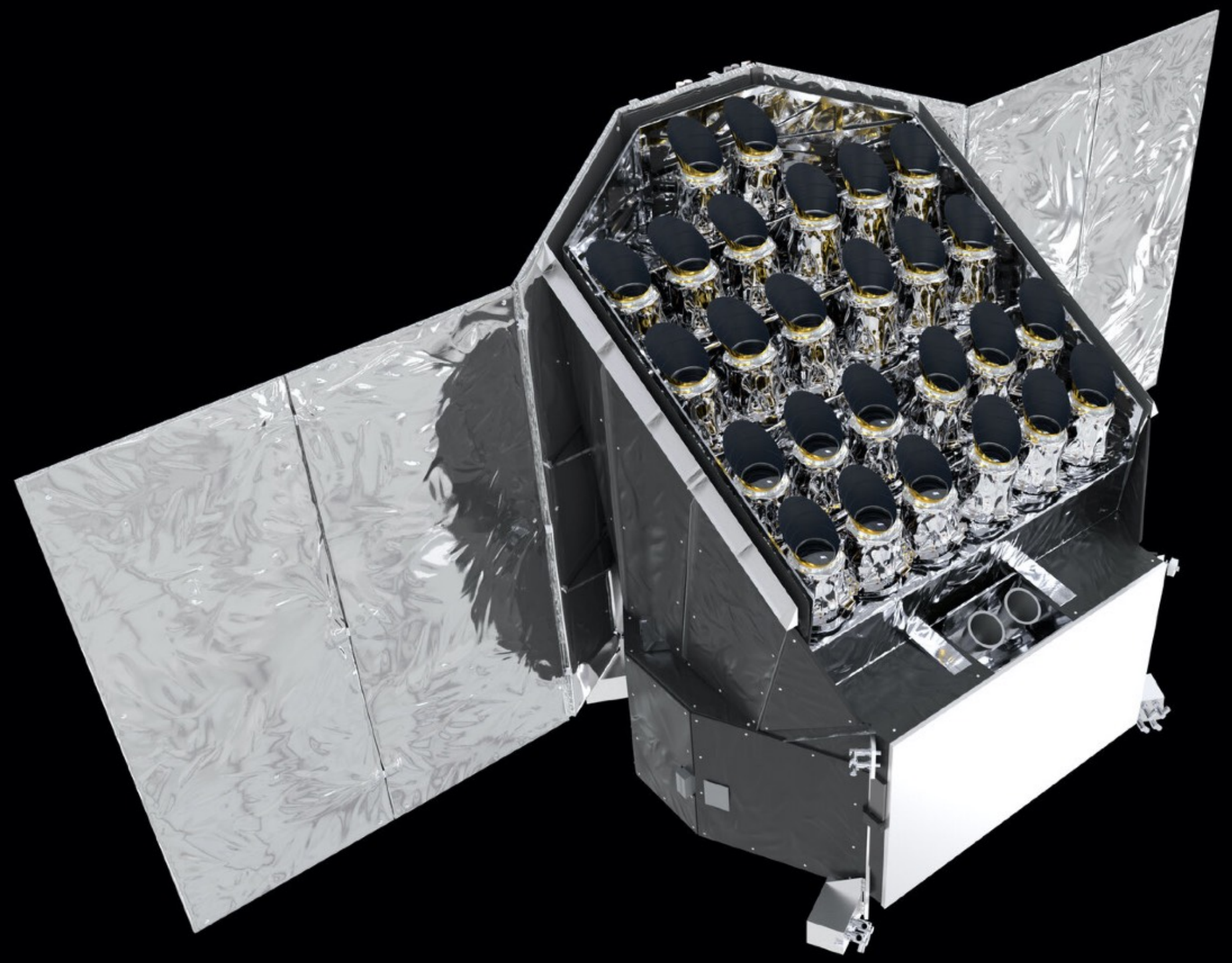


Figure 4. Relative astrometric positions of the planets (red filled circles), orbital arcs for the best-fitting model IVa (black curves), and stable solutions within the 3σ confidence level of the best-fitting model (green curves).

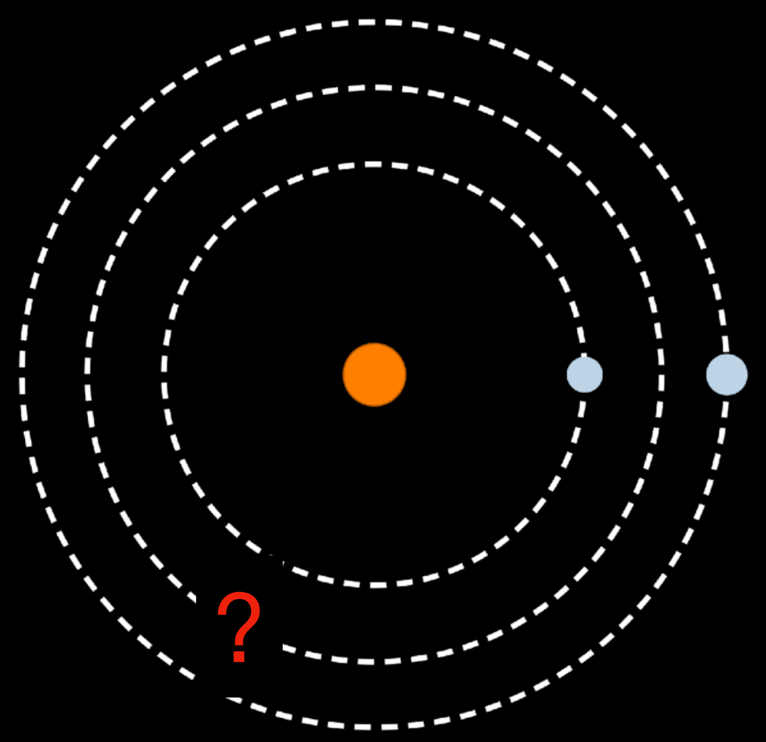
	$m\,[\mathrm{m_{Jup}}]$	$a\,[\mathrm{au}]$
HR 8799 e	9 ± 2	15.4 ± 0.2
HR 8799 d	9 ± 3	25.4 ± 0.3
HR 8799 c	9 ± 3	39.4 ± 0.3
HR 8799 b	7 ± 2	69.1 ± 0.2

Gozdziewski and Migaszewski (2014)

Case 1

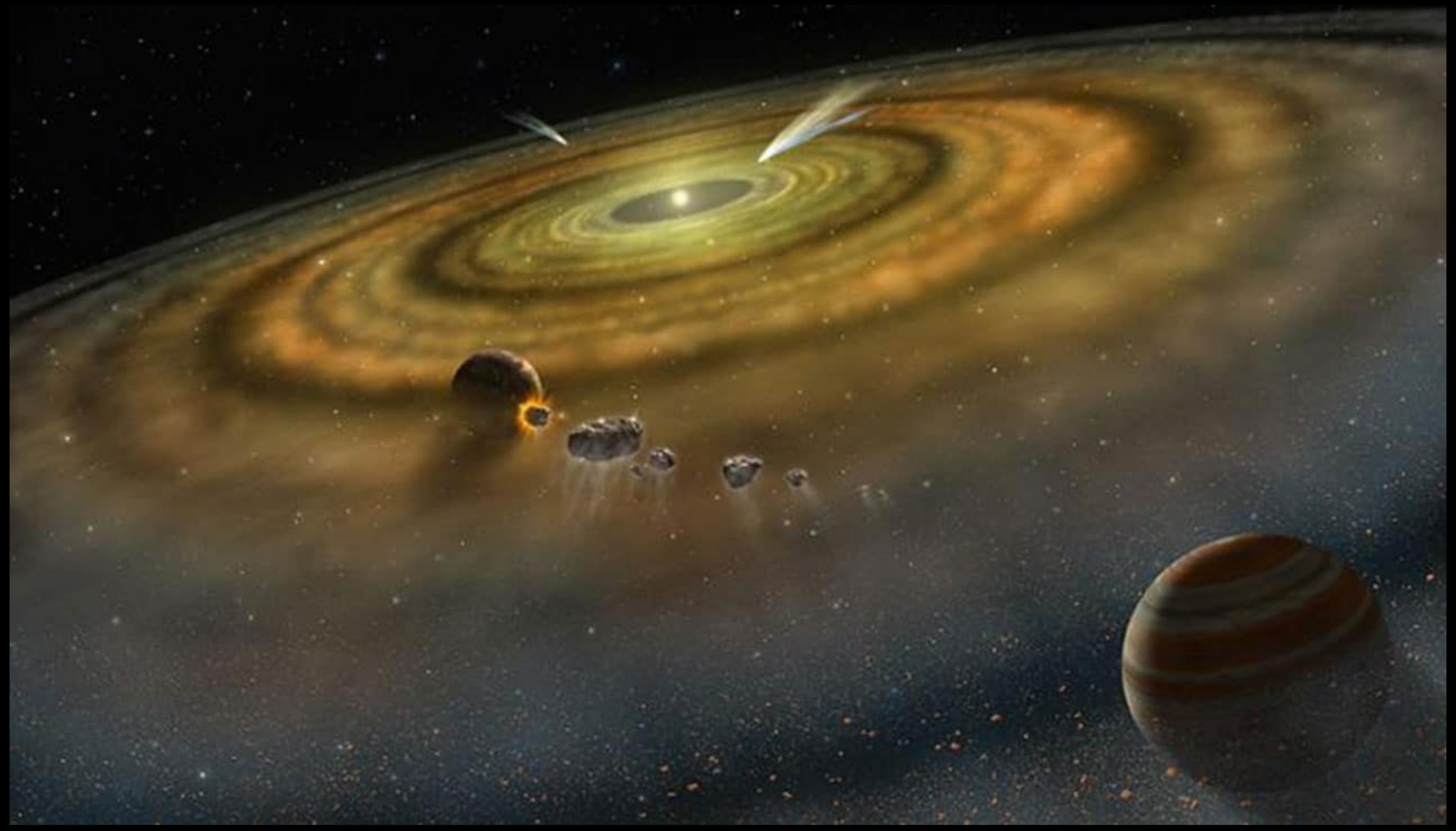


veryfing a
potential resonant chain



Mean motion resonances (MMRs)

Byproduct of the formation of planetary systems



Two planets MMRs (first order):

$P_2/P_1 \approx (k+1)/k$ = 2, 3/2, 4/3, etc. k integer

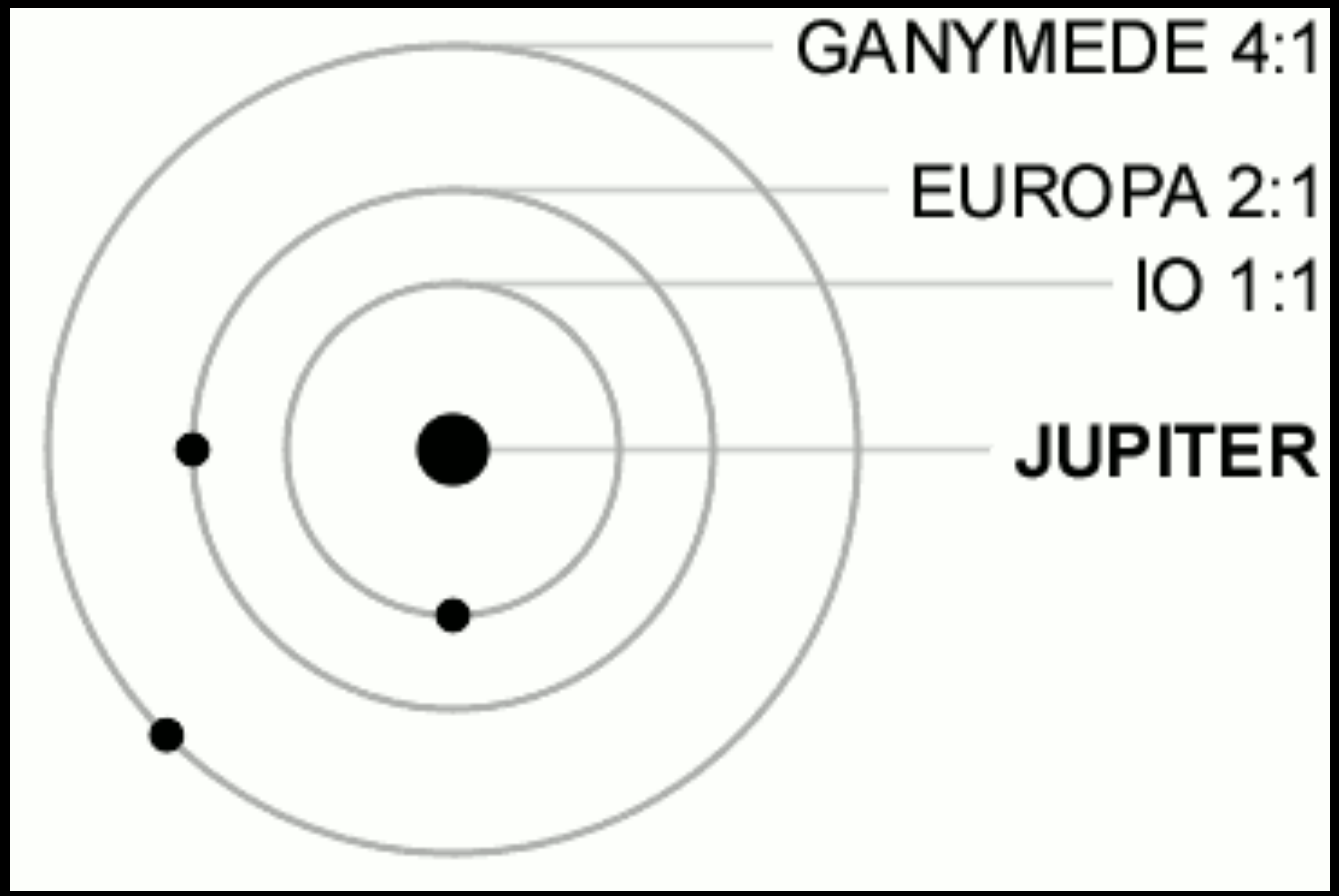
Pluto - Neptune :

247.94 yr / 164.8 yr = 1.504 $\approx 3/2$

Mean motion resonances (MMRs)

(Generalised) Laplace resonance :

$$l/P_1 - (l+m)/P_2 + m/P_3 \approx 0$$ Laplace relation



Io - Europa - Ganymede :

3/(3.52 hour) -1/(1.76 hour) - 2/(7.15 hour) $\approx$ 0

Chain of 2-planet MMRs:

$$k_1/P_1 - (k_1+1)/P_2 \approx 0$$

$$k_2/P_2 - (k_2+1)/P_3 \approx 0$$

$$\Longrightarrow k_1/P_1 - (k_1+k_2+1)/P_2 + (K_2+1)/P_3 \approx 0$$

Laplace relation

Completion of a Laplace chain

First order MMRs are most commons:

$$P_2/P_1 \approx (k+1)/k$$

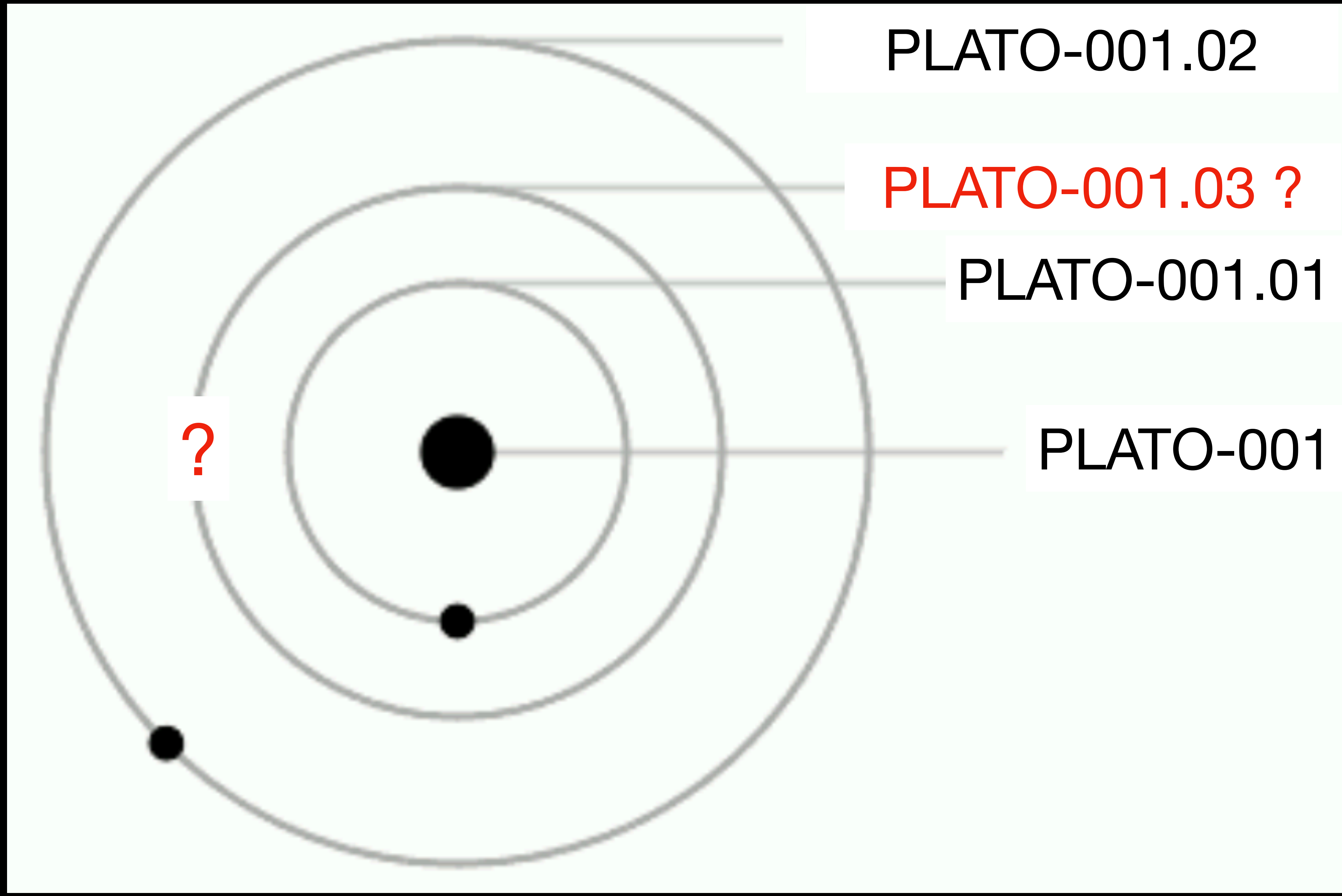
Second order MMRs are more rare:

$$P_2/P_1 \approx (k+2)/k$$

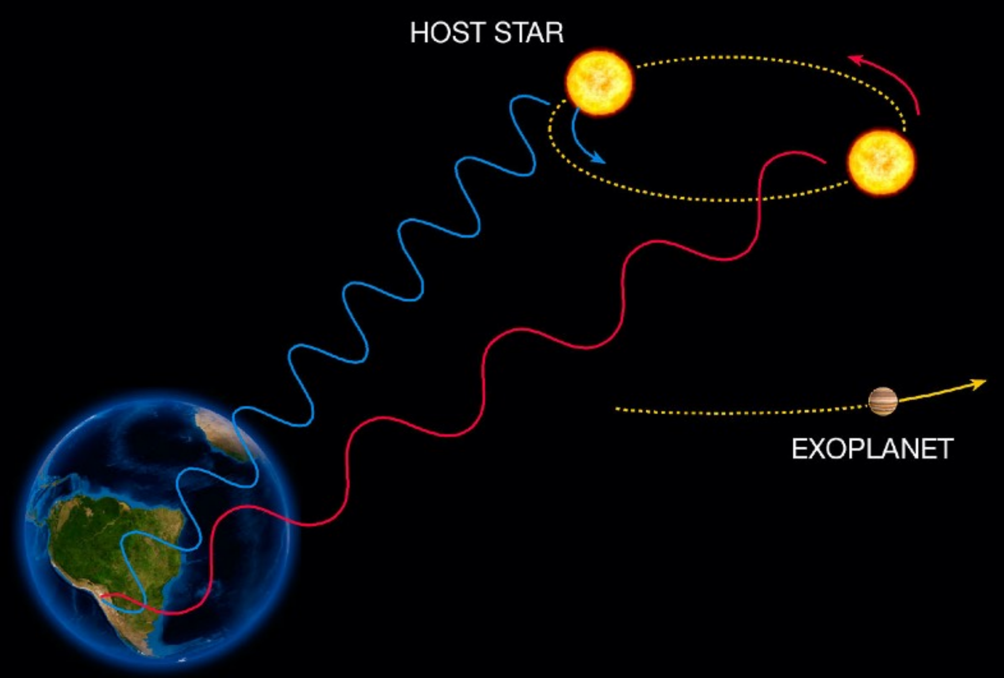
Higher order MMRs $P_2/P_1 \approx (k+q)/k, \quad q \geq 3$ ex. 9/4, 4/1, 5/2

are very weak at low eccentricites.

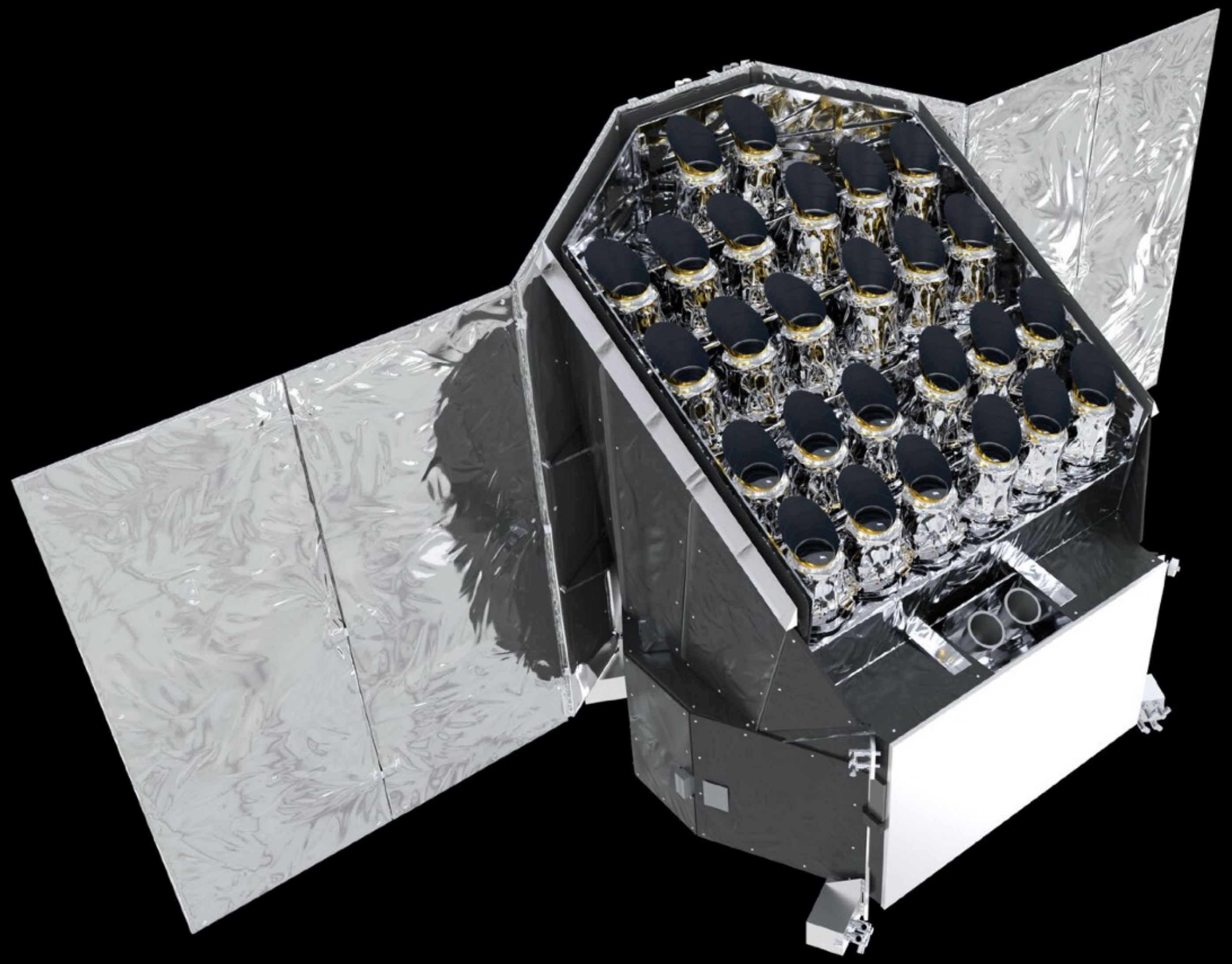
Example, if $P_2/P_1 \approx 4.004$ if might hide and intermediate planet…



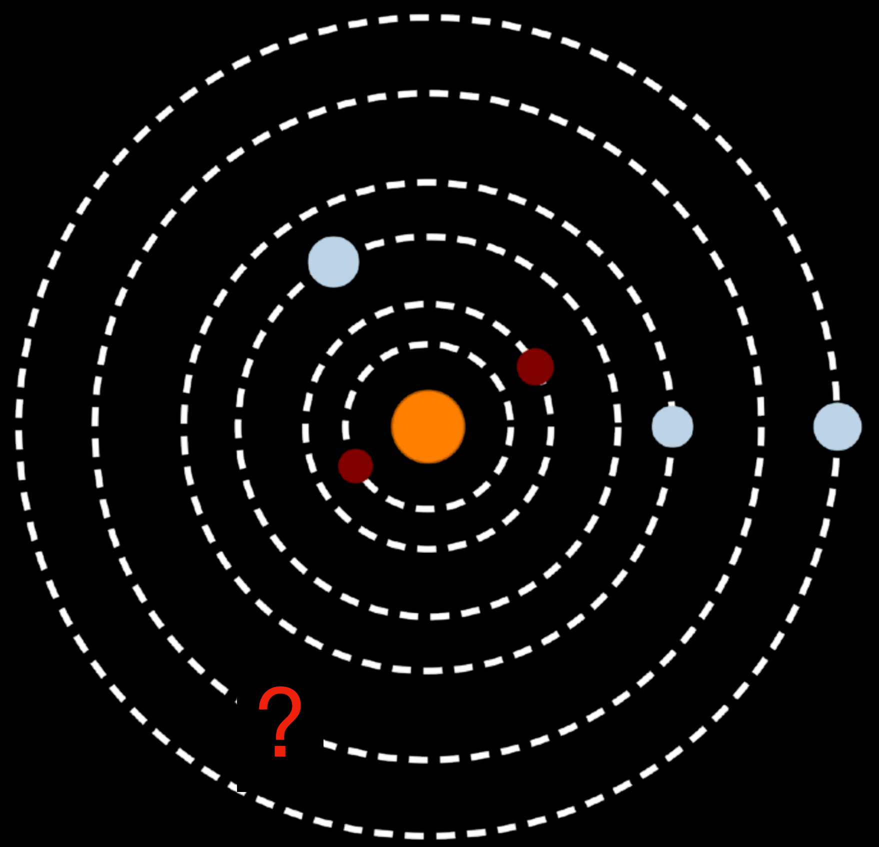
Radial velocity follow-up



Case 2.1



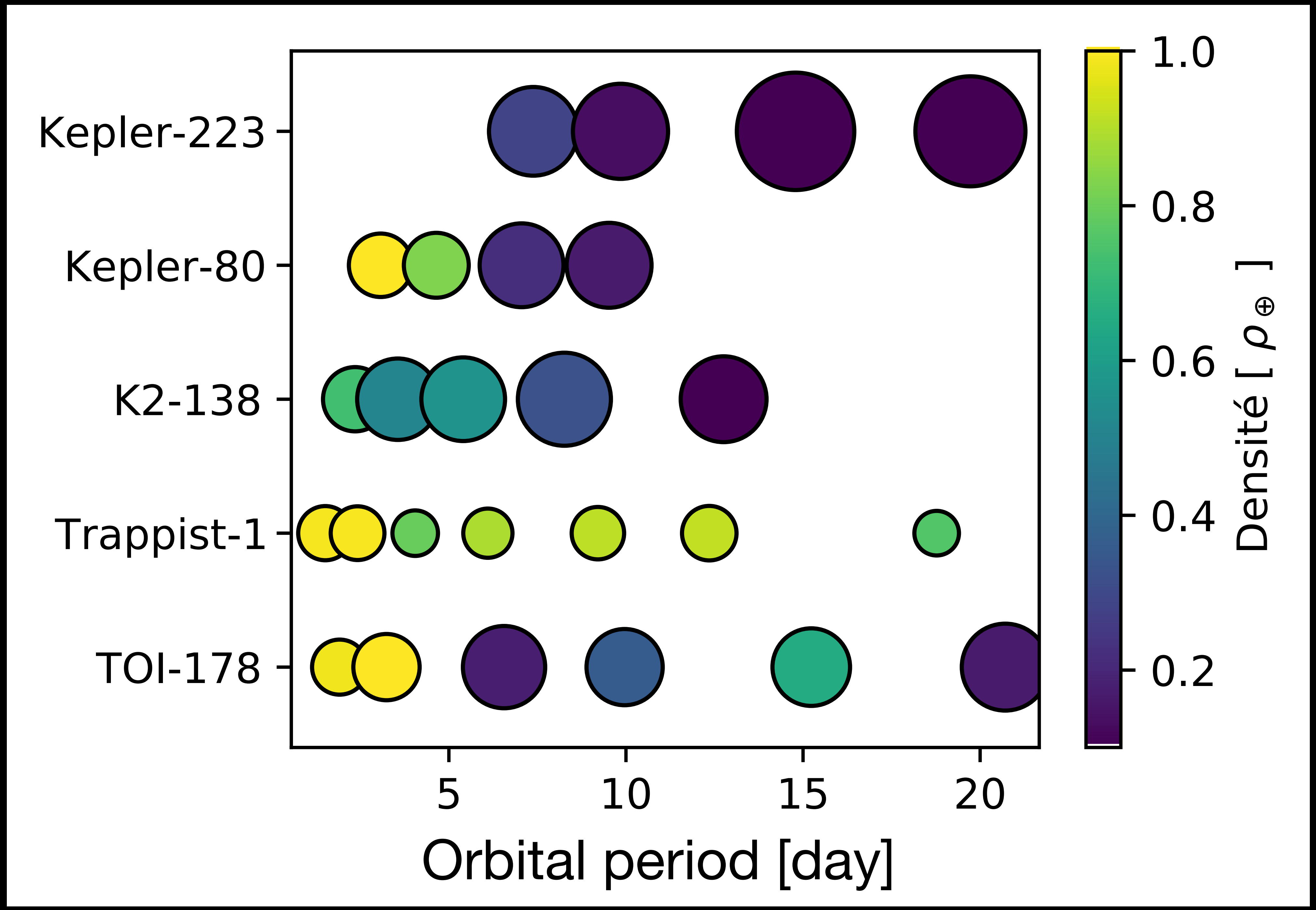
Completing a
resonant system



Planet inside the chain,
single transit

Chains of (generalised) Laplace resonances

4+ planets

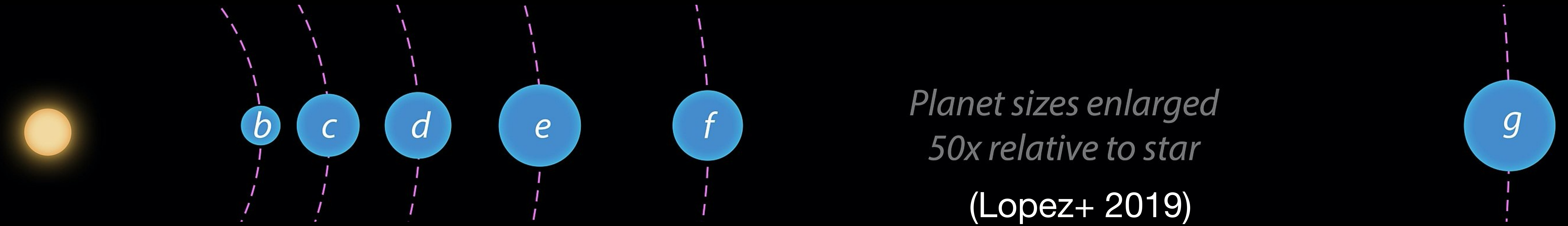


Use of the resonant chain to predict the period of a planet

Trappist-1 : prediction of Trappist-1 h (Luger+ 2017)
TOI-178 : prediction of TOI-178 f (Leleu+ 2021)

TOI-1136 this morning (Dai+ 2022) !!

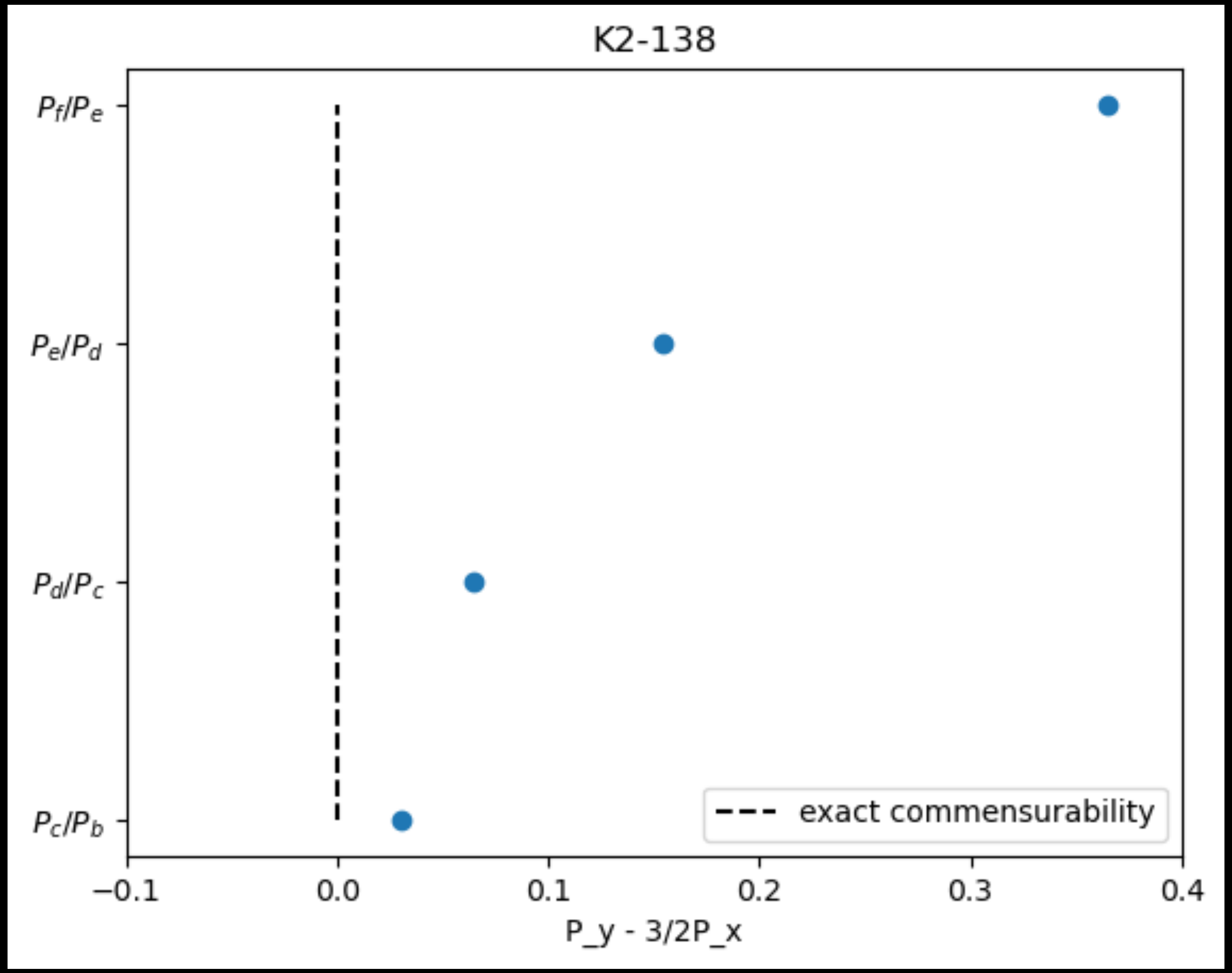
K2-138 distance to 2-body MMRs



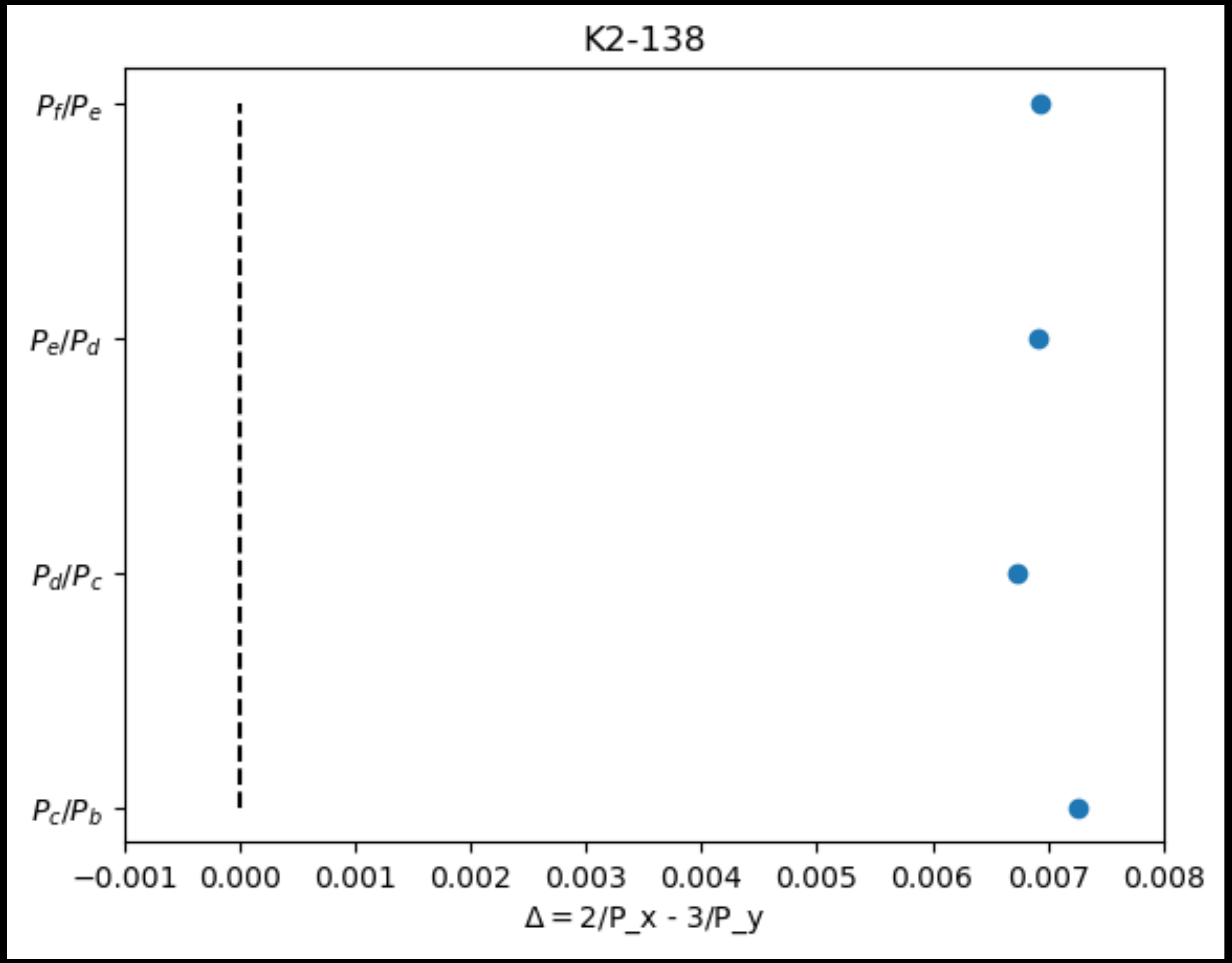
(Lopez+ 2019)

$\frac{P_c}{P_b} \approx 3/2$ $\approx 3/2$ $\approx 3/2$ $\approx 3/2$

Distance in period

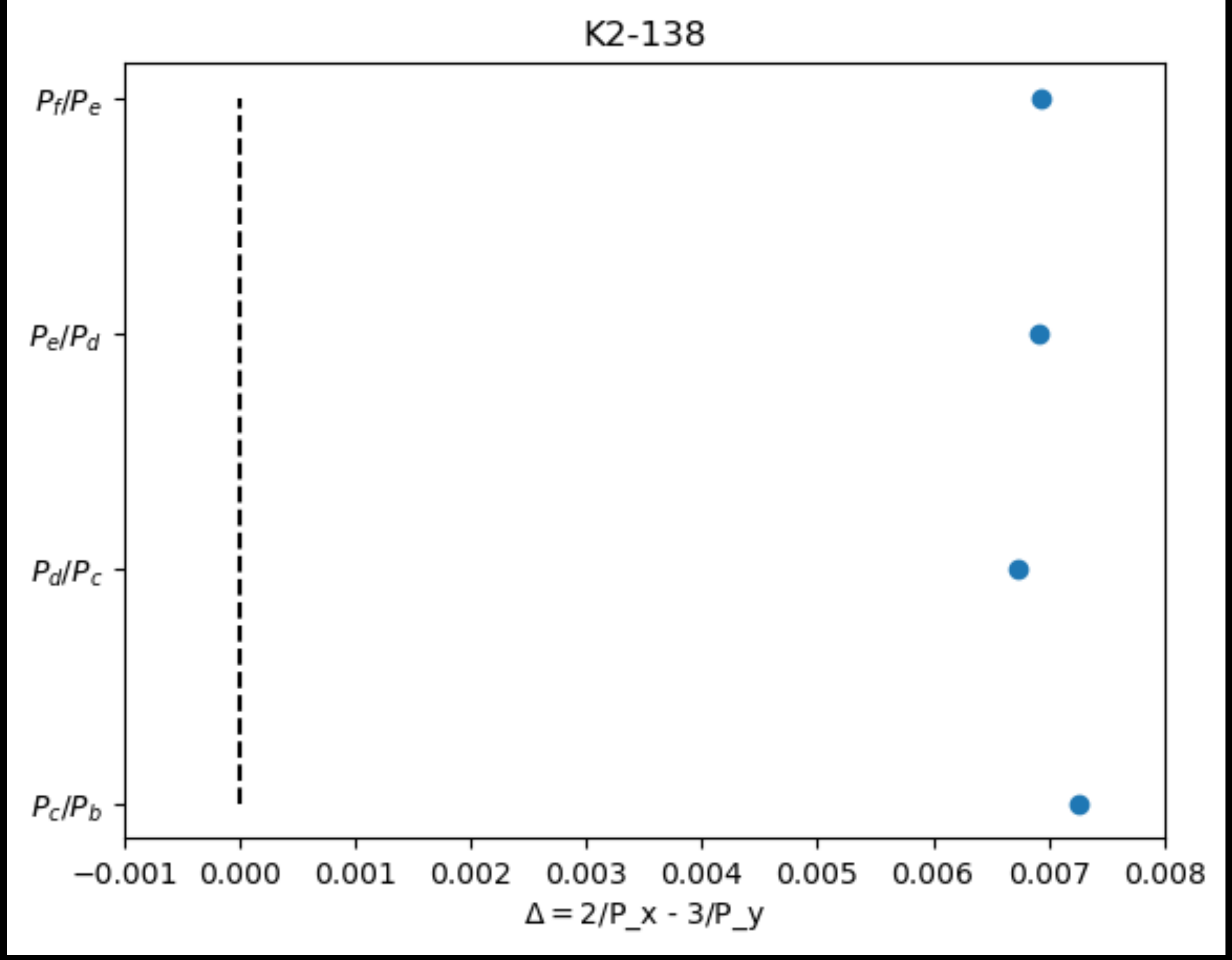


Distance in frequency



Laplace resonances

Distance in Frequency



Distance to the exact commensurability in frequency

$$k_1/P_1 - (k_1+1)/P_2 = \Delta_1$$

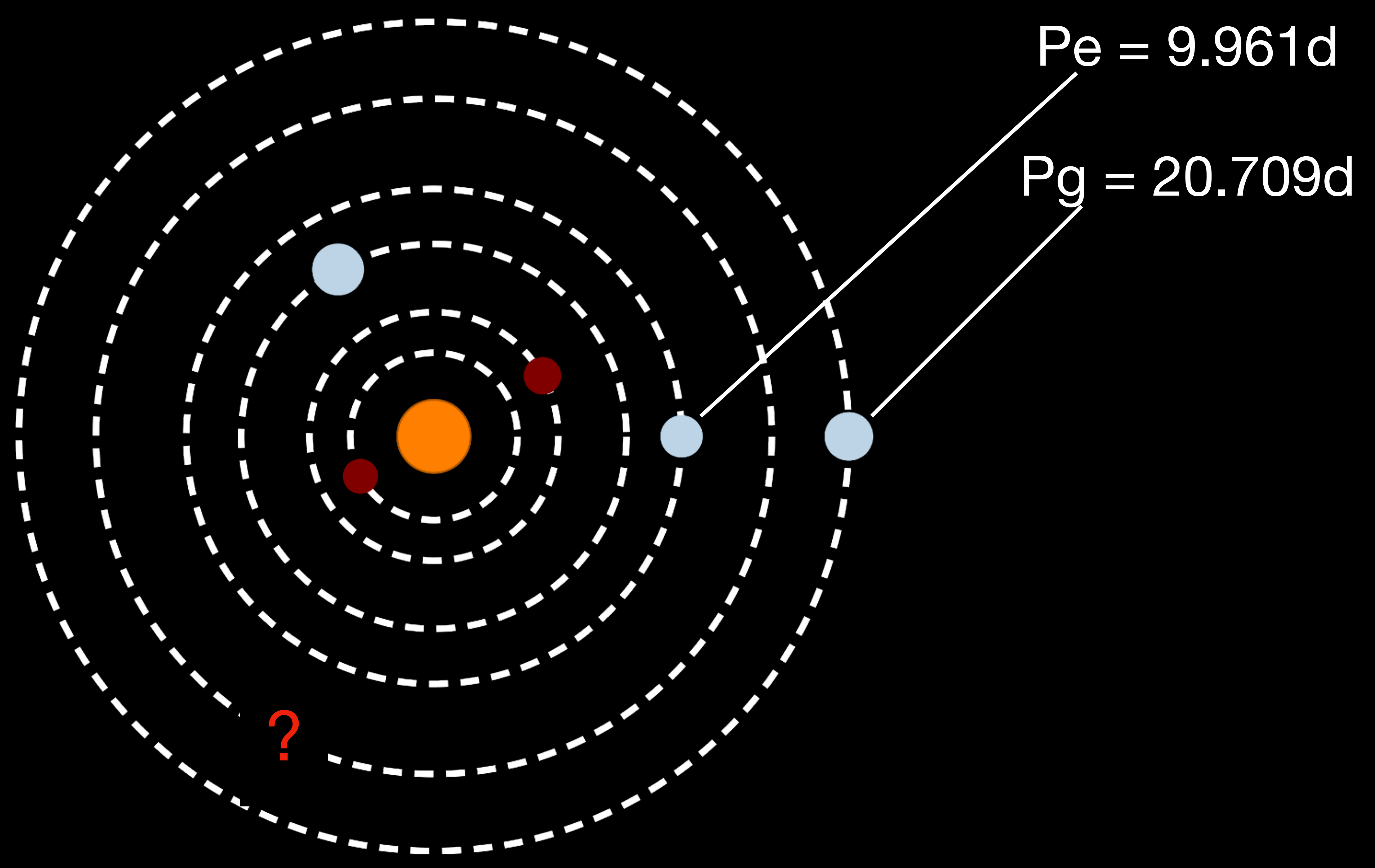
$$k_2/P_2 - (k_2+1)/P_3 = \Delta_2$$

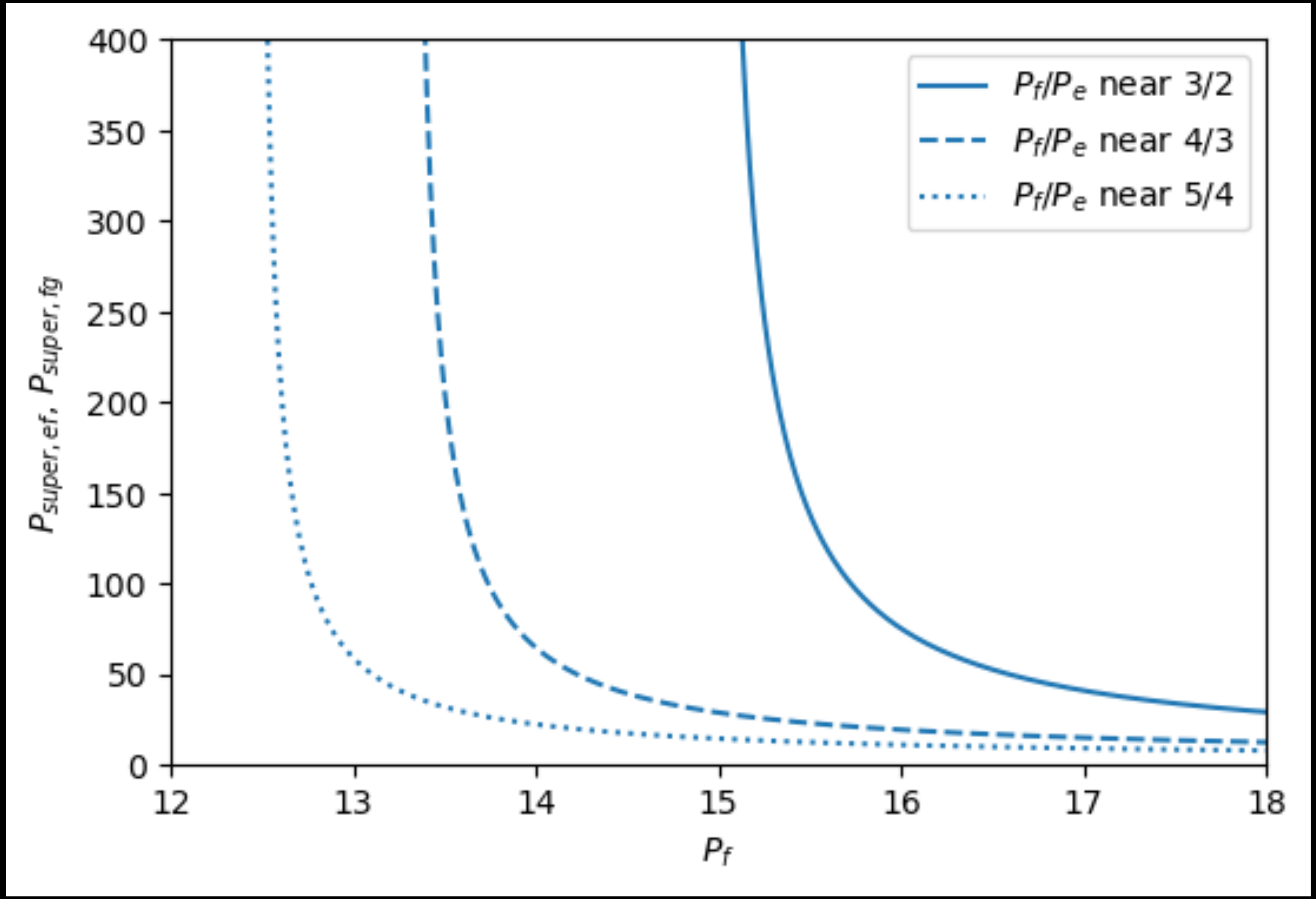
$$\Delta_1 = \Delta_2 \implies k_1/P_1 - (k_1+k_2+1)/P_2 + (K_2+1)/P_3 = 0$$ Laplace relation

Associated super period: $$P_{Super,12} = \frac{1}{\Delta_1} = \frac{1}{k_1/P_1 - (k_1+1)/P_2}$$ (Lithwick+ 2012)

Have to be the same for all pairs in the chain

prediction of the period of TOI-178f





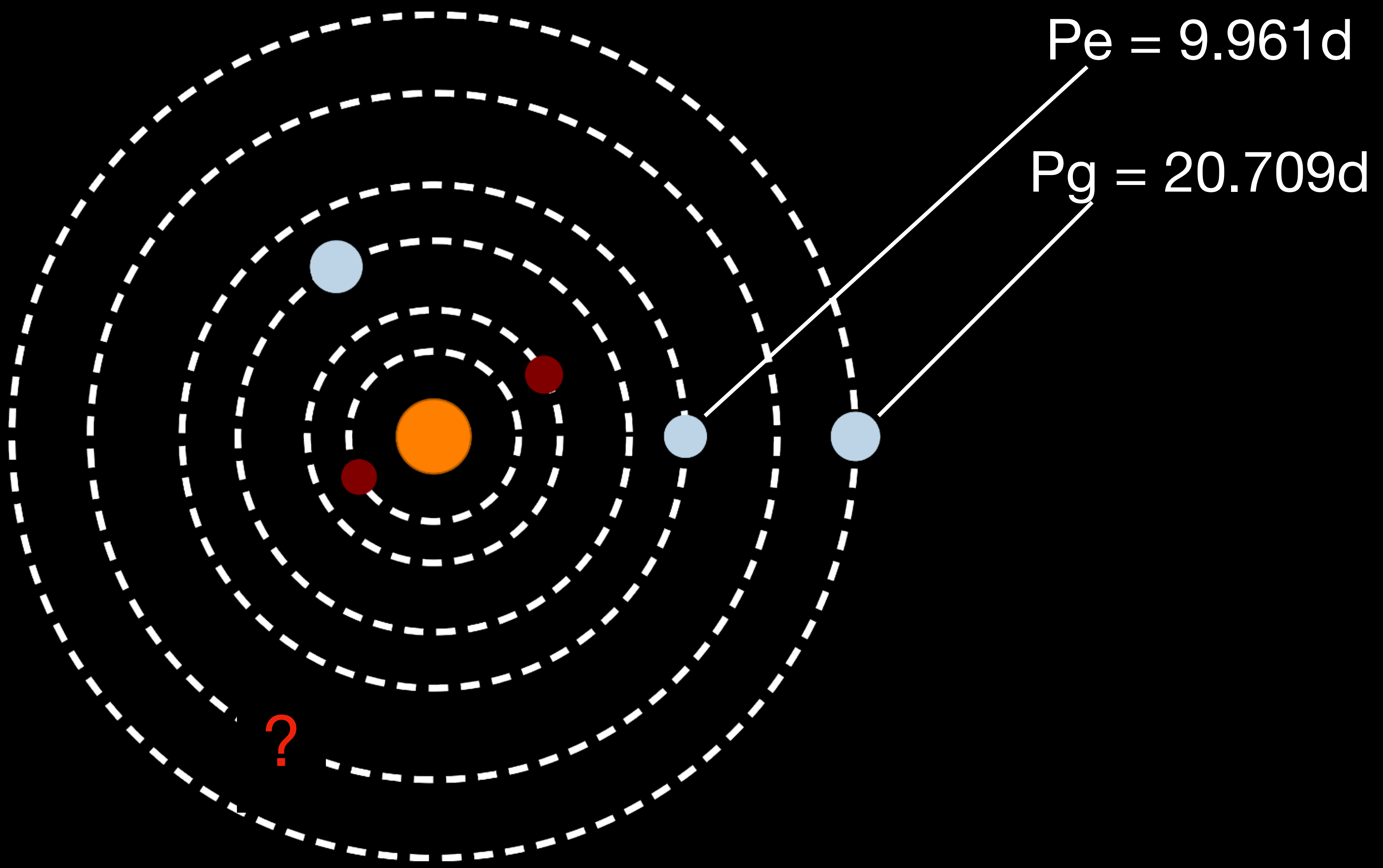
Laplace relation if:

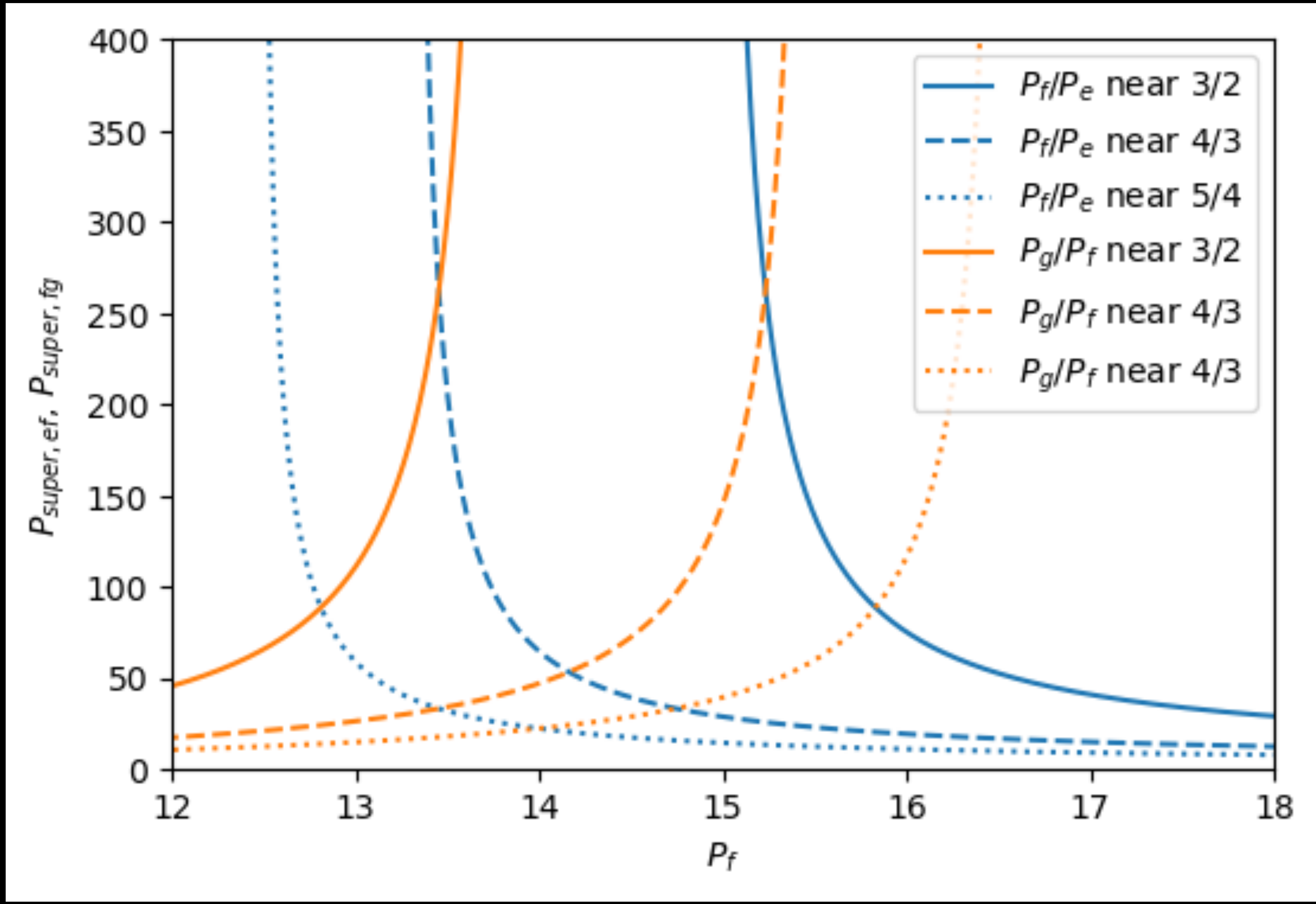
$$\Delta_1 = \Delta_2$$

$$P_{Super,ef} = P_{Super,fg}$$

$$P_{Super,12} = \frac{1}{\Delta_1} = \frac{1}{k_1/P_1 - (k_1+1)/P_2}$$

prediction of the period of TOI-178f





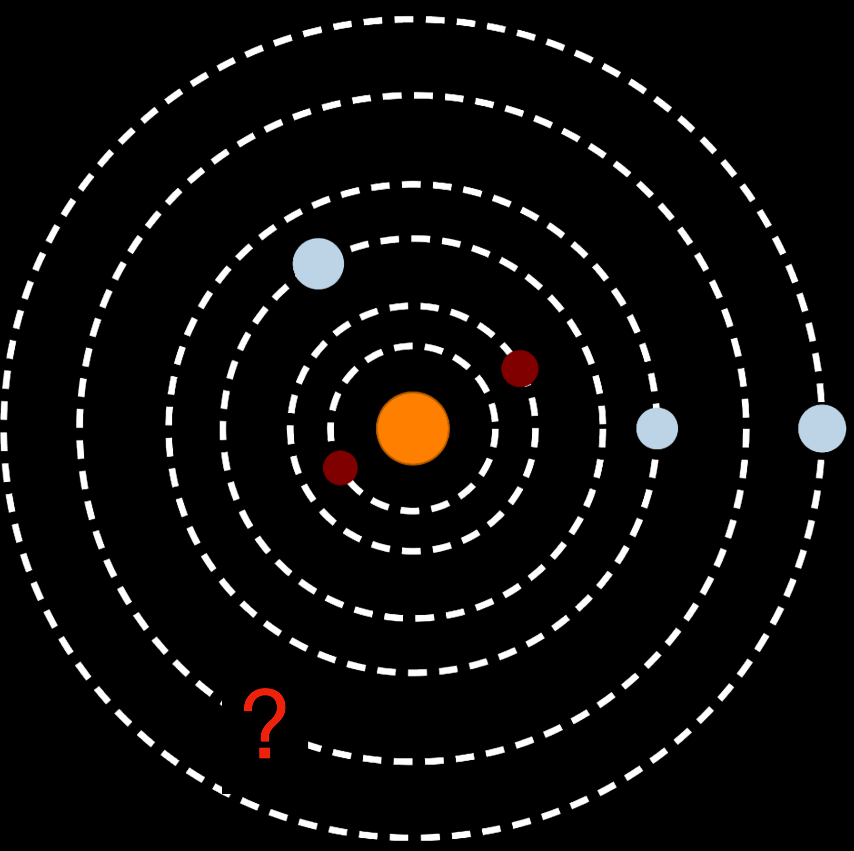
Laplace relation if:

$$\Delta_1 = \Delta_2$$

$$P_{Super,ef} = P_{Super,fg}$$

$$P_{Super,12} = \frac{1}{\Delta_1} = \frac{1}{k_1/P_1 - (k_1+1)/P_2}$$

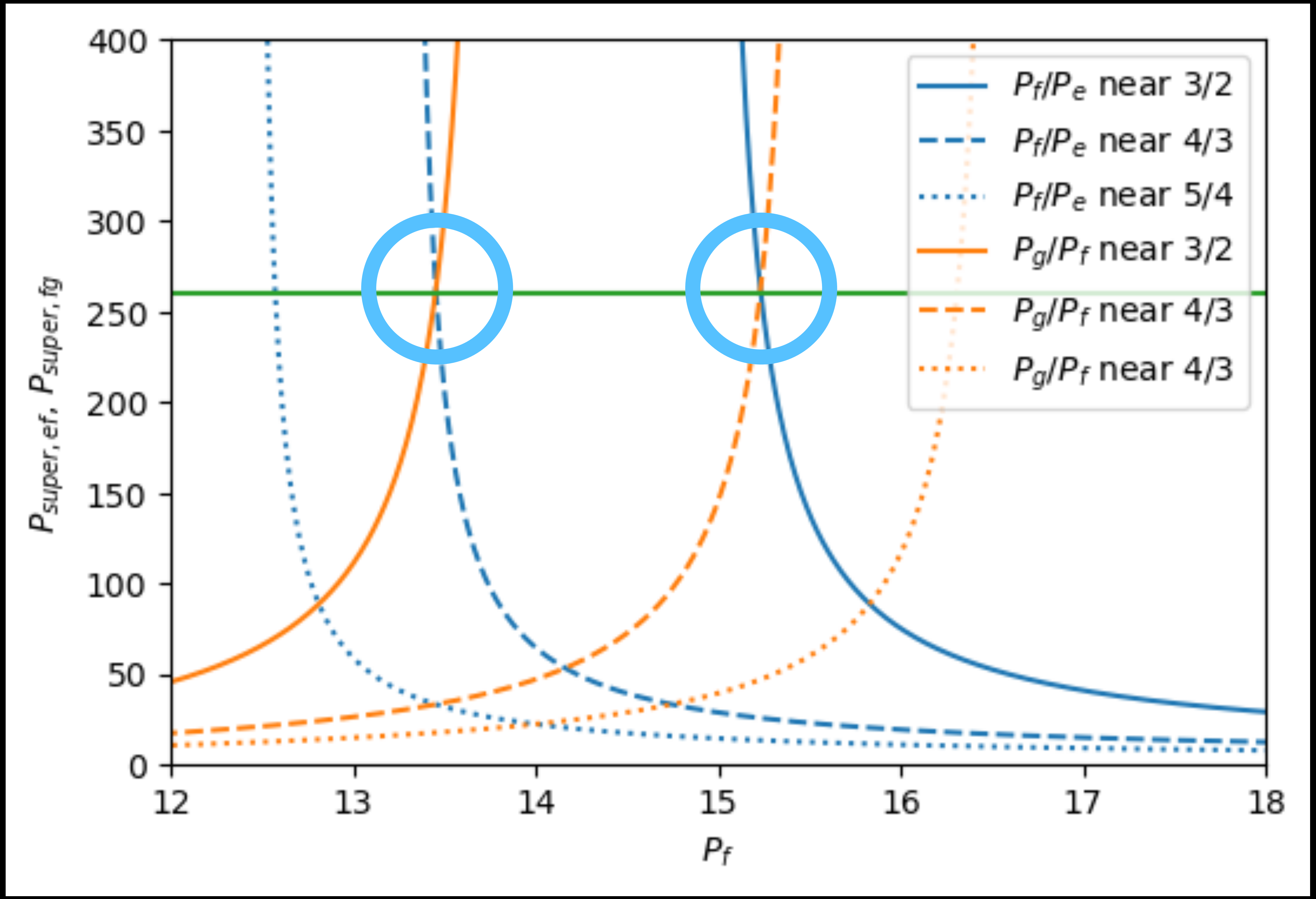
prediction of the period of TOI-178f



$$P_{Super,cd} = 263d$$

$$P_{Super,de} = 260.5d$$

$$P_{Super,eg} = 262.6d$$



Laplace relation if:

$$\Delta_1 = \Delta_2$$

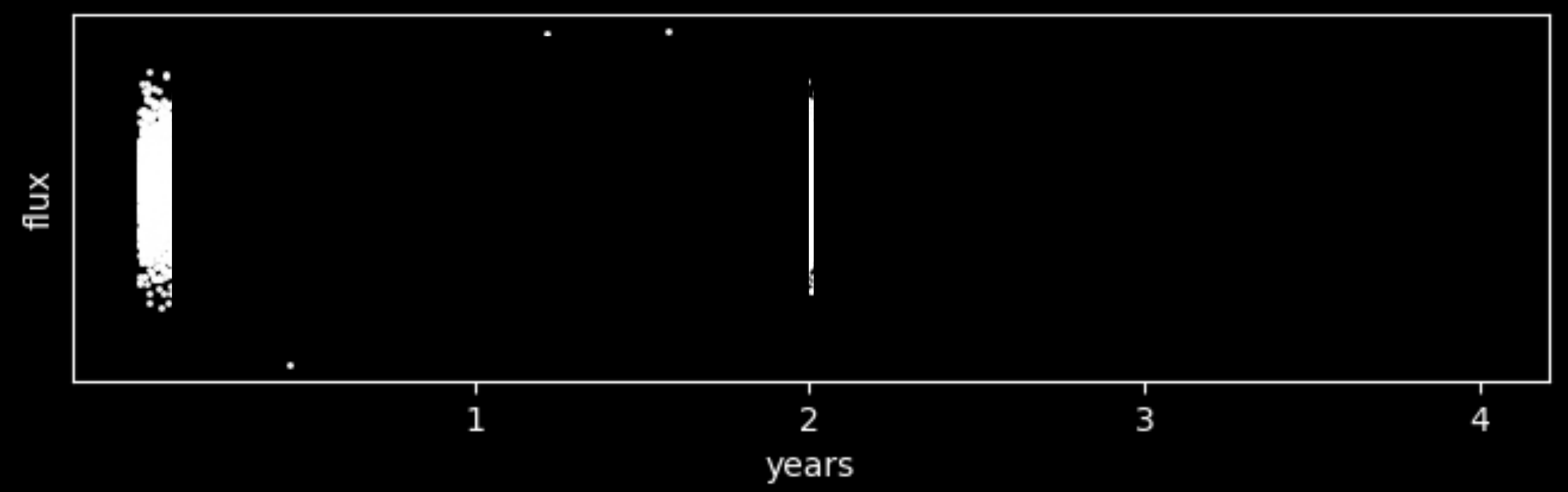
$$P_{Super,ef} = P_{Super,fg}$$

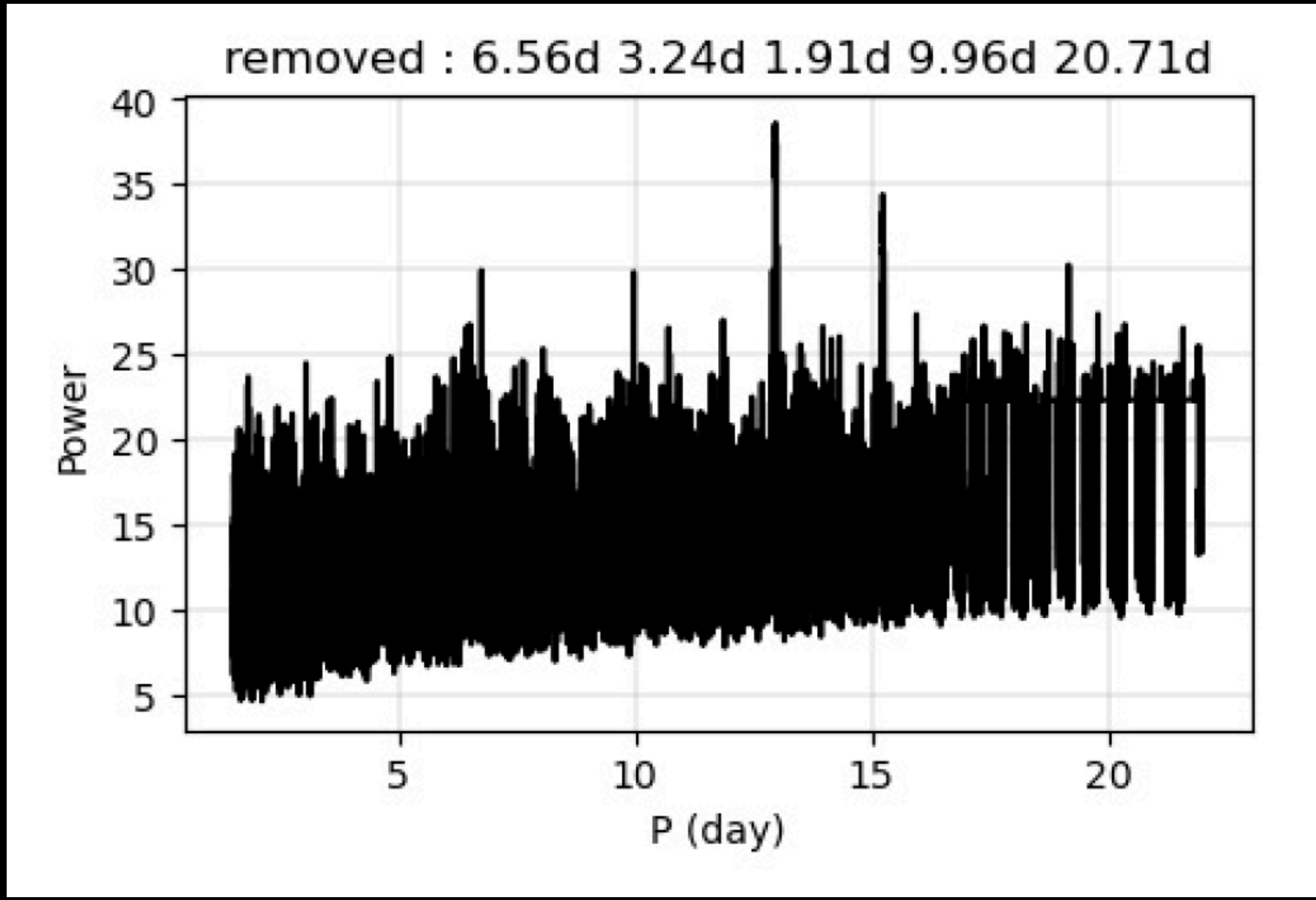
$$P_{Super,12} = \frac{1}{\Delta_1} = \frac{1}{k_1/P_1 - (k_1+1)/P_2}$$

Laplace resonances yield two possible periods:
P = 13.4527 d, or P = 15.2318 d

TOI-178 (Leleu+ 2021)

completion of a resonant chain: TOI-178

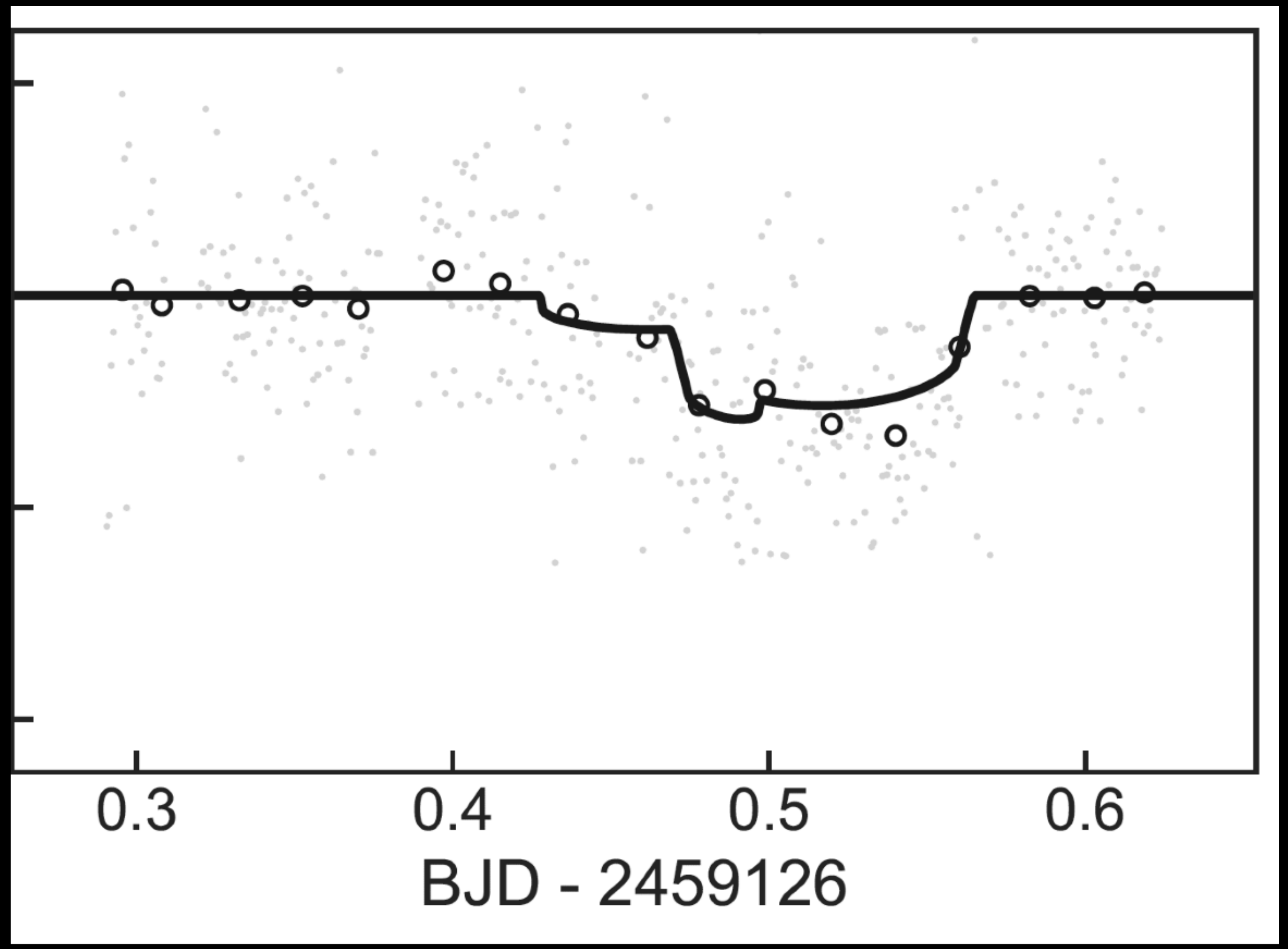


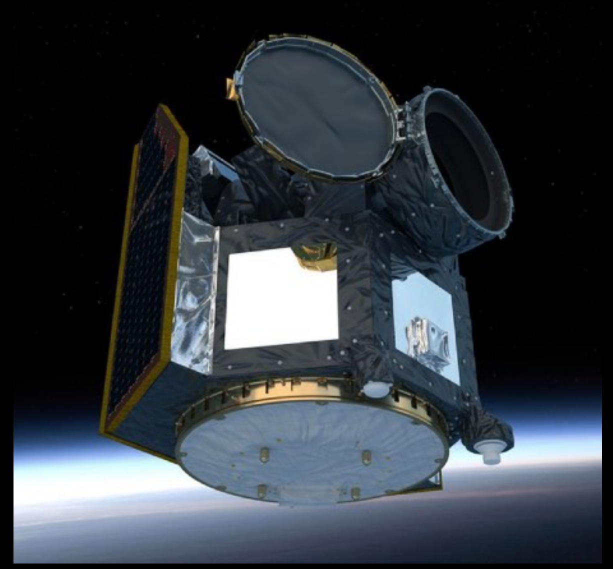


TOI-178 (Leleu+ 2021)

Laplace resonances yield two possible periods:
P = 13.4527 d, or P = 15.2318 d

BLS shows peaks at ~12.9d and ~15.2d
With predicted transits uncertainties of ~1 day
due to a 2 year error propagation

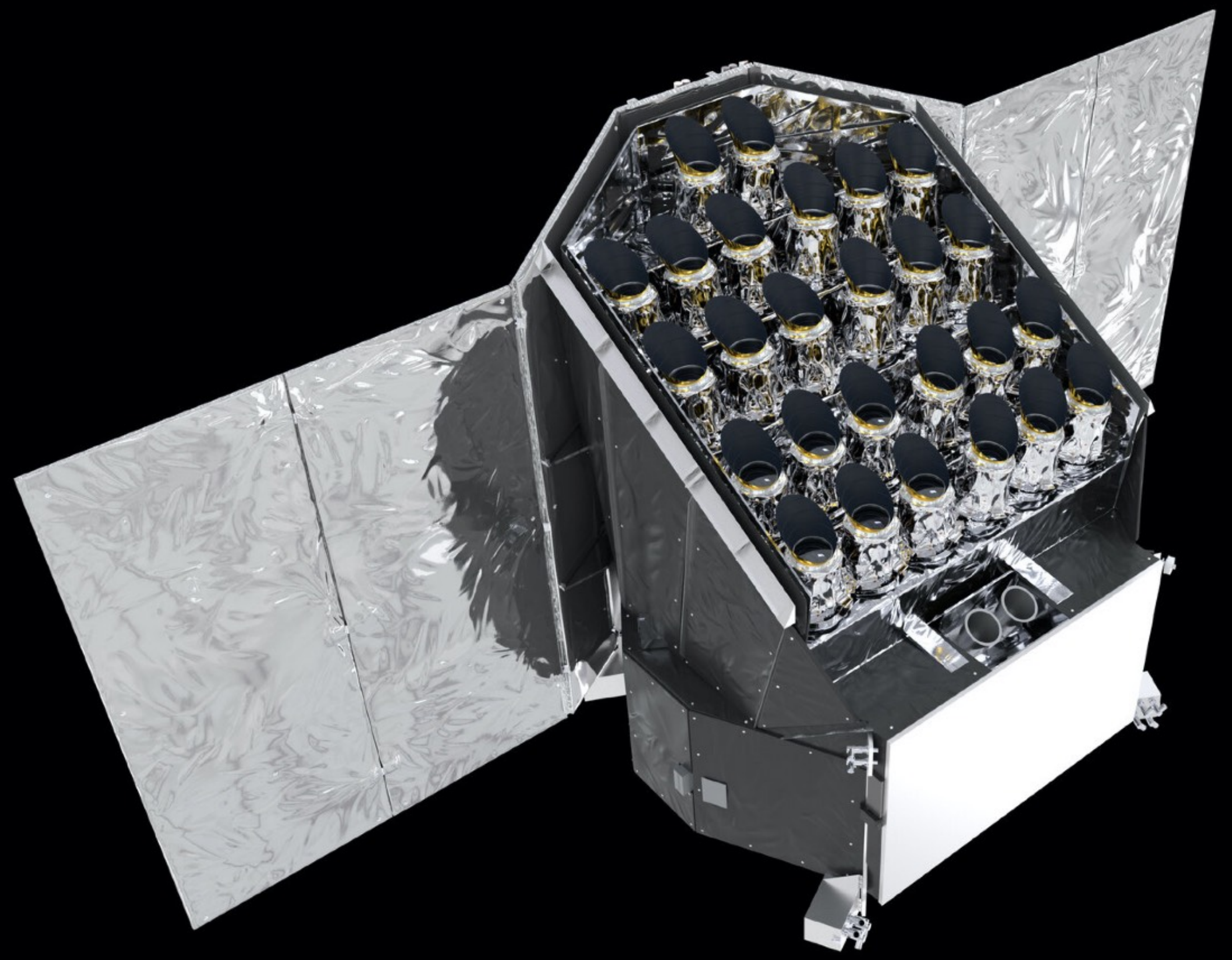




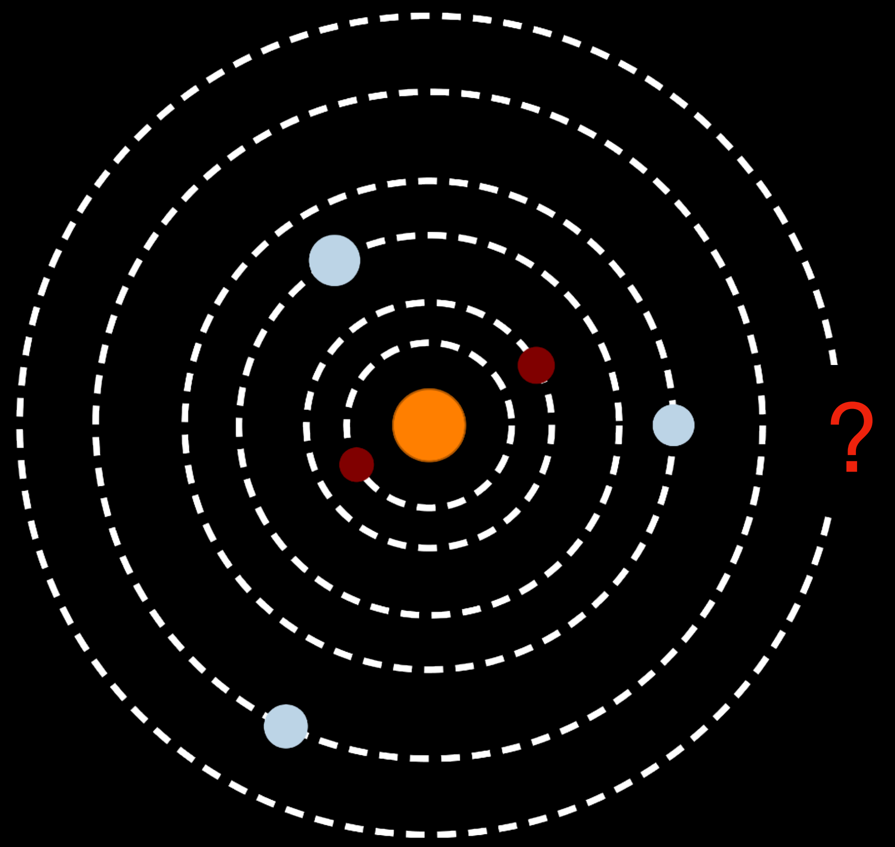
CHEOPS

Predicted period of 15.2318 d,
confirmed period of 15.231915d +/- 0.0001

Case 2.2



Completing a
resonant system



Planet continuing the chain,
single transit

Generalised Laplace Angle

mean longitude

$$\Psi = l\lambda_1 - (l+m)\lambda_2 + m\lambda_3$$

slowly evolving $\dot{\lambda} = 2\pi/P$

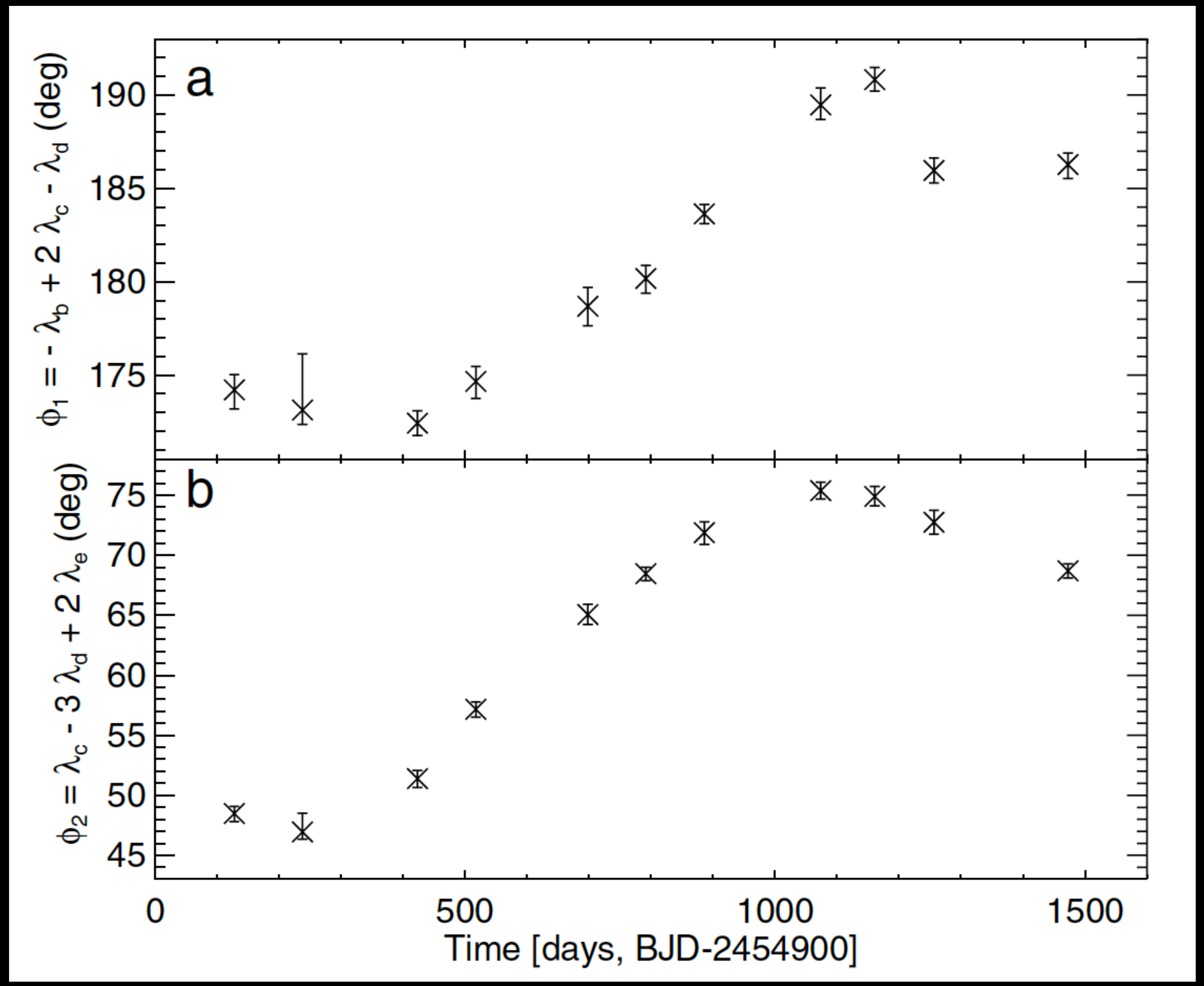
$$\dot{\Psi} = l/P_1 - (l+m)/P_2 + m/P_3 \approx 0$$

Laplace relation

$$\lambda = l - 2e\sin(l - \omega)$$

true longitude

Kepler-223 (Mills+ 2016)

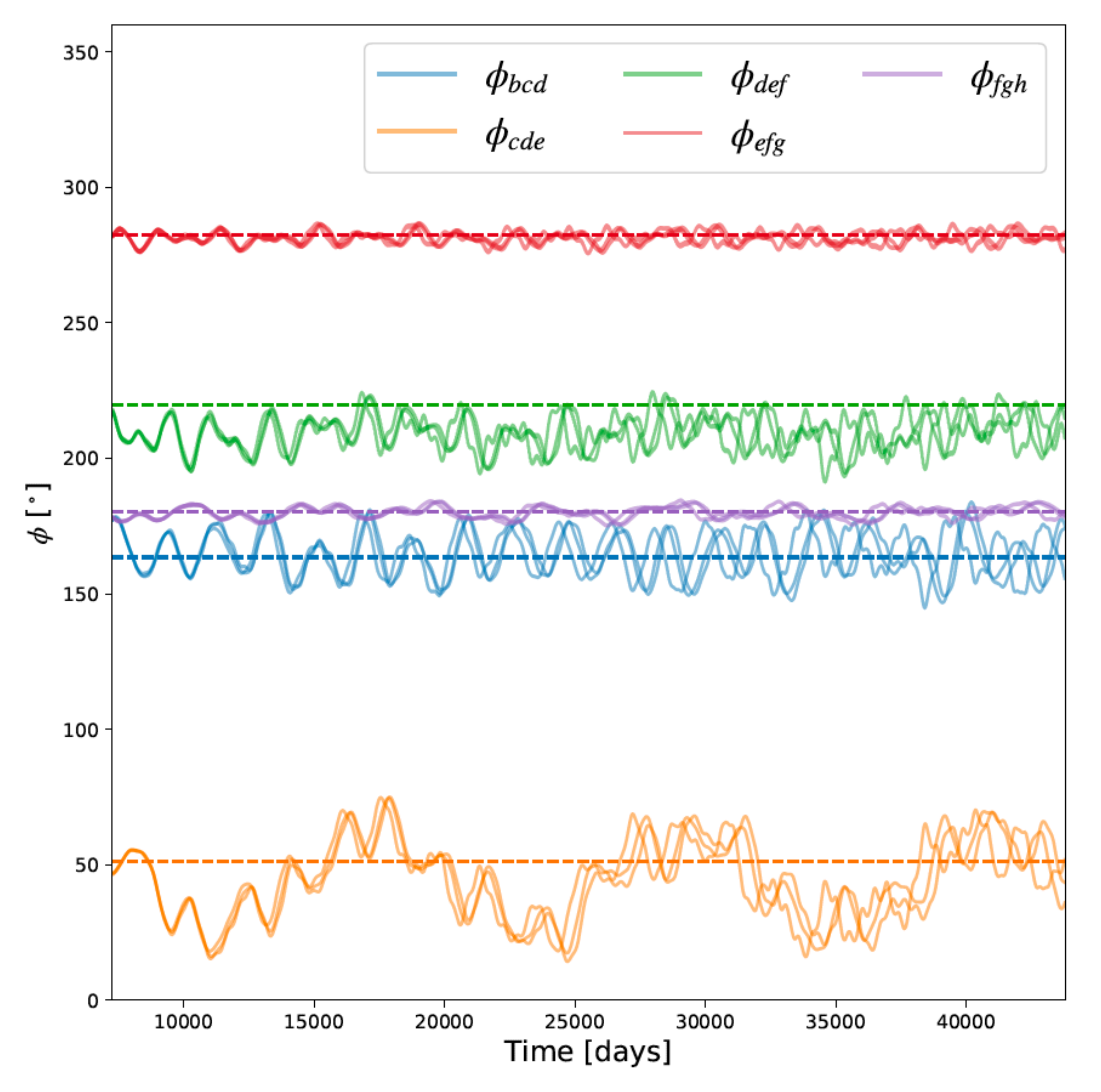


Assuming 0 eccentricities

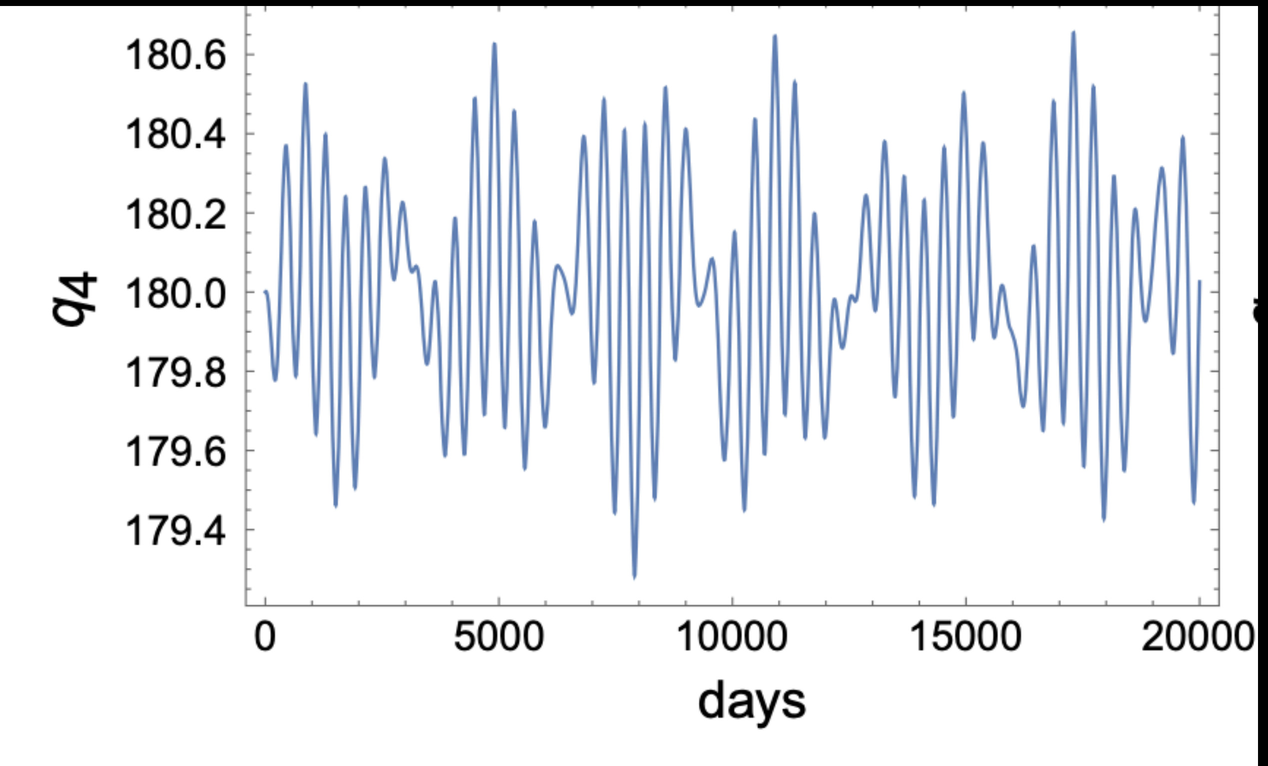
transit occurs when $l = \pi/2$

Assuming 0 eccentricities, Ψ can be computed from times of transit and periods.

Evolution of generalised Laplace angles

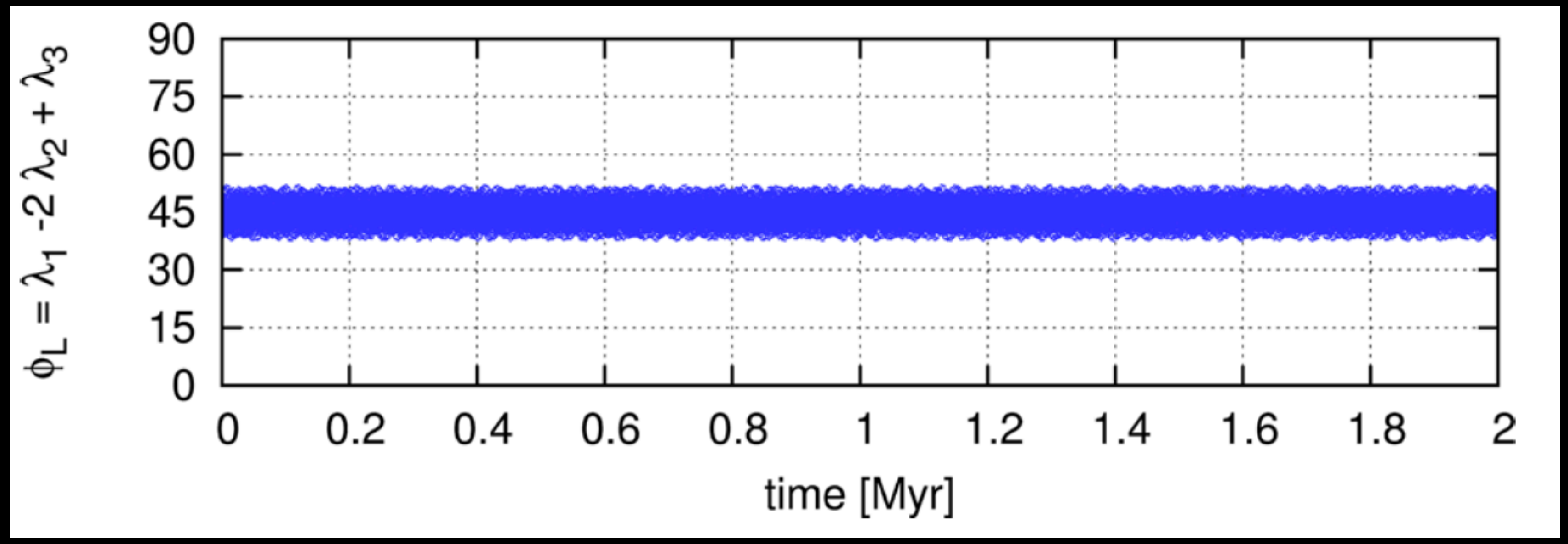


Trappist-1 (Agol 2021)

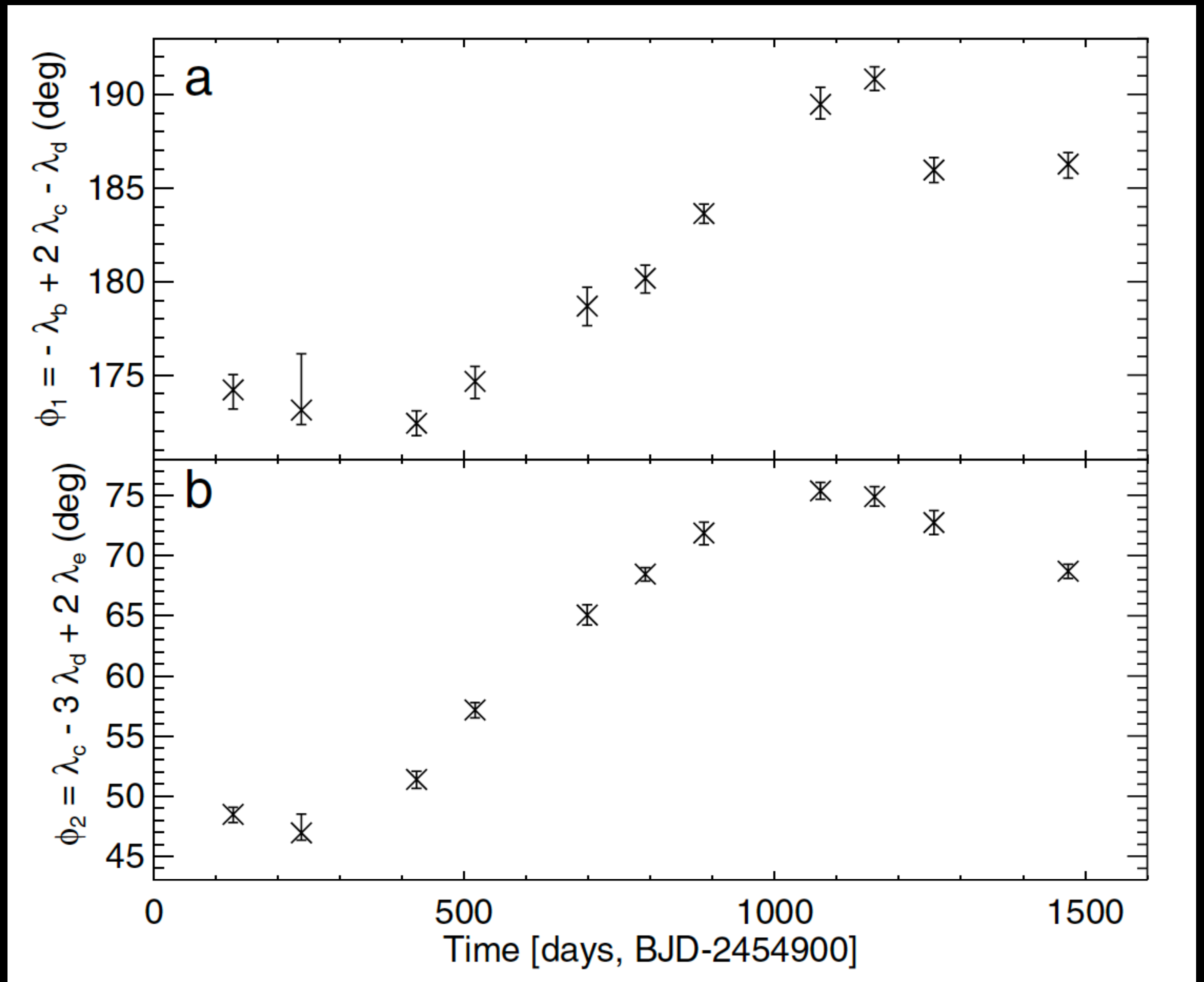


Gallilean statellites (Celletti+ 2018)

Typical oscillation semi-amplitude $\lesssim 20°$



Kepler-60 (Gozdziewski 2016)



Kepler- 223 (Mills+ 2016)

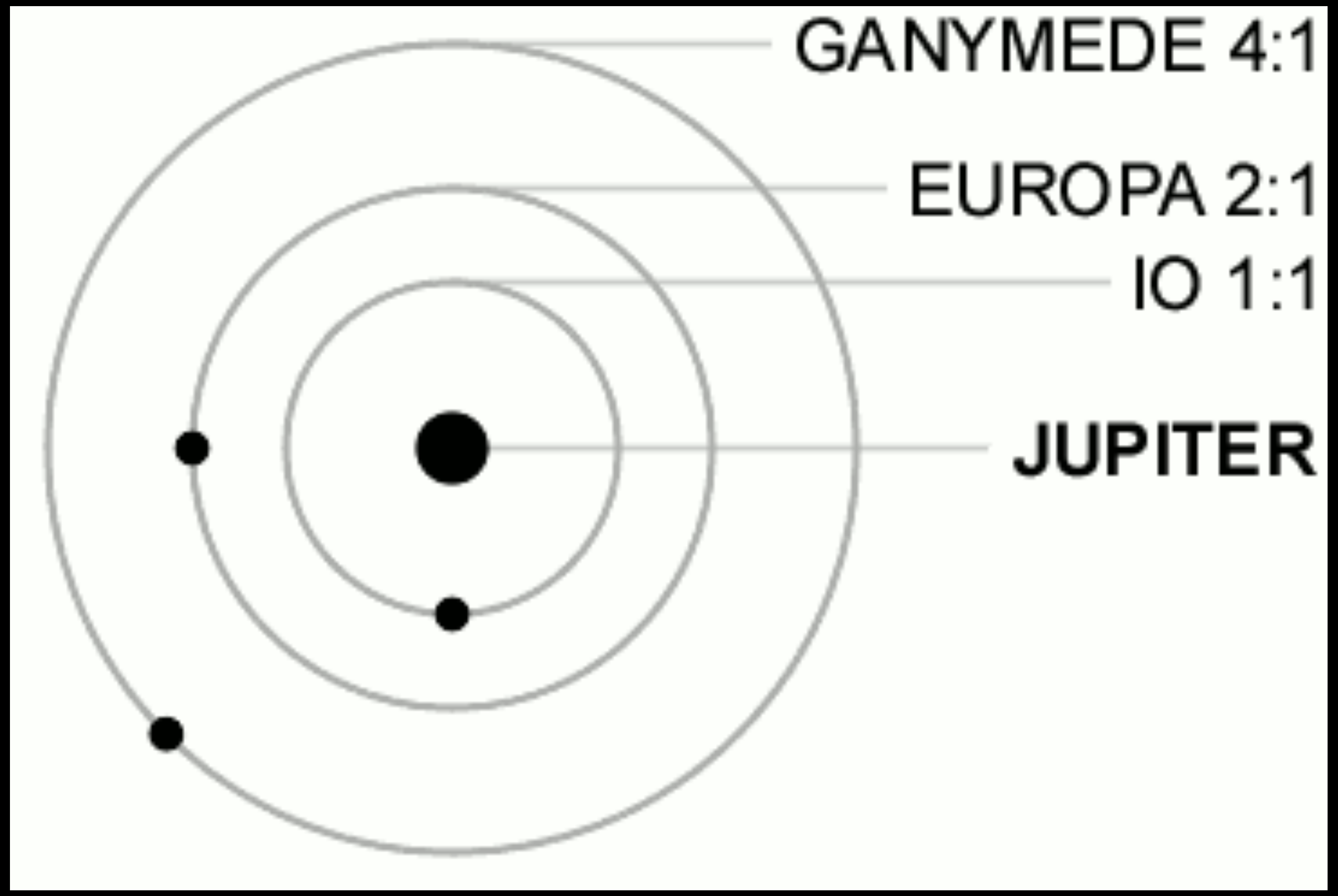
Laplace (Generalised) Resonant Angle

$$\Psi = l\lambda_1 - (l + m)\lambda_2 + m\lambda_3$$

In the case of a chain of two first-order MMRs

$$\Psi_{eq} = 180^\circ$$

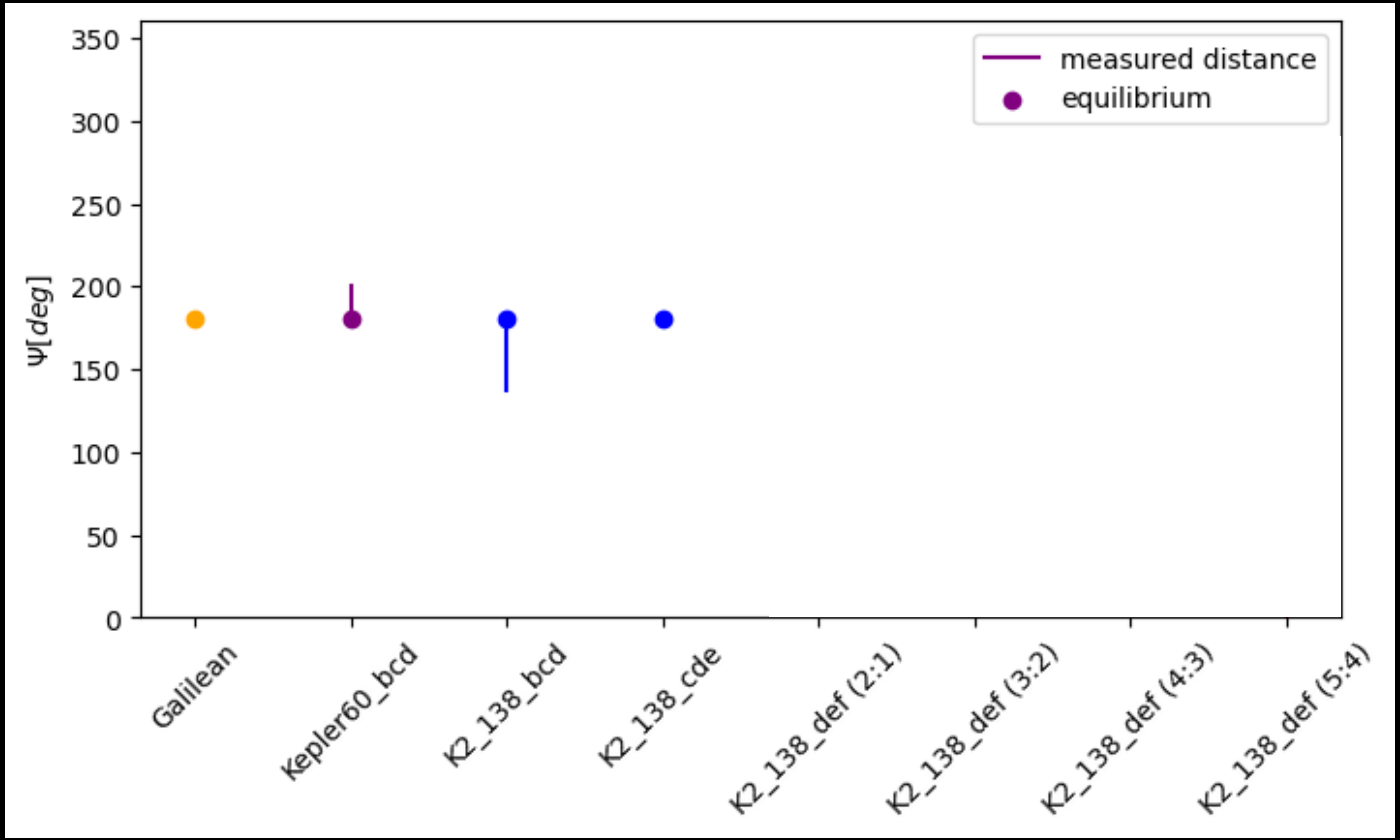
Sinclair (1975)



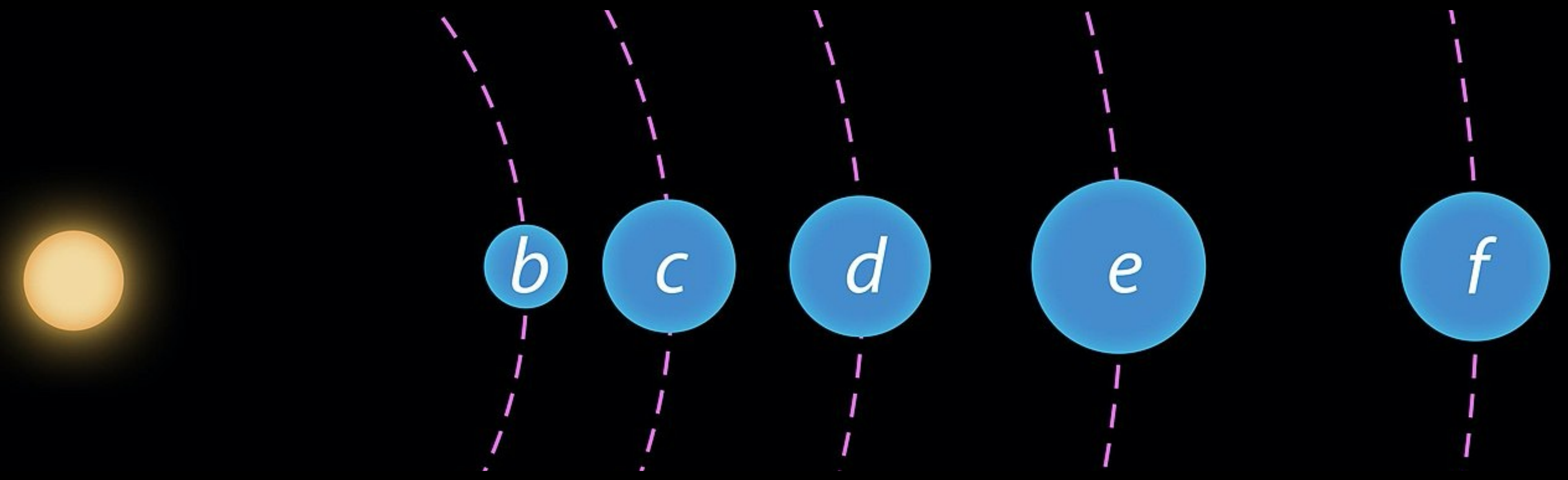
Distance to equilibrium

Typical oscillation semi-amplitude $\lesssim 20^\circ$

$$\Psi_{eq} = 180^\circ$$

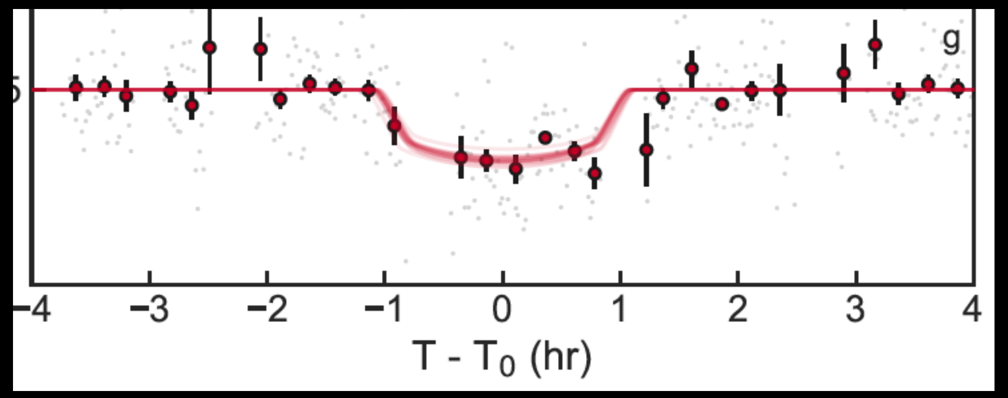


Where is K2-138f ?



$\frac{P_c}{P_b} \approx 3/2 \quad \approx 3/2 \quad \approx 3/2 \quad \approx ?$

Transit of f - single event



$$P_{Super,ef} = \frac{1}{k_1/P_e - (k_1+1)/P_f} = 145$$

$$P_{Super,bc} = 138d$$

$$P_{Super,cd} = 148d$$

$$P_{Super,de} = 145d$$

$$P_f/P_e \approx 2/1 \implies P_f = 15.88d$$

$$P_f/P_e \approx 3/2 \implies P_f = 12.77d$$

$$P_f/P_e \approx 4/3 \implies P_f = 11.23d$$

$$P_f/P_e \approx 5/4 \implies P_f = 10.48d$$

Where is K2-138?

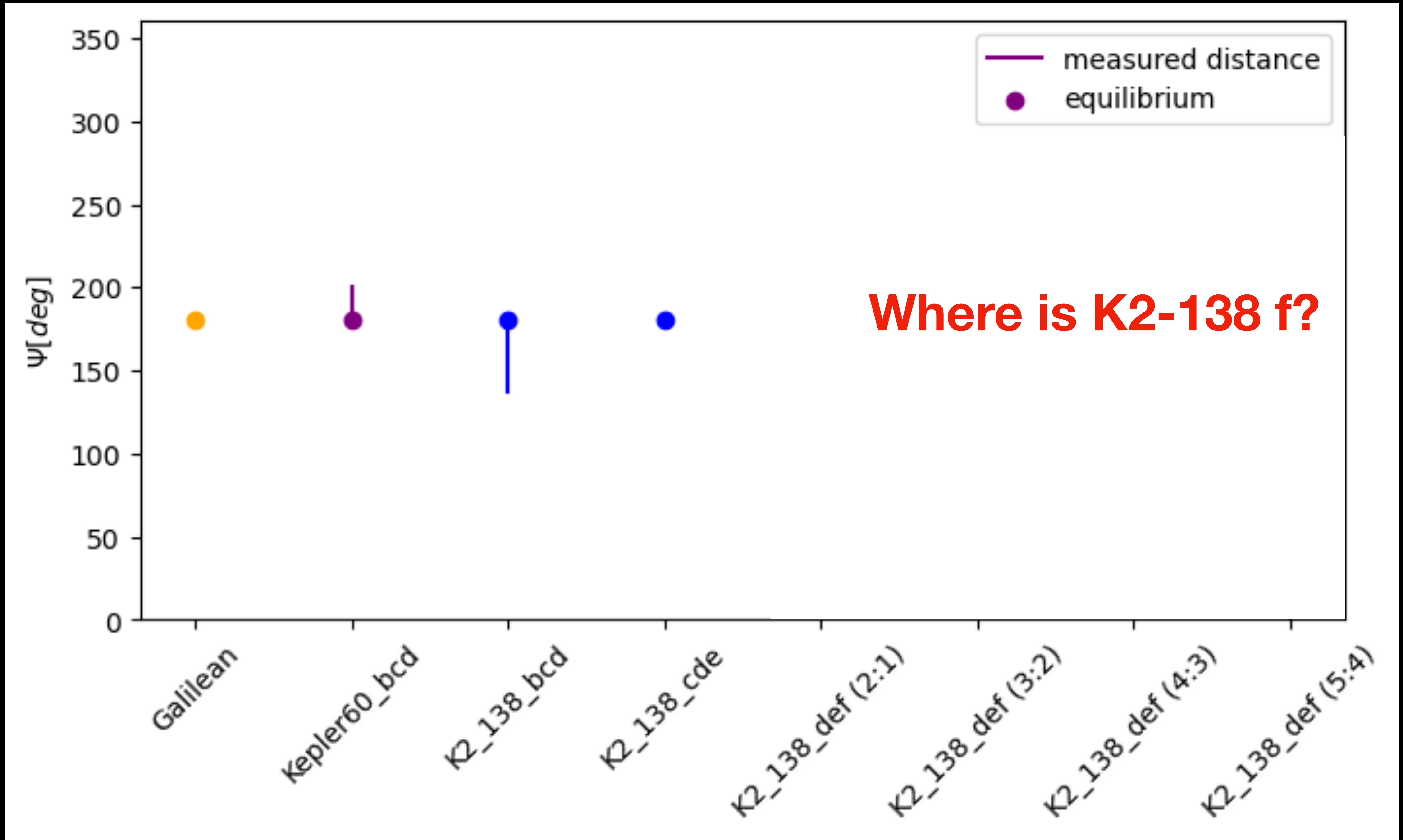
$$P_{Super,ef} = \frac{1}{k_1/P_e - (k_1+1)/P_f}$$

$$P_f/P_e \approx 2/1 \implies P_f = 15.88d$$

$$P_f/P_e \approx 3/2 \implies P_f = 12.77d$$

$$P_f/P_e \approx 4/3 \implies P_f = 11.23d$$

$$P_f/P_e \approx 5/4 \implies P_f = 10.48d$$



Typical oscillation semi-amplitude $\lesssim 20°$

$$\Psi_{eq} = 180°$$

Distance to equilibrium

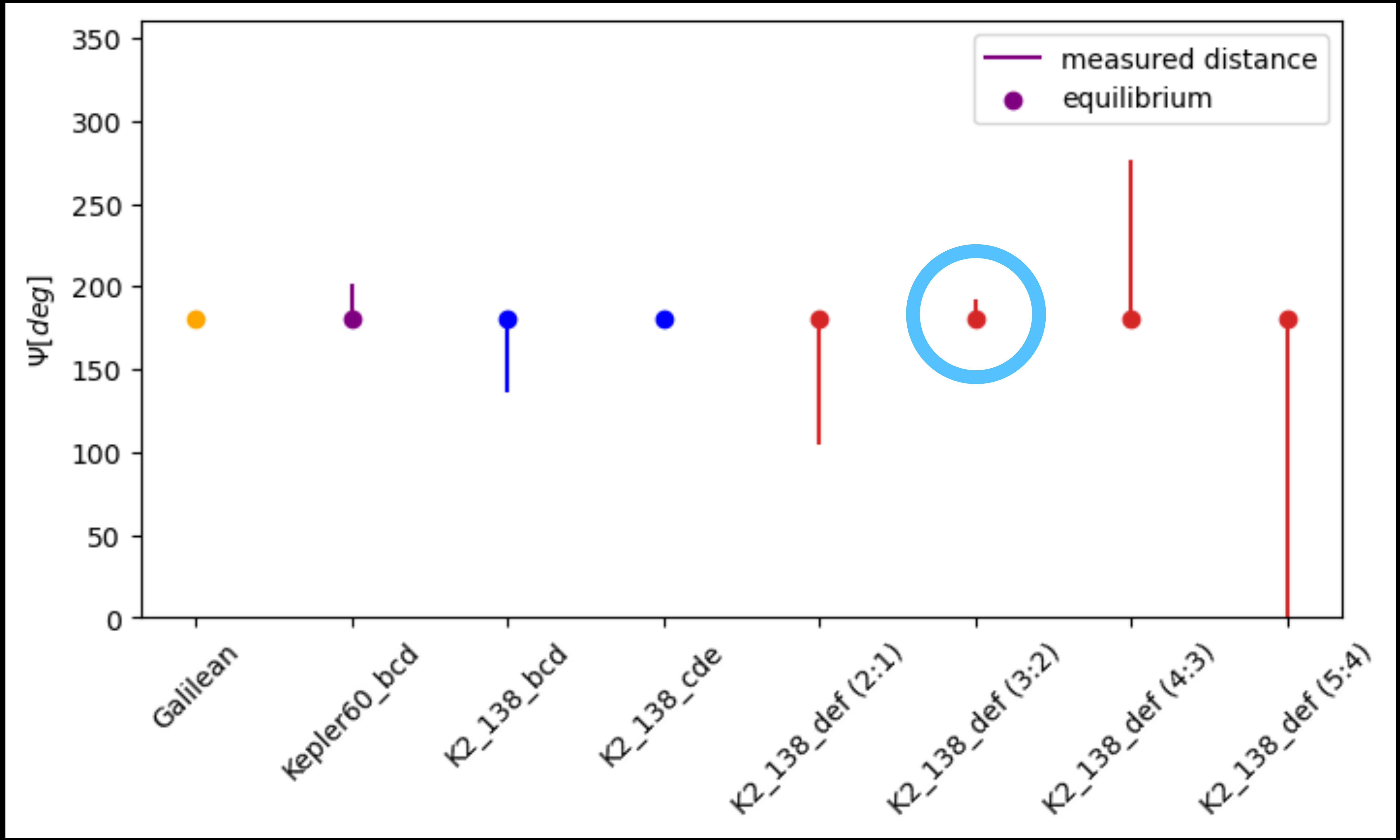
$$P_{Super,ef} = \frac{1}{k_1/P_e - (k_1+1)/P_f}$$

$$P_f/P_e \approx 2/1 \implies P_f = 15.88d$$

$$P_f/P_e \approx 3/2 \implies P_f = 12.77d$$

$$P_f/P_e \approx 4/3 \implies P_f = 11.23d$$

$$P_f/P_e \approx 5/4 \implies P_f = 10.48d$$

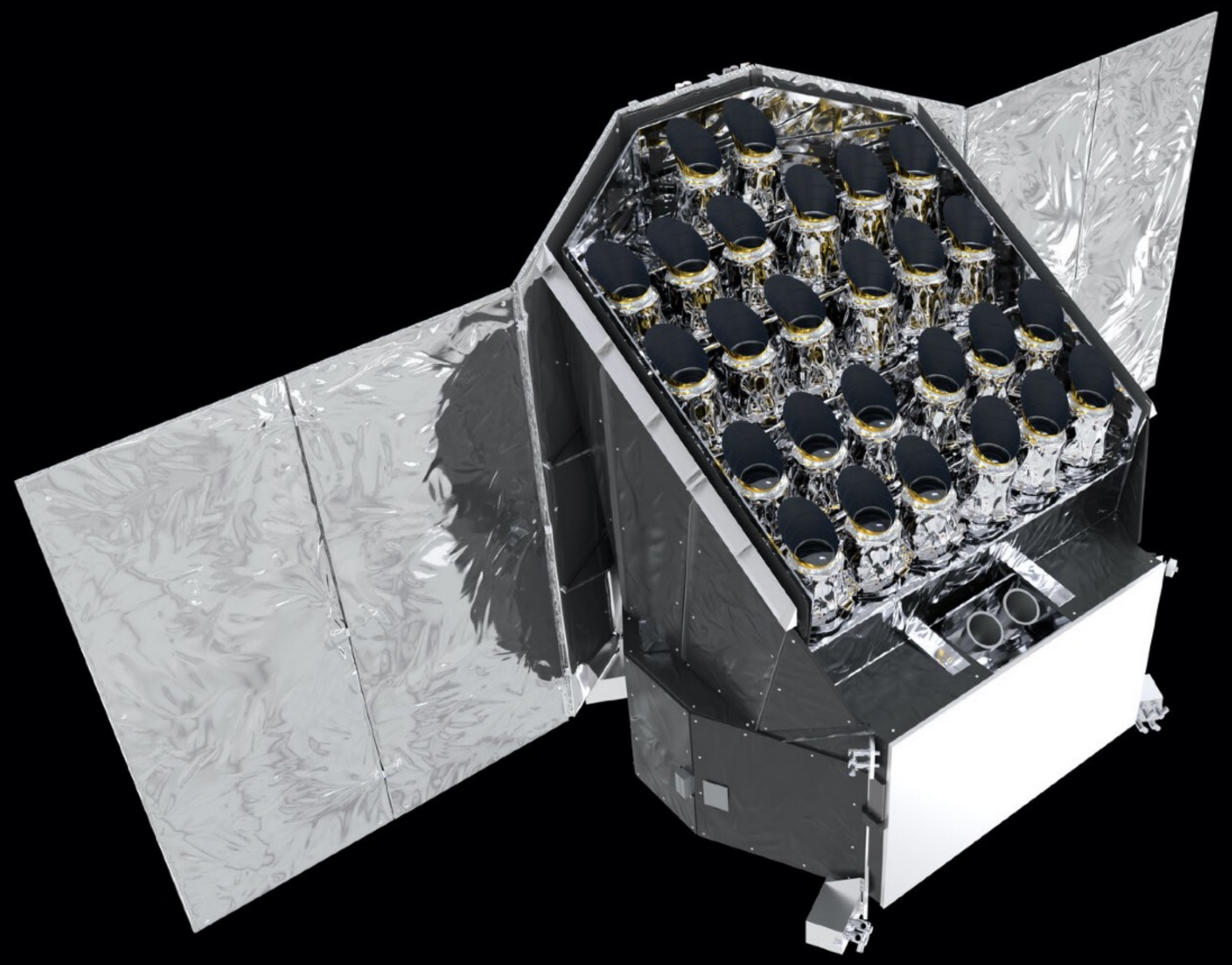


Typical oscillation semi-amplitude $\lesssim 20°$

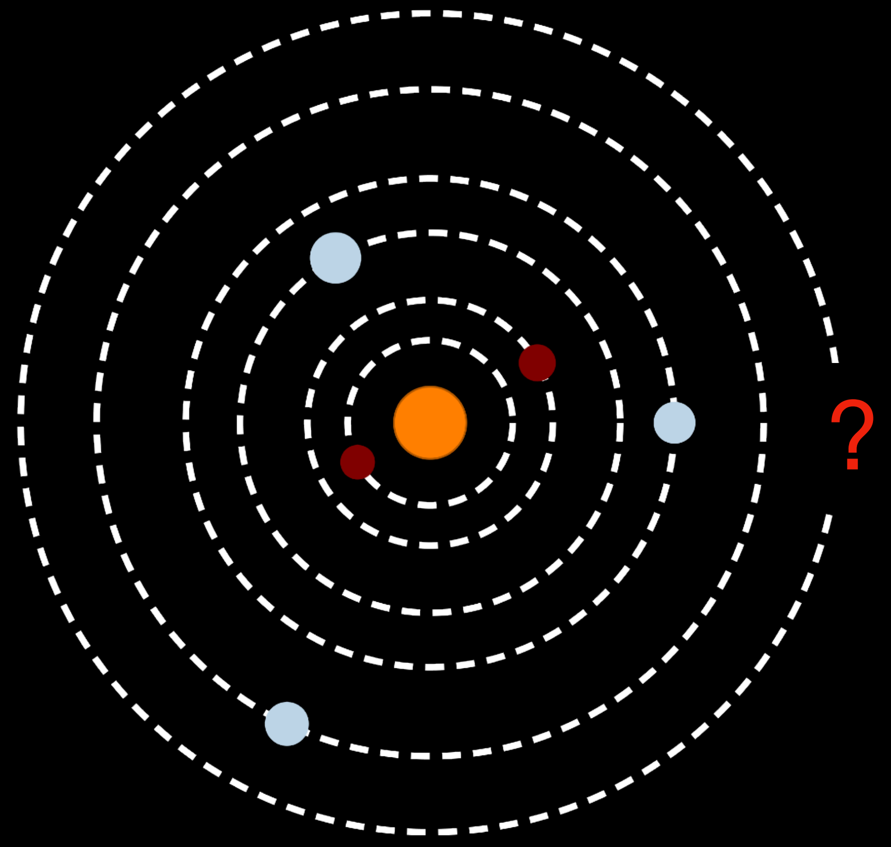
$$\Psi_{eq} = 180°$$

Case 2.3

Continuous observation /
Step and stare



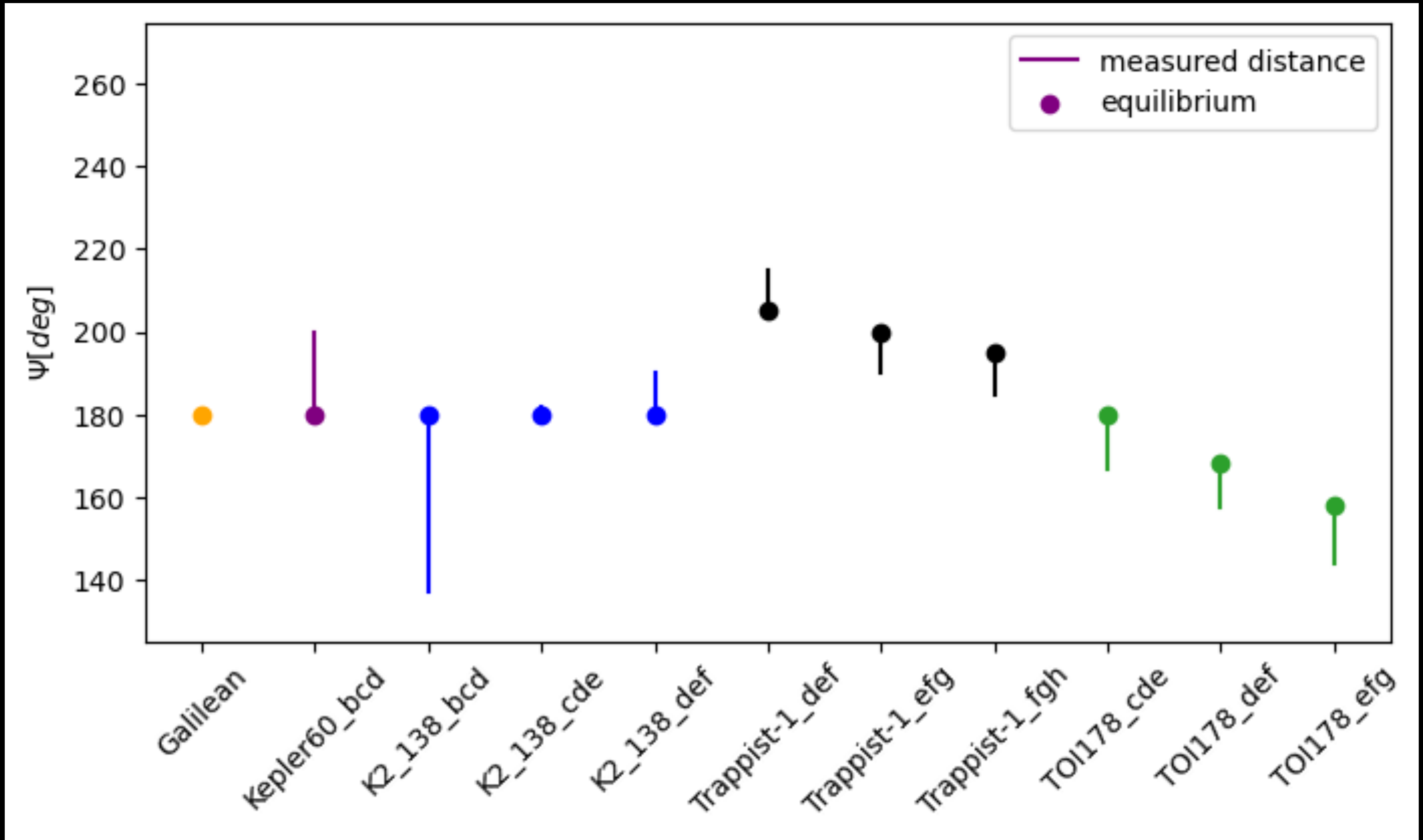
Completing a
resonant system



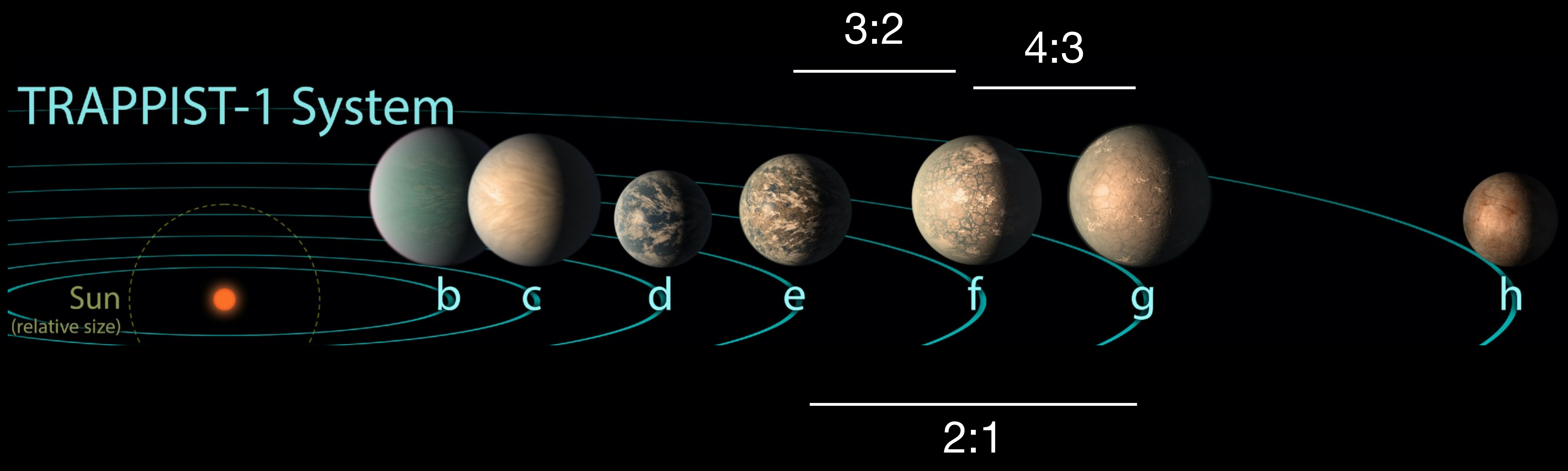
Planet continuing the chain,
no transit

Distance to equilibrium - general case

Typical oscillation semi-amplitude $\lesssim 20°$



Trappist-1

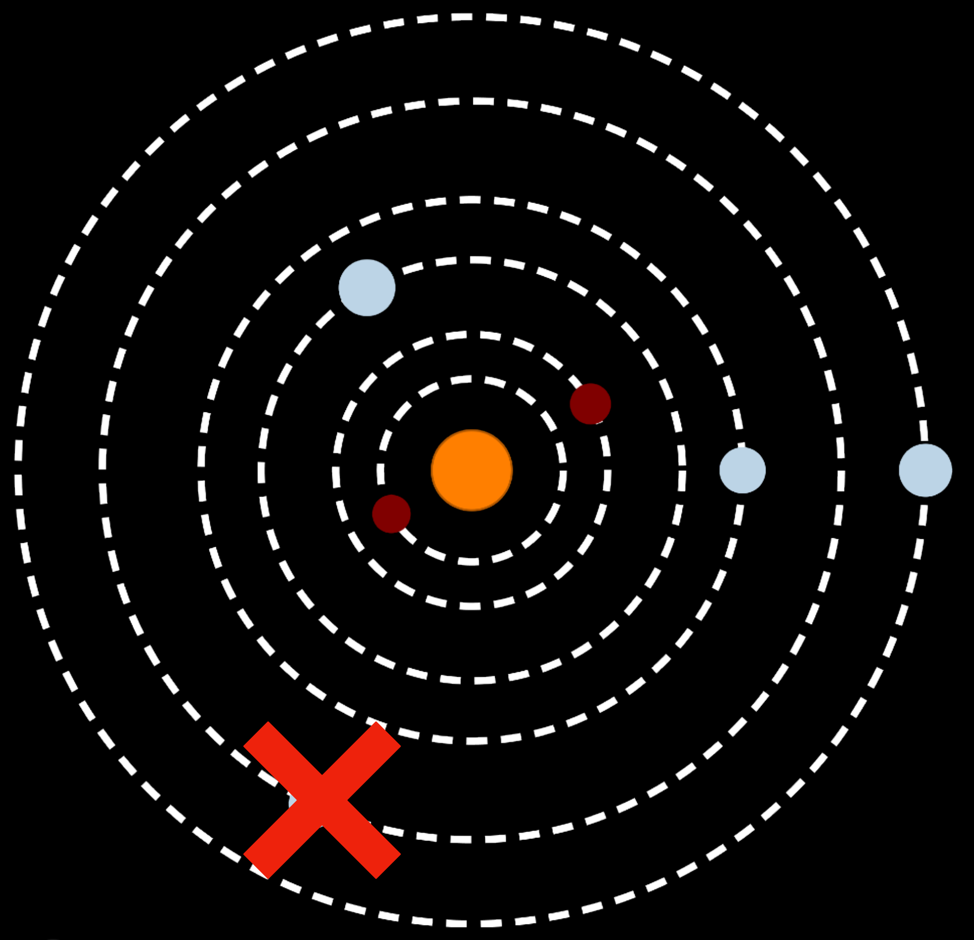


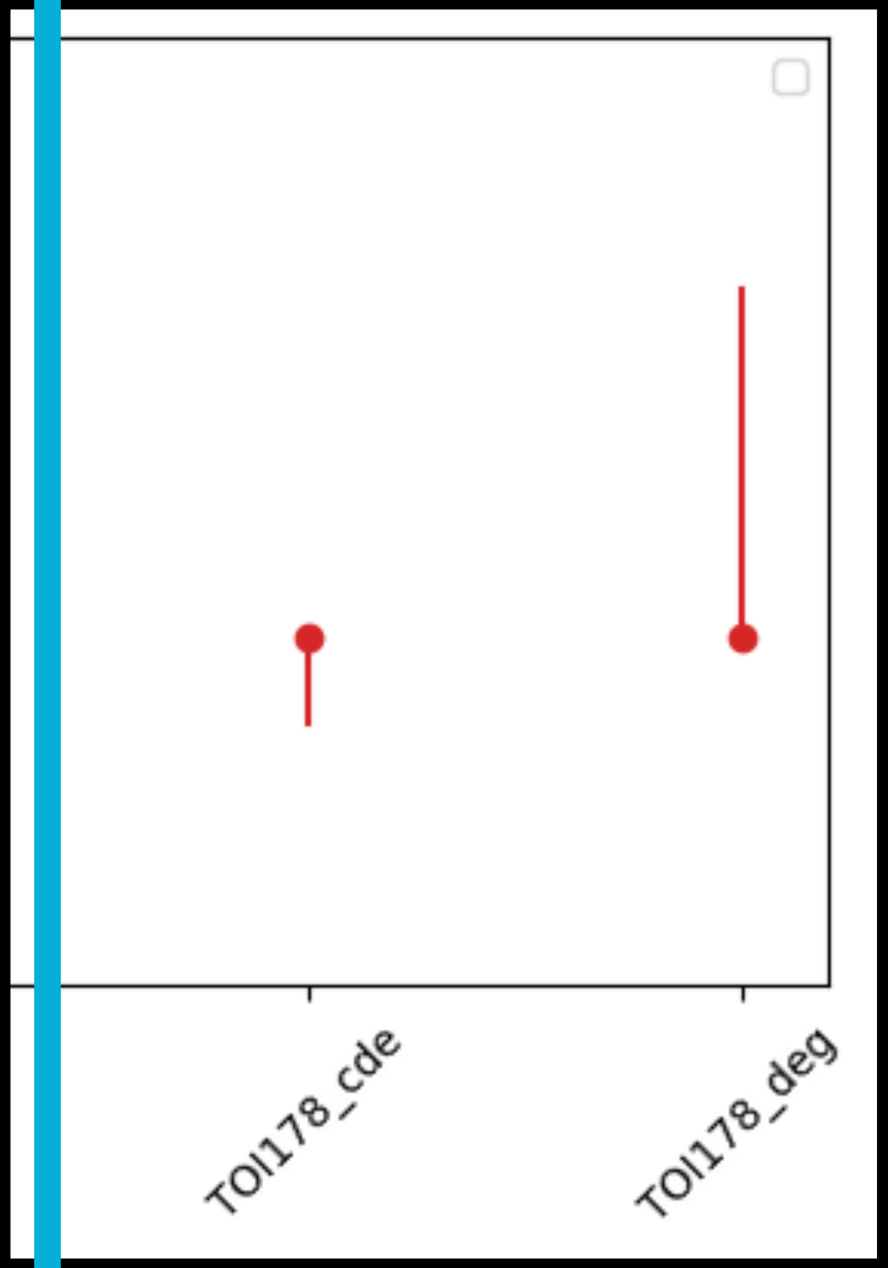
Trappist:
$P_f/P_e \approx 3/2,\ P_g/P_f \approx 4/3$
$\implies P_g/P_e \approx 2$

Computation of the fixed points Delisle (2017)

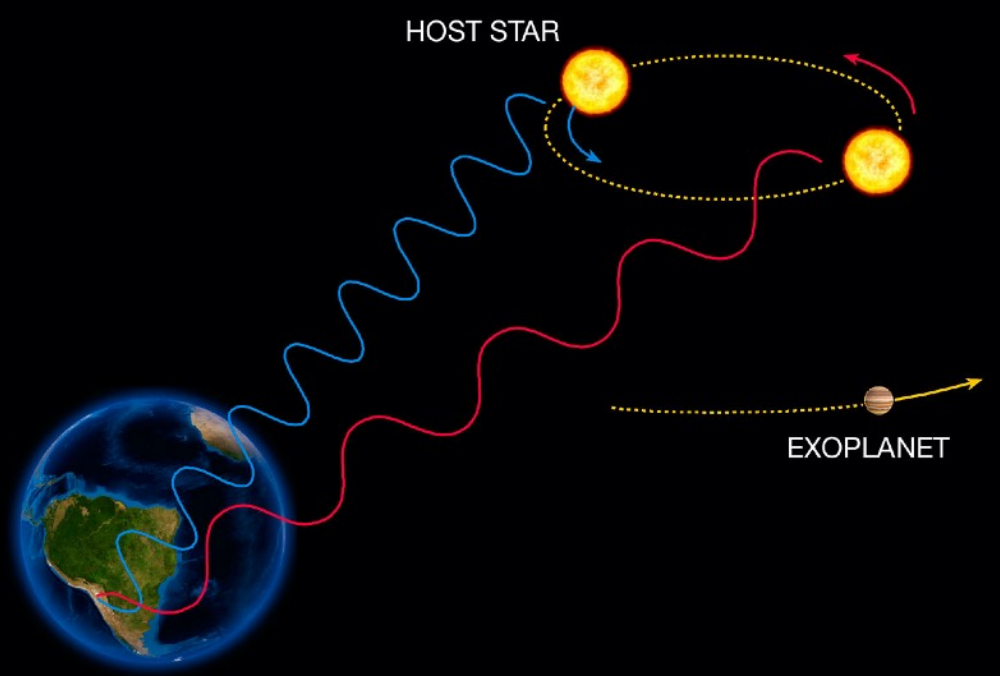
TOI-178 Missing planet

TOI-178 with 5 planets



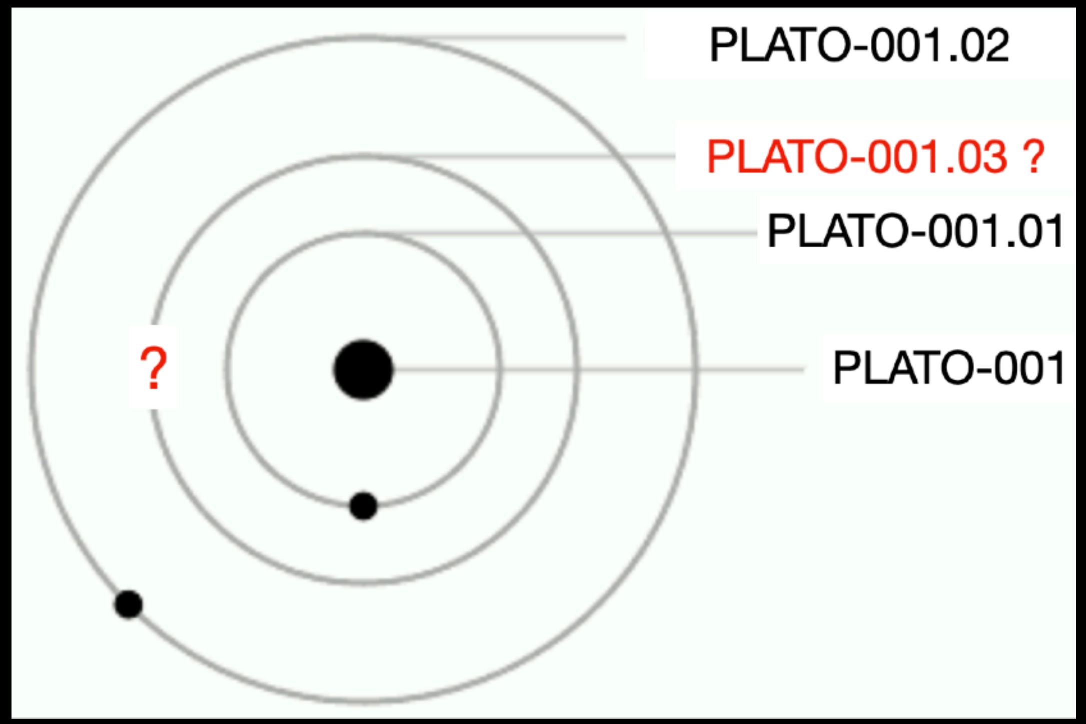


Radial velocity follow-up

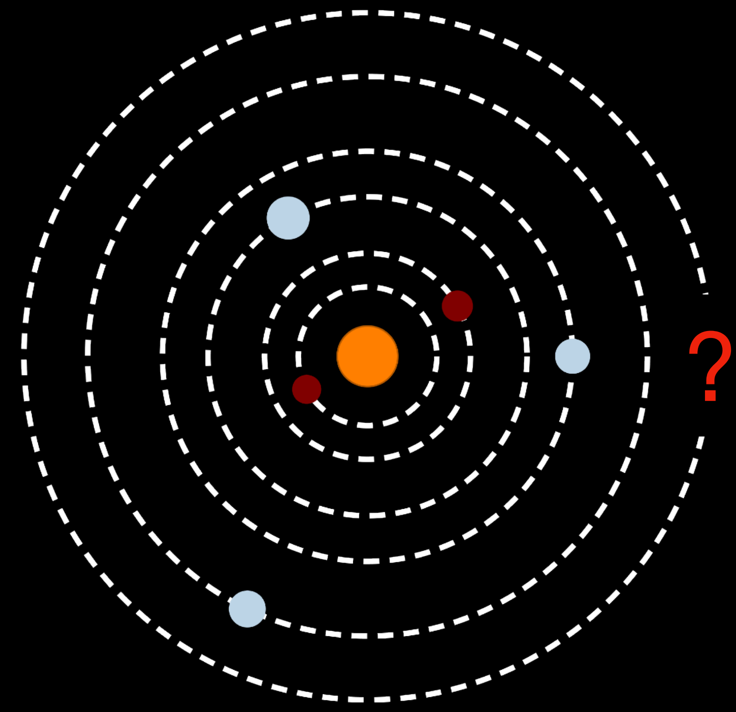


Conclusions

Atypical mean-motion resonances can hint at planets



In case of known chain, a single transit is enough to predict the period of an additional planet



The hint of a totally missed planet can be recognised by odd Laplace angles